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THE HORIZONS OF THOUGHT

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THE HORIZONS OF THOUGHT

A STUDY IN THE DUALITIES OF THINKING

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TO

AGNES HULBURD CONGER

PREFACE

This book, although it may be considered by itself, is designed as a companion (not a sequel) to the author's *World of Epitomizations,* which was published by the Princeton University Press in 1931. Each of the two studies involves the other; this is held to be the proper relationship between a metaphysics and an epistemology. When the world's epitomizations, or successive repetitions of significant structures and processes, are under scrutiny, the fact that perception and thinking are implicitly dual is among the epistemological presuppositions.[1] Among the structures and processes thus seen to be characteristic of the various levels and realms of the universe are some which at the level of thinking, or pattern-reactions, issue in the limitations and dualities which were formerly, for the sake of the argument, presupposed. In other words, we live in an epitomizing world, which we can study only from within the horizons of thought.

The foregoing statement can not be justified without a detailed analysis, and a detailed analysis, as in the case of the *Epitomizations* book, calls for careful and systematic work. The systematic arrangement of the new material is, again, indicated by rather elaborate subdivisions. Each subdivision in Chapters II, V, and VI, and after that in Division E of Chapters VII to XX of the *Horizons* book corresponds to something of the kind in *Epitomizations.* The correspondence is indicated, especially in Chapter VI, by retaining some of the decimal numbers used for reference in the former work, but in order that the new study may be considered independently, another system of lettering and numbering is introduced. This system, which is merely for purposes of reference and comparison, and which may be disregarded if it is not found helpful, is summarized on page 96.

This book, also, has been rather thoroughly documented in order to facilitate further study, and to indicate points of agreement and disagreement with other writers. The material cited is usually close to the words of the sources, although some of the symbols have been modified for the sake of approximate uniformity.

A preliminary account of the main thesis of the present work appeared under the title "The Implicit Duality of Thinking," as

[1] See *A World of Epitomizations,* pp. 2 *f.*

Appendix A of the text-book, *A Course in Philosophy,* in 1924. Portions of this account are incorporated here, by permission of the publishers. Harcourt, Brace & Co. The present study shows several important changes in terminology, especially as regards "denotation," and includes many new considerations. Much of the material of the first chapter was published in the *Monist,* Volume 43, 1933, pp. 73-87, and is used here with the permission of the Open Court Publishing Company.

This book discusses practically no problems of religion and ethics, inasmuch as these are reserved for further treatment in accordance with the principles which it has been sought to establish as regards the world's epitomizations and our horizons.

The author's acknowledgments are again due to students, colleagues, and friends, a number of whom have watched the work grow through a period of years. The members of the staff of the Princeton University Press have again been especially helpful.

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ARGUMENT

WHEN WE THINK OF THE UNIVERSE OR ANY OF ITS PARTS, OUR THINKING, WHATEVER MAY BE ITS RELATIONSHIP TO OUR PERCEPTION OR OUR LANGUAGE, PROCEEDS AS PERCEPTION DOES, BY SELECTION AND AT THE SAME TIME BY A CORRELATIVE NEGLECT. THIS DUALITY OF SELECTION AND NEGLECT IS GROUNDED NOT MERELY IN THE STRUCTURES AND PROCESSES OF OUR MINDS, BUT ALSO IN THE STRUCTURES AND PROCESSES OF THE UNIVERSE. ANYTHING WHICH AT A GIVEN MOMENT IS SELECTED IN OUR THINKING IS TREATED IN WAYS WHICH HAVE BEEN MADE FAMILIAR BY THE TERMS "DENOTATION" AND "CONNOTATION," BUT ANYTHING ELSE WHICH AT THE GIVEN MOMENT IS NEGLECTED IS PROPERLY TREATED ONLY IN WAYS WHICH WE SHALL DESIGNATE BY THE TERMS "ENOTATION" AND "INNOTATION." MANY DIFFICULTIES IN CLASSICAL AND CONTEMPORARY PHILOSOPHY ARE DUE TO MISTAKEN ATTEMPTS TO TREAT NEGLECTED, ENOTATIVE OR INNOTATIVE BACKGROUNDS, WHICH ARE BEYOND THE HORIZONS OF OUR THINKING, AS IF THEY WERE, OR IN ANY ULTIMATE SENSE COULD BE MADE, PARTS OF ITS SELECTED, DENOTATIVE OR CONNOTATIVE CONTENT. THE DISTINCTION THUS TRACED IN THE DATA OF PSYCHOLOGY BECOMES IMPORTANT IN WORK ON TRADITIONAL PROBLEMS OF LOGIC, MATHEMATICS, ONTOLOGY, COSMOLOGY, AND VALUES. THE EFFECT OF THE STUDY IS A SYSTEMATIC CRITICISM OF MANY EXTRAVAGANT STATEMENTS IN CURRENT THOUGHT, ALTHOUGH THE RESULTING RELATIVISM, BECAUSE OF ITS GROUNDING IN THE STRUCTURES AND PROCESSES OF THE UNIVERSE, HAS AMPLE ROOM FOR CONSTRUCTIVE AND COMPREHENSIVE VIEWS CONCERNING THE GREAT PROBLEMS, AND MAY EVEN APPROACH THEM WITH A MEASURE OF STRENGTH DERIVED FROM INTUITIVE KNOWLEDGE.

CHAPTER I

Epitomization and Epistemology

"It follows that for the empirical method the problem of knowledge, the subject-matter of epistemology, is nothing but a chapter, though an important one, in the wider science of metaphysics, and not its indispensable foundation."
—S. Alexander, *Space, Time, and Deity,* 1920, Vol. 1, p. 7.

1. *This book is an attempt at a detailed study of some of the processes of our minds in the universe, when both are described in accordance with the "hypothesis of epitomization."* In this hypothesis, which has been developed at length elsewhere,[1] it is argued that the universe is a vast system of systems which strikingly resemble one another in the details of their structures and processes. To these resemblances the term "epitomizations" is applied. Among the systems, or realms, are those of matter, life, and mind. The structures and processes of matter, the physical world (the cosmogonic realm), when taken level by level and characteristic by characteristic —*i.e.,* in their individuations, interactions, reproductions, and segregations—resemble and are epitomized in those of life (the biotic realm), and both matter and life resemble and are epitomized in mind (the neuropsychological realm). Below or beyond these existent realms there appear to be three other systems which, by the similarities of their structures and processes to those of the foregoing, are also identifiable as realms. These three are the realms of logic, number, and geometry-kinematics, which together are held to form a great subsistent substructure for the existent cosmos.

The six realms, taken in the order logic, number, geometry-kinematics, matter, life, and mind, are in transitive relationships of inclusion, or "container and contained," most easily visualized in a diagram of circles in this relationship, with mind represented by the inmost circle. The circles need not be concentric,[2] nor in any fixed ratio of area. The structures, or monads, of each level and realm selectively interact with monads of environing or containing

[1] See G. P. Conger, *A World of Epitomizations,* 1931—hereafter cited as *Epitomizations.*

[2] While it is simplest and for some purposes most suggestive to picture the successive realms in concentric circles (see *Epitomizations,* pp. 337, 339), such a diagram is not adequate for some of the relationships between levels.

prior levels and realms. As the successive epitomizations develop, the interactions become more and more intricate and mark a "cumulative coordination" in the universe, so that, for example, the living organisms are cumulative coordinations of the structures and processes in their environment, and minds (which, at least for the sake of the argument, are defined as "nervous systems at work") are cumulative coordinations of the structures and processes in their bodies *and* environments. Cumulative coordination makes differences in successive levels and realms, while epitomization makes underlying and essential resemblances.

Our minds, then, belong within the inmost and most highly coordinating of the six realms. The interactions of our minds with other realms containing them constitute at least the groundwork of our experience. But because of their predicament of occurring within the inmost of the realms, our minds apprehend the data of outer realms only as these data are refracted through realms which are intermediate in the series. Thus, our minds, first, apprehend the material world only intermediately, through our bodies (*i.e., via* our sense organs) ; second, apprehend geometry only through matter (*e.g.,* figures on blackboards or paper) *and* our bodies, with their sense organs; third, apprehend numbers only through geometry, etc. (*i.e.,* only through arrangements in columns, series, or temporal sequences), as these data are represented in material media and in our sense organs; and, finally, apprehend the data of logic—terms, propositions, classes, etc.—only through numbers (*i.e.,* as singulars, plurals, manifolds, etc.), as these data are represented in spatial or temporal arrangements, material media, and sensations. All these representations, or refractions, are in the psychological sense "conditioned" by language, which complicates the situation without essentially altering it.

Structures and processes of prior realms persist characteristically in later realms. It is for this reason that data of psychology, when taken analytically or reductively, can after a fashion be described in terms of physiology, and physiology in terms of chemistry and physics, and chemistry and physics in terms of mathematics and logic.

All this, in brief, describes the mind-world situation, with which our present study is concerned.

2. *The present study accepts, in principle, certain realistic theories of the relationships of mind and world.* Without arguing again the

questions which have been in debate between the epistemological idealists and the realists,[3] it may be said at once that neither of the two schools is able completely to refute the other. The differences between them are seldom thoroughgoing, and often reduce to differences of emphasis. The idealists emphasize the rôle played by our minds in constituting our world, while the realists emphasize the independence of the world from our minds which function selectively within it. The illustration is in some respects inadequate, but, for the idealists, mind may be compared to a Zeiss projector and its world to the planetarium sky studded with projected points of light in their marvellously synchronized motions. For the realists, on the other hand, mind is to be compared rather to an ordinary astronomical telescope, sweeping the heavens and revealing here and there stars or systems which, but for the telescope, would simply have gone on radiating without the added fact of being seen.

3. *The epitomization hypothesis, approaching problems of epistemology from the broader field of metaphysics, may offer some suggestions concerning the varieties of realism and the civil war which rages intermittently between them.* From the point of view of a realistic and empirical metaphysics, as Alexander says in the passage above quoted, epistemology is only a chapter. It deals with one set of structures and processes (albeit a distinguished set) among others. In all these structures, according to the epitomization hypothesis, the relationship of container and contained is very important. Especially as regards the existent realms, the cosmogonic contains the biotic and the biotic contains the neuropsychological. Now in spite of his mention of the five senses, Locke and after him the classical epistemologists treated the problem of mind and world much as if we with our minds *confronted* objects before us, especially in the visual field; the break between mind and world occurred, we might say, "right before our *eyes*." But the fuller account of the situation is that no one ever succeeds in separating his mind from the world, except in some illusory or fragmentary way. Pressure stimuli from floor or armchair or atmosphere, and, still more, kinaesthetic stimuli from the interior of the body, do not cease while the epistemologist seeks to disentangle the contributions of the mind from those of the world, for example, in a visual field. To phrase the problem as if he confronted the world is to take only

[3] On these questions, see G. P. Conger, *A Course in Philosophy*, 1924, Chap. 25. *cf.* Report of Committee on Definitions, American Philosophical Association, *Journal of Philosophy*, etc., 8, 1911, p. 703.

a sector of it; the whole problem is much more that of the relationships of mind to a world which contains it. And none of us walks up to his world; each of us wakes up in it, if he does anything at all with the problem of knowledge.

Between mind and the world which contains it there is constant interaction, *via* the body and its sense organs. The definition of mind as nervous system at work, and the fact that the body contains the mind as thus defined, appears at first sight to be a variant of the "hypodermic philosophy" which Lovejoy so roundly scores.[4] It should be noted, however, that the nervous system is *at work*; the process of interaction means essentially that to be contained in the body is not to be insulated from the world. The epistemologist has always engaged in the process of knowing long before he studies it, just as the physiologist has breathed before he studies respiration. The epistemologist, with respect to the process of knowing, may raise the question of the possibility of knowing just as the physiologist may hold his breath, but both procedures are forced, if not injurious. To put it another way, the question of the epistemologist to the world is not the challenge of a sentry halting the world at mind's outposts; it is more like the question of a householder asking a burglar in the dining room how he got in. Probably few householders ask such questions, but it is equally true that few knowers ask the questions of epistemology. What the epistemologist does is to study, describe, and modify processes of interaction which are essential to his investigation.

According to the epitomization hypothesis, or any other realistic metaphysics, the structures and processes of mind are only a few of the myriad structures and processes of the cosmos, and the bifurcations which are either favored or feared by various realists are only a few of the bifurcations which can be detected. In fact, if it be recalled that individuation is always relative, the structures found in other realms as well as those of mind and world may be said to exhibit bifurcations in great variety. Bifurcations seem to occur more readily, or in a different way, or at least to be more conspicuous, in structures than in processes. Of course structures and processes are correlative and inseparable, but there is a difference between them; somehow structures (and objects, too), whether in mind or world, most naturally bifurcate spatially, while processes most naturally bifurcate temporally. According to this view, time is

[4] A. O. Lovejoy, *The Revolt Against Dualism*, 1930, pp. 12 *f.*

not, as for Alexander,[5] the mind of space, but the processes of mind get a more adequate analysis in terms of time than in terms of space. Because the processes, with all their "warmth and facility,"[6] differ from the structures which, so to speak, mark their course, confusions arise when one tries to describe them. Bergson has long championed the view that real duration, inwardly apprehended as a flow of time, can not be adequately described in terms of space, where the units of measurement are external to one another.[7] Eddington regards it as very important that there are two ways of regarding time—the metrical or scientific way, and the inner way of intuition or mysticism.[8] Lovejoy seems to be concerned with the difference between structures and processes when he mentions, among the claims or pretensions which man makes in the attempt to know, the claim that he can somehow reach beyond the confines of his own transient being and bring external existences within the compass of his own life, yet without annulment of their transcendence. Again, the assumption is that the human animal does not for the most part live where its body is—if an organism's life is made up of what it really experiences; it lives where the things are of which it is aware, upon which its attention and feeling are directed.[9]

All these views seem to converge upon the fact that while the *structures* of mind and world may be called numerically or qualitatively two, the *processes* going on between mind and world can not be so neatly dissected into parts proceeding "in here" and other parts proceeding "out there." This means that if I am to conduct any comparison or contrast of different parts of the process, as is imperative for the problem of error, I ought to distinguish not between my mind on one side of a "break" and the world on the other side, but between my interaction with the world at one moment of time and my interaction with the world at another moment. The presence of other minds complicates this situation, without essentially modifying it; the essential distinction is not between mind and mind in space, but between interaction and interaction in time, no matter in what minds the interactions occur. By such comparison and contrast, errors, which are essentially maladjustments between structures belonging to different levels or realms,

[5] S. Alexander, *Space, Time, and Deity,* Vol. 2, p. 38.
[6] A. O. Lovejoy, *op. cit.,* p. 35.
[7] H. Bergson, *Time and Free Will,* transl. F. L. Pogson, 1910, pp. 90*f.*
[8] A. S. Eddington, *The Nature of the Physical World,* 1929, pp. 51, 100.
[9] A. O. Lovejoy, *op. cit.,* pp. 11 *ff.*

tend progressively to be minimized and in practice at least to be eliminated.

From the point of view of our hypothesis, then, it appears that, as the naïve realist and the neo-realist have maintained, the knowing process is overt and direct, and that it is an interaction which the critical realist is justified in calling more fundamental than inference. Moreover, as the neo-realists also maintain, there are subsistent realms, although, being composed of entities of logic, number, and geometry, they are not as colorful as some of the artists and aestheticians think, and they have more majestic functions than merely that of affording room for our errors. But our predicament in the inmost of these interacting realms, and the fact that anything which we get from the outer realms must be refracted through those which intervene, go to show that there's many a slip 'twixt the stimulus and the response, and, especially in view of the developments of language, make some form of epistemological dualism almost inevitable.

We said that because of our predicament, we know the realm of matter and the subsistent realms only as they are refracted, and not as they may be "in themselves." We know the entities of matter only in sensations or ideas based on sensations, and we know the entities of geometry, number, and logic only in certain appropriate material representations, as given in our sensations or ideas. We infer, however, and since we react to them with mental organizations more inclusive than idea-patterns, we probably also intuit that the other realms are objectively real; their levels and structures and processes, which so strikingly resemble those that we find in the neuropsychological, are too unperturbed and imposing to be regarded as our projections, constructions, or conveniences. Any attempt to discuss objectivity involves objectivity.

4. *Acceptance of the realistic theories in principle does not prevent a partial agreement with one form of idealism.* The varieties of epistemological idealism agree that mind constitutes the world, but there are at least three different senses in which the word "constitutes" may be understood. It may be taken, first, to refer to the stuff or elements of which something is made, as when we say that students and instructor constitute a university class. In this sense of the word, the idealist statement that mind constitutes the world amounts to Berkeley's mentalism. Or the word may be taken to mean "builds up," "sets up," or "puts together," in the sense in which the university administration constitutes the aforesaid class.

In this second sense of the word the idealist statement reflects Kant's doctrine of the constitutive activity of the mind. Or, again, the word may be used to mean "calls to order" and in the sense in which the instructor, when he enters the room where the students have already gathered, constitutes the class. In this sense of the word, the idealist statement amounts to the "speculative idealism" of Creighton.[10]

Now the idealism last named, in spite of some of the phraseology, makes a notable concession to realism.[11] There is, or perhaps was, a hulk of a world before the advent and distinctive work of mind. Mind calls that chaotic world to order, to be sure imparting to it everything which the idealist has learned to regard as important, but not putting the chaotic world there in the first place. And since, as we shall see, disorder can not be described except as a form of order, and since no precise line can be set between mind and world, there is only a difference of emphasis here by which to distinguish idealist and realist.

That there is a world prior to mind is one of the cardinal principles of the epitomization hypothesis; the neuropsychological realm is relatively late in development, in fact the sixth realm in the list. For the hypothesis, however, the world prior to mind is not chaotic. The structures and processes of the universe at those infra-mental levels can be described in orderly fashion, in fact in the same orderly fashion as that of the neuropsychological realm itself. But along with all this the neuropsychological realm exhibits the highest known degree of cumulative coordination and is also to a unique degree retroactive[12] upon the realms prior to it. In its recognition of these last two relationships between the realm of mind and other realms, the epitomization hypothesis at least establishes contact with speculative idealism. The connection may become even more apparent as applications of the hypothesis to problems of personal and social values are worked out.

5. *From the point of view of the epitomization hypothesis, pragmatism is a method mistakenly magnified into a metaphysics.* If we may call sensation, perception, and conception or ideation the cognitive functions of mind, then the pragmatic method of obtaining knowledge is essentially reliance upon the non-cognitive functions

[10] See J. E. Creighton, "Two Types of Idealism," in *Studies in Speculative Philosophy,* ed. H. R. Smart, 1925, pp. 257, 268.

[11] *cf. ibid.,* pp. 258 *f.,* 266, 280.

[12] For retroaction, see *Epitomizations,* p. 340.

such as emotion, volition, satisfaction, action, or control. It is no wonder that pragmatism is often vague and shifting: in its very nature it discounts those functions of mind by which clearness and consistency are most conspicuously achieved. Now, as has been pointed out many times, the so-called pragmatists have no monopoly of the pragmatic *method*; the method is the common property of everybody. The earlier pragmatists begin to take their distinctive position when they insist that once having obtained satisfaction (not always clearly defined), we shall, by a kind of forced baptism, call that result by the name of truth, and shall seek for truth nowhere else. Later on, pragmatic instrumentalism insists that cognition is essentially prospective rather than retrospective and tends not merely to reduce the environment to the measure of our control of it, but also to disregard the universe in the interest of the environment. Thus pragmatism becomes a restricted metaphysics, and in its emphasis upon certain functions of mind affords a non-cognitive idealism.

In its beginnings, pragmatism was justified by the extravagances of the older idealisms. Its more recent gospel of control is best authenticated, where perhaps it originated, on university campuses. On university campuses, even if philosophers can not be kings, pragmatists can be administrators and formulators of educational policies. Granting of course a certain measure of freedom from tradition, any sorry scheme of things can be smashed to bits overnight and moulded nearer to next morning's heart's desire. But these utopian conditions certainly begin to be modified in the off-campus fields of the social sciences, economics, and politics, where reality, although it sometimes yields to treatment, is assuredly not as plastic as among the sophomores and freshmen. In industry the measure of our control is impressive as regards products, but it is not impressive as regards cosmic principles. We sometimes reform the appearance or the distribution of things, but only because we conform to their characteristic structures and processes. In the natural sciences, our control begins still more to dwindle. We can, for example, produce certain chemical compounds, but only by paying heed to the fixed relationships of atomic weights or numbers. All this is obvious, the moment our attention is called to it; our attention needs to be called to it, and to many a feature of the structure of the world, because the pragmatists so often obscure it by placing their emphasis elsewhere. To have our attention directed upon ends and the issues of future activities is as laudable and as natural as walking. Walking, too, is prospective, and it is

achieved by continually pitching forward. But in the attainment of pragmatic ends we do not achieve anything by disregarding the structures around us, nor even those antecedent to our activities, any more than we could say that we were walking if at each step we picked up the ground under our feet and threw it forward as if it were to be stepped on again in some new form.

The weakness of pragmatism, when pragmatism is magnified into a metaphysics, appears nowhere so clearly as in the pragmatic treatment of astronomy. Dewey says that of course we can not introduce variation into remote heavenly bodies (one wonders what has happened to prevent us in those which are nearer!), but that we can deliberately alter the conditions under which we observe them, which is the same thing in principle of logical procedure.[13] Thus do we loose the bands of Orion, and thus is the stupendous universe around us reduced to logical procedure!

It is clear when we stop to think of it that pragmatic control, while sometimes very real and important, operates on a sliding scale, especially as we pass from campus to Canopus. And the necessity for this sliding scale weakens pragmatism, taken as a criterion of reality, and to this extent weakens it as a general metaphysical principle. In proportion as any modern philosophy becomes cosmic in its scope, it will seem less and less likely that the structures of nature are set up or that the laws of nature are imposed by any variant of the practical reason. And in particular, if the hypothesis of the subsistence of realms of logic, number, and geometry-kinematics can be sustained, it will seem more and more inadequate to regard such entities or processes as merely the results of our thinking, counting, and measuring.

6. *The theory of "epitomizing interaction" accommodates features of each of the rival classical theories of the relationships of mind and body.*[14] When mind is defined as nervous system at work and the neuropsychological is regarded as a new realm contained within the biotic, mind, as thus defined, differs from body not by being animistic, but by belonging to a realm later than the biotic, and exhibiting the different degree of cumulative coordination characteristic of a later realm. Interaction between the monads of different realms is precisely to be expected; no such monad is even an individual unless it is an interactor, exhibiting processes of appro-

[13] J. Dewey, *The Quest for Certainty,* 1930, p. 82.
[14] See G. P. Conger, *Course in Philosophy,* Chap. 26.

priation, rejection, etc., as regards other monads and realms. Thus the classical interactionism of mind and body, when mind is divested of its animistic aura, appears as only a special case of the widespread interactions between monads and realms.

Now the neuropsychological realm, according to our hypothesis, not only interacts with the biotic, but epitomizes it, and, level for level and process for process, can be set in parallelism with it. This, again, is not the classical parallelism, according to which the processes of mind and brain run along concomitantly without interacting. But, like the classical parallelism, this view ascribes to mind a certain distinctiveness (the distinctiveness of a new realm), as well as at least a kind of concomitance (the occurrence in two different realms of processes, for example, ingestion of food and reception of stimuli, illustrating the same monadic characteristic of appropriation).

With the epiphenomenalist and the later emergent views our hypothesis has much in common, although the fact that the later realms so consistently epitomize the earlier tells against any haphazard or random occurrence of mind in the cosmos. Mind evolves in the course of an orderly process, the major levels and characteristics of which can be described. However "jumpy"[15] the universe may be, mind is not altogether eccentric in it, but tends at least to be not merely apical but focal.

7. *According to the hypothesis, we are free to take a matter of fact view of the presence and operation of other minds.* There is no denying that if any one wishes to insist upon it he can reduce the whole world to a solipsism of the present moment;[16] but if any one insists upon it, he can practise other fantastic forms of immolation, too. The healthier view shares the world with others. It is a commonplace of recent thought that minds develop in social groups, under the influence of mutual interactions. Among these interactions, language is important enough to be studied in detail, as in Chapter III. No collective mind or will of a higher order or level than that of the individual is necessary; at every level there are a number of cognate monads effecting interchanges with one another, and minds at their level (*our* level) are no exception to the rule. Only an over-epistemologized metaphysics, or an over-individualized epistemology, needs to announce the arrival of other minds on

[15] See C. Lloyd Morgan, "Emergent Evolution," *Mind, N.S.*, 34, 1925, p. 71.
[16] See B. Russell, *Philosophy*, 1927, p. 291.

the scene where an argument is taking place. And it is a matter of indifference whether the study of the processes of mind in the world is phrased in the singular or in the plural, or considered on the individual or the social scale, as long as eventually provision is made for the inclusion of the one along with the other.

8. *In the epitomization hypothesis, several types of duality are discerned in the characteristics of the monads at all levels. Some of these dualities are explicit, with both members prominent; some are implicit only, with one member prominent and one left obscure. The implicit dualities discerned in the patterns involved in thinking are the principal object of study in this book.* They may be listed as follows:

(I)[17] There is, first, in general, the duality of a monad individuated relatively (00.111) to its background, or milieu; examples are a star in space, an organism in its environment, and a pattern in its organism. The milieu of a given monad usually includes (00.113) other monads cognate—*i.e.*, of the same level—with the given monad. Among all the monads of a given level there usually appears some (00.1131) dyadic relationship, interpretable as a priority, or difference of rate, or of inclusiveness; examples are the proton and electron (or perhaps the more recently discussed positron and negatron), the organisms of slower and those of more rapid metabolism, and the tonic and clonic reflexes or patterns.

(II) Sometimes dualities are discernible in the fact that monads of a given level are (00.133) of different degrees of complexity.

(III) Another implicit duality appears in the fact that (00.20) the interactions of monads are selective; each monad appropriates certain prior or cognate monads, but by the very fact of selection rejects others.

(IV) A monad in its interactions with cognate monads may (00.231391) preempt something from another monad, or (00-.231392) enter into conflict with it, or (00.231394) exchange constituents, etc., with it.

(V) It is a point sometimes of importance that (00.2543) certain peripheral or boundary conditions appear where a monad is in contact with its milieu.

[17] The roman numerals correspond to those used in Chaps. II, V, VI, and Division E of Chaps. VII-XX of this book. The arabic decimal numbers correspond to those in *Epitomizations,* pp. 7 *ff.,* and *passim.*

(VI) The process of (00.27) growth, especially growth (00.271) toward or to a limit, also involves a duality, since the growth of a monad is, after all, an adjustment to its milieu.

(VII) There are various dualities in (00.33) biparental reproduction and (00.3324) processes subordinate to it, at the levels where these occur.

(VIII) The extremely important process of integration often (00.431) occurs between monads in the dyadic relationship, as with protons (or positrons) and electrons in atoms, or tonic and clonic impulses in complexes.

(IX) Finally (00.44) differentiation within such integrates involves dualities of segregated constituent monads relatively to their milieu within the new monad. These dualities are in some respects similar to the dualities of relative individuation as above considered.

We find all these dualities important in the discussions concerning thinking and all in one way or another necessary for understanding the nature and scope of the process. There seems, however, to be widespread confusion and looseness in the use of the terms "duality," "polarity," etc. We consider, for example, the work of M. R. Cohen in his book *Reason and Nature.*

For Cohen, the principle of polarity means that the opposites such as immediacy and mediation, unity and plurality, the fixed and the flux, substance and function, ideal and real, actual and possible, etc., like the poles of a magnet or the blades of a pair of scissors, involve each other when applied to any significant entity. Familiar illustrations are that physical action is not possible without resistance or reaction, and that protoplasm can not live except by continually dying. The principle prevents us from identifying such opposites, in Hegelian fashion.[18] It warns us against the assumption that the negative can be dispensed with or even entirely separated from the positive (p. 59). Especially in metaphysics and the social sciences, what is necessary is not uncritical adherence to simple alternatives, but an integration of opposite assertions by finding the proper distinctions and qualifications (p. 166).

Cohen distinguishes between contradictions, which, strictly speaking, are always dialectical, holding only in a logical universe; antinomies, which arise when contradictory propositions do not annihilate each other, since they refer to a complex of incompletely determined existence, as in the case of the total universe; and aporias or practi-

[18] M. R. Cohen, *Reason and Nature,* 1931, pp. 165 and n. 4, 166. Other references are parenthesized in the text.

cal difficulties, where nature seems to prevent man's achieving opposite results at the same time. The Hegelian view, for Cohen, is inadequate particularly because of its explicit identification of the historical and the logical, the real and the rational (pp. 166*f.*).

Cohen's statement that he offers only the barest hints of a possible metaphysics that might be developed along these lines (p. 168) makes it difficult to estimate his work to date. The instances of polarity cited in various passages, however, suggest that he does not sufficiently distinguish what we should call implicit and explicit dualities, and furthermore that he does not recognize what seem to us to be the relationships between the subsistent realms of logic and mathematics and the existent realms of matter, life, and mind.

Thus he includes, as instances of polarity, identity and difference (p. xi); affirmation and negation (p. 166); the particular and the universal (p. 161), and individuality and universality (pp. xi, 399, 426); the actual and the possible (p. 165); the ideal and the real (p. 165), and the ideal and the actual (p. 426); being and becoming (p. 426); unity and plurality (pp. 165, 426); rest and motion (pp. xi, 426); change and constancy (pp. 18, 412); the fixed and the flux (p. 165); action and reaction (p. 165); life and death (p. 165); substance and function (p. 165); activity and passivity; rational and sensory elements (p. 196); individual and collective responsibility (p. 394); determinate form and the boundless (p. 165); immediacy and mediation (p. 165); universality and contingency (p. 151); value and existence (p. 385); the concrete and the abstract (p. 368); form and matter (p. 135); nature and reason (p. 135); the empirical and the mathematical (p. 239); and positivism and idealism (p. 426).[19]

[19] Other writers have given what seem to be almost random instances of polarity or duality. Charles Peirce called any distinction between two equally decided characters to which no third seems to be coordinate (although a neutrality separates them) a polar distinction, but thought that in the external world polar distinctions are few. Those of past and future; the two ways of passing over a line, with right- and left-handed spirals, etc.; possibly magnetic and electric poles; the right and left side of our bodies; and the two sexes seem pretty much to exhaust the list of them. They abound, however, in the much smaller universe of psychology, where most of them refer to volition (*Principles of Philosophy: Collected Papers,* ed. C. Hartshorne and P. Weiss, Vol. 1, 1931, p. 165).

W. H. Sheldon, in his *Strife of Systems and Productive Duality,* 1918, groups together the subject and predicate of propositions or judgments; the

Obviously no detailed discussion of these topics can be undertaken at the close of a chapter; in fact, detailed discussion of most of them is the object of this book. There is real need for a more precise and discriminating study. A basis for just such a study seems now to be afforded by the detailed working out of the hypothesis of epitomization, which accords each of the nine dualities we have listed a cosmic status in relationship to that of the others, and sees apparently isolated structures and processes of epistemology as instances of the systematic data of metaphysics.

family, with its sex contrast; the binomial theorem; and positive and negative electric charges (p. 506). Sheldon's book is considered at several points below.

It should be added that the present work, concerned with such dualities of thinking as have been indicated, is not a discussion of the epistemological dualism of perceived object and real object, ably championed by Lovejoy in *The Revolt Against Dualism.*

PART ONE

Some Problems of Psychology

"This roundabout progress through all things is the only way in which the mind can attain truth and wisdom."
—Plato, *Parmenides,* 136.

CHAPTER II

Some Dualities Involved in Perception

"[There is] the general law of economy which causes us to select for conscious notice only such elements of our object as will serve us for aesthetic or practical utility, and to neglect the rest."

—William James, *Principles of Psychology*, 1890, Vol. 2, p. 13.

Any study of thinking as related to other mental processes must consider perception, and the present study of the dualities of thinking must consider the dualities of perception. We may take it for granted that thinking has its roots in perception. Even if there were innate ideas in any given generation, they would be traceable to ancestral experiences in which perceptions were important elements.

The problem of this chapter is to consider in some detail the principal dualities of perception, according to the hypothesis of epitomization. Although it is fair enough to say that the whole organism is involved, we may distinguish first of all between (64.10)[1] the pattern directly involved in a given perception and (64.20) the actual process of perception. Like any structure and its correlated process, they are inseparable; our concern will be sometimes with the one, sometimes with the other.

(1) The point of first importance is that a perceptual pattern, considered as a monad, is (64.10) individuated (64.11) relatively to (64.111) the physiological organism. Köhler says that experience tells us about its own functional incompleteness. Nothing else can be expected if, of a larger functional whole, only a restricted field has experience as a correlate.[2]

There is also individuation relative to (64.113) other patterns, as is plain from various theories, whether they are phrased in terms of subconsciousness, automatisms, or unverbalized tendencies. Among cognate perceptual patterns there are often (64.1131) pairs in the "dyadic relationship" of tonic and clonic, etc. This is, properly speaking, the relationship of polarity, but the word, as we saw, is often applied to a number of other relationships. Other dualities

[1] Decimal numbers are those used in *Epitomizations.*

[2] W. Köhler, *Gestalt Psychology*, 1929, pp. 270 *f*.

between cognate monads appear when there are conflicts between patterns. These processes are considered in IV, below.

(II) Some cognate patterns may be (64.131, etc.) of different degrees of complexity of organization, and in some cases differences between two such patterns may become conspicuous as dualities, as when a more complex pattern inhibits a simpler pattern.

(III) In considering the principal dualities of perception one may, as above, emphasize the fact that the perceptual pattern is relatively individuated in the nervous system of the percipient, or one may emphasize the fact that the object perceived is relatively individuated or selected in the midst of the environment. For our purposes it makes no great difference which treatment is chosen, since there must be at least a formal correspondence between any pattern and its object, whether the two are similar in content or not. Thus for Köhler there appear complex unities possessing specific structures in the physical world and these are copied in the configurations of perceptions.[3]

Turning now to the processes characteristic of perceptual patterns considered as monads, we find that perceiving is (64.20) a *selective* interaction, consisting primarily of (64.2111) appropriations from the environment. Although analysis of perceptions into their more elemental sensations is difficult or impossible, there is every reason to believe that sensations are also selective. "Differential sensibility" marks a beginning of specialization of function, far down in the scale of animal development. Perceptions in the lower grades of human mentality are sometimes notably selective. In the case of savages, Malinowski says that when moving with them through any natural milieu he was often impressed by their tendency to isolate the few objects important for them and to treat the rest as mere background.[4] But we do not need to go far for examples of such selection: the most immediate example is that you or I perceive words on this page. The words are perceived selectively against the white background of the page. Each letter is or may be perceived selectively as regards other letters. The page as a whole is or may be perceived selectively in the book; the book on a desk or in the hand, and so on.

Perception is selective not merely as regards space but also as regards time. According to Eddington, that order of events which

[3] H. Helson, in *Am. Jour. Psy.*, 36, 1925, p. 367.

[4] B. Malinowski, in C. K. Ogden and I. A. Richards, *The Meaning of Meaning*, 1923, pp. 502-4.

is parallel with an individual's track through the world appears in his experience to be differentiated from all other orders of events.[5]

It is clear that (64.22) responses, especially movements and forms of overt behavior, are important in preparing the way for perceptions, either because they are actual stimuli, or because they bring perceptions to their full development. According to R. M. Ogden, a perceived object is a member of a perceived response.[6] But on the side of response the process is selective. Even an amoeba selects one among movements which relieve it of stimulation.[7] Ogden holds that the "principle of dichotomy" is applicable to all primitive modes of behavior. Concretely, an act of behavior stands off as a pattern or figure upon an undifferentiated ground. In this way it becomes a particular. However vague or lacking in precision, that part of whatever it might be which emerges to become what it is stands out against a less differentiated and less articulate ground.[8]

Perception in its most conspicuous form, as attention, involves inhibition, and is best treated at IV, below; but the basic fact that attention is selective needs no elaboration. For Baldwin, an object of attention is so far a whole that some content is "inside" it and the rest of the psychic field at the time is "outside" it. What is sometimes called the span of consciousness or attention has its whole of content thus defined by limitation. Attempting a genetic account of such developments, he says that the extreme case, the one of least meaning, would seem to be that of the limitation set to the original grasp or span of function whereby an object as such is constituted. The panorama of presented matter is always larger than the content grasped as object. He finds the first form of opposition in that penumbra of indeterminate otherness which is the background or margin of limitation of a presented or projected content.

This, for Baldwin, passes into a mode of negative meaning called privation, where the sphere of actual indeterminateness is much more restricted, and, we should say, the selection becomes that of one object in the midst of other similar or cognate objects. The essence of privation, according to Baldwin, is that in respect to the matter or meaning of which the object is deprived, it is indeter-

[5] A. S. Eddington, *Space, Time, and Gravitation,* 1921, p. 57.
[6] R. M. Ogden, *Psychology and Education,* 1926, p. 134.
[7] H. S. Jennings, *Behavior of the Lower Organisms,* 1915, p. 22.
[8] R. M. Ogden, *op. cit.,* p. 27.

minate, simply other or whatever else it might have been or have tended to be.[9]

Even where attention is seen to involve opposition and inhibition, it may be interpreted in terms of selection among similar elements or objects. The motor theories of attention, according to which its essential feature is assumed to be the occurrence of tentative movements belonging especially to the stimulus attended to,[10] show this; the responses, like the excitations, are selective. Holt says that the concentration of attention is traditionally supposed to involve some sort of selection among afferent (sensory) impulses, and this implies a rejection, exclusion, or inhibition of many of the afferent impulses. In fact, however, except in the case of the eye with its lid, etc., afferent impulses can not be excluded, and exclusion or inhibition of afferent impulses apparently has no meaning. The selective aspect of attention is to be found altogether on the motor side.[11]

This differential or selective feature of perception has been investigated particularly by the *Gestalt* psychologists in their work on "figure" and "ground." When one field is surrounded by another, the smaller enclosed field is usually taken as figure[12] and the other as ground. The distinction is phenomenological,[13] a matter of experience.[14] We deal with a selective agency the effects of which are felt, not comprehended, and the conditions of which are compulsory, not chosen.[15] According to Koffka, the first phenomena are qualities upon a ground.[16]

Figure, often treated as a positive field,[17] and ground, referred to as a negative, are thus the simplest configurations observable in perception.[18] The distinction between them is not primarily a matter of attention, but in general a visual figure is more impressive than its ground. It has a higher degree of clearness,[19] and a higher threshold for color induction.[20] It is more highly differentiated.[21] It

[9] J. M. Baldwin, *Thought and Things . . .* , 1906, Vol. 1, pp. 187-9.
[10] M. F. Washburn, *Movement and Mental Imagery,* 1916, pp. 104 *f.*
[11] E. B. Holt, *Animal Drive and the Learning Process,* Vol. 1, 1931, p. 215.
[12] E. Rubin, *Visuell wahrgenommene Figuren,* Teil 1, 1921, p. 79.
[13] H. Helson, in *Am. Jour. Psy.,* 36, 1925, p. 496.
[14] E. Rubin, *op. cit.,* pp. viii, ix, 37.
[15] R. M. Ogden, *op. cit.,* p. 136.
[16] K. Koffka, *The Growth of the Mind,* 1924, p. 131.
[17] E. Rubin, *op. cit.,* p. 5.
[18] H. Helson, *op. cit.,* pp. 496, 498.
[19] E. Rubin, *op cit.,* pp. 67, 99, 101.
[20] H. Helson, *op. cit.,* p. 496; W. Kohler, *Gestalt Psychology,* p. 220.
[21] E. Rubin, *op. cit.,* p. 47.

usually seems to protrude from the ground,[22] which, when the figure is in three dimensions, seems to extend behind it without interruption.[23] A figure is more meaningful than a ground, and has more value for knowledge.[24]

Figures are not merely visual but also auditory, etc., and the distinction of figure and ground applies to behavior as well as to phenomenological data.[25] It applies also to the creations of dreaming[26] and is of considerable importance in memory.[27] Many think that the distinctions represent physiological realities in the brain,[28] or in the objective world,[29] or both. The explanation advanced for the difference between figure and ground in vision is that of different ionic concentrations in the retina, involving differences of electrical potential and concentrations of energy in the smaller area, making the figure predominate.

With respect to any given figure, the ground is much less definitely perceived. A figure can be connected with other figures more easily than connections may be formed between one ground and another.[30] We saw that a ground is sometimes referred to as negative, but at other times certain rather vague positive statements may be made about it—for example, that as was said above, it seems to extend behind the figure. Such statements are really made with some relationship to the figure. For example, we say that the ground has a certain size, that it is a rim or surface, and that it shares with the figure a contour or boundary.[31]

Sometimes the ground is regarded as including not merely the visual background but the entire psychological setting of the experiment; the tonus or organic readiness for response is said to serve as the ground from which the so-called phasic pattern of specific behavior arises.[32] Lorimer says that in the organization of any response some elements must be included and other elements excluded, but the excluded elements continue to be more or less

[22] H. Helson, *op. cit.*, p. 495.
[23] W. Kohler, *op. cit.*, pp. 219 *f.*
[24] E. Rubin, *op. cit.*, pp. 74, 96.
[25] H. Helson, *op. cit.*, p. 498, and *Am. Jour. Psy.*, 37, 1926, p. 32.
[26] W. Kohler, *op. cit.*, p. 275.
[27] E. Rubin, *op. cit.*, pp. 13 *ff.*; W. Köhler, *op. cit.*, p. 221.
[28] *cf.* W. Kohler, in C. Murchison, ed., *Psychologies of 1930*, p. 149.
[29] *idem*, in C. Murchison, ed., *Psychologies of 1925*, p. 173.
[30] H. Helson, in *Am. Jour. Psy.*, 36, 1925, pp. 366 *ff.*, 497.
[31] E. Rubin, *op. cit.*, pp. 43 *f.*
[32] R. M. Ogden, *op. cit.*, pp. 114, 131, 237.

functional as an organic context or ground, against which the newly organized pattern emerges as distinct.[33]

There are some positive statements about the background of perceptions in the work of Baldwin, who maintains that after the stage which he calls "privation" is reached, we tend, with developing experience, to select one thing in the midst of other things which, for one reason or another, we assume that we know. By the aid of memory, we go about among even the excluded things with skill and safety, having little actual need of revising our expectations as to what they will be. There is therefore in the background or penumbra of the psychic process little that can be called strictly indeterminate.[34]

Other positive statements about the background of perceived events are found in Whitehead's theory of duration, and of the discernible. He says that the simultaneity of the whole of nature comprising the discerned events is the special relation of that background of nature to the percipient event. Such a complete whole of nature is called a "duration." The whole continuum of nature "now-present" means one whole event (a duration) rendered definite by the limitation "now-present" and extending over all events now present. A duration is in a sense unbounded; for it is, within certain limitations, all that there is. It has the property of completeness, limited by the condition now-present. It is a temporal slab of nature.

Nature as we know it is a continuous stream of happening immediately present and partly dissected by our perceptual awareness into separated events with diverse qualities. Within this present stream the perceived is not sharply differentiated from the unperceived; there is always an indefinite beyond of which we feel the presence, although we do not discriminate the qualities of the parts. This knowledge of what is beyond discriminating perception is the basis of the scientific doctrine of externality, involving the "constants of externality." There is a present whole of nature of which our detailed knowledge is dim and mediate and inferential, but capable of determination by its congruity with clear and immediate perceptual facts.[35]

Again, the general fact that something is going on at once yields for our apprehension the "discerned" and "discernible." The dis-

[33] F. Lorimer, *The Growth of Reason,* 1929, p. 82.

[34] J. M. Baldwin, *op. cit.,* Vol. 1, p. 189.

[35] A. N. Whitehead, *Enquiry Concerning the Principles of Natural Knowledge,* 1919, pp. 68 *f.,* 72. This work will hereafter be cited as *Principles of Natural Knowledge.*

cerned is the field directly perceived, but it is related to other entities not particularly discriminated in this individual way, but known merely as the relata in relation to the entities of the discerned field. Significance is relatedness, with emphasis on only one end of the relation.[36] Every event is a factor in a larger whole and significant of that whole.[37] A discernible entity is merely a "something" which has such and such definite relations to some definite entity or entities in the discerned field. The complete general fact comprises both sets of entities. The factors in nature of which we have this peculiar sense awareness are known as not comprising all the factors which together form the whole complex of related entities. This peculiarity of knowledge Whitehead calls its unexhaustive character. Nature as perceived always has a ragged edge—for example, there is a world beyond the room to which our sight is confined, known to us as completing the space relations of the entities discerned within the room.[38]

It is clear that in these passages more is claimed for the background than in the *Gestalt* psychology. Statements are made concerning the whole, the whole continuum, unbounded duration which is a temporal "slab" of nature, completeness, the beyond of which we feel the presence, knowledge dim and mediate and inferential, congruity with clear perceptual facts, the discernible as disclosed while the discerned is discriminated, the world beyond the room known to us as completing the space relations of the room, and so on.

The *Gestalt* work on figure and ground suggests more caution in dealing with backgrounds, and gives fresh point to the problem fundamental for this book. According to the view which we have indicated, perception is selective, but, as William James saw, *selection involves neglect. The process of perception is, in fact, best described by the hyphenated term selection-neglect.* It is a noticing-ignoring.[39] Each case of this selection-neglect, or noticing-ignoring, is typically momentary, if not instantaneous. At any given moment I select the perceived object in the midst of a neglected background. At each new moment, what was previously a part of the background, like the book or the desk in the case of the white paper, may be selected, but if selected it is to the neglect of some further background, like the remainder of the room, which even at the new

[36] A. N. Whitehead, *The Concept of Nature*, 1920, pp. 49-53.
[37] *cf.* A. N. Whitehead, *The Principle of Relativity . . .*, 1922, pp. 21, 26.
[38] *idem*, *Concept of Nature*, pp. 49 *f.*, 53.
[39] *cf.* W. H. Sheldon, *Strife of Systems*, pp. 475 *f.*

moment is still indefinite and unexplored. Even when expanded or extended by memory, our mass of perceptions in time continues to be selective and neglective. These statements concerning selection-neglect will need to be somewhat qualified as we proceed, but will remain fundamental. And the problem then arises, how we shall treat the neglected background, and how, if in any way, we have or can acquire knowledge concerning it.

(IV) If one pattern were operating alone, it would, as we have just said, proceed by selection-neglect, and we should have that peculiar kind of duality. But in the higher organisms the multiplicity of stimuli and responses makes the whole process more complicated, and we encounter another duality. This is the duality not of a single pattern and a relatively undifferentiated milieu, but typically of *two cognate patterns* in a relatively undifferentiated milieu. It is the duality involved in inhibitions and probably in the formation of images. It has been traced, too, in the more general processes of consciousness and, as we said, in attention.

According to Rignano, there can be an inhibition of one physiological activity by another only if, either directly or indirectly, by means of other activities intimately associated with the first, there happen to be two nervous activities specifically different which tend to arise at the same point or in the same tract of the organism. One affectivity can inhibit another different from it if both have a part of their seat in common.[40] Herrick says that inhibitions occur even in the nerve nets of the lower organisms. Two excitations which tend to produce opposite effects may act on the same muscle cells with resulting blocking of the response.[41] Sherrington holds that, among the higher animals, inhibition usually involves two simple reflexes of opposite synaptic action at the final common paths. The final common path is handed from some group of a plus class of afferent arcs to some group of a minus class or a rhythmic class, in virtue of interaction between rival reflexes, or interference.[42] According to Sherrington, there is evidence that arrangements exist in the motor cortex for reciprocal innervation,[43] and, according to Brücke, there is inhibition between intracortical centers.[44]

[40] E. Rignano, *The Psychology of Reasoning*, transl. W. A. Holl, 1923, p. 65.

[41] C. J. Herrick, *Neurological Foundations of Animal Behavior*, 1924, p. 92.

[42] C. S. Sherrington, *The Integrative Action of the Nervous System*, 1906, pp. 65, 232 *f.*

[43] *ibid.*, p. 291.

[44] E. T. Brucke, *Handbuch der normalen und pathologischen Physiologie*, Vol. 9, 1929, pp. 645, 647, 659.

Whatever theory of the nature of inhibition,[45] or whichever combination of them, is favored, it is clear that inhibition involves duality. It is essentially an interaction between two cognate patterns. Since the drainage theory involves transfer of energy, it may be said to consider the interaction as (64.231391, etc.) incorporative, while theories of blocking, etc., regard it as (64.232391, etc.) non-incorporative. If one pattern in its processes both of stimulation and response makes some other pattern with its processes for the time being completely impossible, their interaction is of the type of (64.231391, 64.232391) preemption. When one of the antagonists secures some part of the structure of the other, as in the processes involving a "final common path," or in certain dissolving figures like the "staircase,"[46] their interaction is of the type of (64.231392) conflict. The many cases where one pattern (64.231393, 64.232393) reinforces another, are not very important as regards the dualities involved. If two centers loosely connected by association fibers be thrown into simultaneous or nearly simultaneous excitement, each has something to give and each is ready to take, and so (64.231394) interchange takes place between them.[47]

Processes of inhibition, etc., may alternate, as in reciprocal innervation, with resultant series of (64.23138) rhythmic or oscillatory actions. According to the *Gestalt* psychology, in certain cases the portion of a field taken as figure and that taken as ground may alternate or oscillate.[48]

According to Washburn, sensations and images depend upon the simultaneous excitation and inhibition of a motor pathway.[49] Theories of conflicting tendencies as the basis of consciousness have been made familiar by Holt and Dewey.[50]

Many psychologists consider attention as essentially or largely inhibition.[51] William James said that attention is the taking posses-

[45] For the theory of qualitatively different processes in the same center, C. S. Sherrington, in Roy. Soc. Lond., *Proc.*, B, 97, 1925, pp. 267-83, and *cf.* E. B. Holt, *Animal Drive and the Learning Process*, Vol. 1, 1931, pp. 200 *f.*; for the drainage theory, C. K. Ogden, *The Meaning of Psychology*, 1919, pp. 50 *ff.*; for the theory of chronaxies, L. Lapicque, *Science*, 70, 1929, pp. 151 *ff.*; and for the theory of "overcrowding," E. B. Holt, *op. cit.*, Vol. 1, index.

[46] For the status of lines in interchangeable figures, see E. Rubin, *op. cit.*, pp. 169, 174.

[47] G. T. Ladd and R. S. Woodworth, *Elements of Physiological Psychology*, p. 619.

[48] E. Rubin, *op. cit.*, pp. 39, 83.

[49] M. F. Washburn, *op. cit.*, pp. 26, 30, 56, 58.

[50] J. Dewey, *Essays in Experimental Logic*, 1916, p. 11, n. 1.

[51] E. Rignano, *op. cit.*, p. 64, and refs.

sion by the mind in clear and vivid form of one out of what seem several simultaneously possible objects or trains of thought. . . . It implies withdrawal from some things in order to deal effectively with others.[52] The motor theories of attention, as we saw, make inhibition one of its prominent features.[10, 11]

Rignano says that conflicts of motor elements induced by affective tendencies are expressions of conflicts of these tendencies. With the advent of distance receptors it became possible for animals to adjust themselves to objects at a distance. But if the "consummatory" adjustment is evoked too soon, the act is a failure; and the repeated deceptions which resulted when the affective tendency released by a distance receptor produced immediately the complete performance of a consummatory act which was necessarily a failure must have had a profound effect in tending to inhibit such hasty actions in future. So there are two affectivities, simultaneously aroused and opposed—one making for the consummatory act, the other causing hesitation for fear of making a mistake. This, for Rignano, is the origin of attention. He says also that conflict is exhibited with great distinctness in certain typical states of attention where it is expressed in exceedingly subtle choice between almost imperceptible modalities of a certain act.[53]

All this is assuredly enough to show that dualities of the kind we are discussing, occurring in the interactions of patterns with other cognate patterns, are of great importance in the processes associated with perception.

(v) We find, next, that any consideration of selected and neglected regions is likely to involve questions concerning a boundary between them. William James made the metaphors of fringes of consciousness and peripheral nebulosities familiar. As Broad puts it, a field of view does not come sharply to an end at its edges. It fades gradually away; details become less and less definite the further they are from the center. It rather presents itself as a fragment of something bigger.[54] Dewey thinks that the classical psychology has been too much occupied with sharp focalizations, whereas we really have shadings off.[55]

Where there are such shadings off, it is easy to suppose, again, that our knowledge extends beyond what is in the foreground of

[52] W. James, *Principles of Psychology,* Vol. 1, 1890, pp. 403 *f.*

[53] E. Rignano, *op. cit.,* pp. 34, 36, 42, 63; also *Monist, 22,* 1912, pp. 5 *ff.*

[54] C. D. Broad, *Scientific Thought,* p. 34.

[55] J. Dewey, *Experience and Nature,* 1925, pp. 305, 311.

perception, and, since it is difficult to fix the limits, it is easy to suppose that there are none.[56] The lack of complete differentiation between figure and ground leads Werner to assign a positive character to the ground.[57]

Evidently a field of perceived objects tends to shade off in this indeterminate way, but the case is often different as regards single or particular objects, relatively isolated within the field. Here the very fact that an object is distinguished visually from others around it or beyond it is bound up with the sharpness of its apparent boundary lines. According to Reiser, the eye tells us about those physical objects upon which the motor organs must act.[58] According to T. Smith, vision is now so constituted in human beings as to bring out as clearly as possible the sharpness of the contours of bodies; if the eye were sensitive to the ultra-violet there would be want of contrast amounting to general fog, and if it were sensitive to the infra-red there would be too much shadow.[59] So the evolution of vision has gone in the direction of providing a mechanism for the sharpening of the meniscus between matter and the field so that bodies take on sharper outlines.[60] Baldwin speaks of the circle of limitation by which a positive meaning is hemmed in.[61]

The most careful work on this problem in connection with visual figures has been done in the *Gestalt* psychology, where it is found that in typical cases the difference between figure and ground is marked by a contour which runs between them as their common boundary.[62] J. Parsons says that no other type of sensory discrimination is so highly developed.[63] It is possible that such a contour is uniquely characteristic of visual fields; how far the principle applies to other senses is uncertain. Rubin says that it is not known what kind of psychical structure belongs with such a contour, although there may be some kind of physiological interchange in the visual area.[64] Köhler assumes that in the optical processes contours are preserved by similar forces of antagonistic contact, probably at surfaces in the nervous network.[65]

[56] *cf.* S. Alexander, *Space, Time, and Deity,* Vol. 2, p. 358.
[57] H. Helson, in *Am. Jour. Psy.,* 37, 1926, p. 208.
[58] O. L. Reiser, in *Monist,* 41, 1931, p. 523.
[59] T. Smith, *Nature,* 121, 1928, pp. 242 *f.*
[60] O. L. Reiser, *op. cit.,* p. 522. [61] J. M. Baldwin, *op. cit.,* Vol. 1, p. 191.
[62] H. Helson, in *Am. Jour. Psy.,* 36, 1925, p. 497; E. Rubin, *op. cit.,* p. 36.
[63] J. Parsons, in *Nature,* 121, 1928, p. 94.
[64] E. Rubin, *op. cit.,* pp. 106, 195.
[65] W. Kohler, *Gestalt Psychology,* pp. 143 *f.*

Precision of contour is largely a matter of erudition, or elaboration. In the beginning, the edges of a visual pattern are nebulous and uncertain.[66] Practice may, in fact, work the other way. Rubin notes that if one tries, a figure and its ground can be seen simultaneously, but this appears to be only a broadening of range of a perception, as in VI, below, whereby a portion of a former ground is incorporated into a new and in this case composite figure, which shades off into a new ground farther out. On the other hand where a contour develops, it seems to be more important for simple figures than for complex ones, and for large figures than for small ones. The contour comes to be thought of as surrounding or enclosing the figure; for its relationship to the ground there seems to be no convenient recognized term. Where there are two adjacent contrasted fields, one white and the other black, the contour appears to belong to the field which is regarded as figure, but by special effort in such a case the contour may appear to acquire a relative independence of the figure. Rubin says that it is difficult to see what would happen if the contour worked outwards, and determined the form of the ground rather than that of the figure. He says we might have limiting cases where both figure and ground are shaped by the contour, or where neither figure nor ground is so shaped. There are also cases where a given field is taken partly as figure and partly as ground, for example, where a polygon has a long sharp-angled "nick" or tongue in it, the polygon as a whole is taken as a positive figure, but the tongue is taken negatively.[67]

Baldwin mentions the exclusion, from what is actually perceived, of that "hypothetical but definite" [we should say, indefinite] other by which we interpret the margin—as if the ground could be used to interpret the margin. He says that when this "other" is replaced by a truer construction of the actual margin, there is an exclusion of this latter[68]—as if there were a duality of figure and contour.

Evidently, then, in certain cases, especially in the visual field, we have to do not merely with a duality of determinate figure and indeterminate ground, but with a more or less indeterminate intermediate boundary between the two.

(VI) Empirically and concretely, there appears to be a kind of reaching forward, an "animal faith," characteristic of our percep-

[66] R. M. Ogden, *Psychology and Education*, pp. 120, 307.
[67] E. Rubin, *op. cit.*, pp. 33, 37, 43, 45, 81 *f.*, 93, 148, 150, 152.
[68] J. M. Baldwin, *op. cit.*, Vol. 1, p. 189.

tions. Patterns exhibit (64.27) processes of growth and depletion, both as regards their nervous structure and the extent or range of the objects which they appropriate. So the relationships between figure and ground, and hence the contour between them, may change by increase or diminution in the course of time. This, too, is involved in Baldwin's view when he says that in the shading out of a context from what is used into a margin or penumbra of what is there but not used we have, as it would seem, a sort of contrast which offers promise of genetic development. In the very process by which indeterminate marginal or penumbral content takes on determinate form it becomes subject to just the contextuating process which enlarges the positive meaning of the object then in the center of interest. The circle of limitation by means of which the positive meaning is hemmed in is pushed back upon what is undetermined.[68]

For the *Gestalt* psychology, figure and ground are in a reciprocal relationship,[69] and it is their dynamic intercourse which decides what is figure and what is ground.[70] In poor light, for example, the color of the ground spreads over the figure.[71] Under certain conditions, figure and ground can even be interchanged. Important for the *Gestalt* psychology is the tendency to complete an imperfect or incomplete figure, for instance by closure of the boundary lines.

(VII) The (64.33) neuropsychological processes which, according to the hypothesis of epitomization, are parallel to biparental reproduction in biology involve notable dualities, but they are difficult to distinguish from processes of (64.231392) conflict and (64.231394) interchange among cognate monads, mentioned above. A "felt difficulty," for example, may be either such a conflict of tendencies or it may be the precursor to the development of a suggestion which, by a process like Pavlov's "mutual induction," will serve as a sort of compromise between them.[72] The view that there are resultant fusions of two tendencies is also held by Janet.[73]

Often when the two conflicting tendencies are clearly distinguished, the very clearness of the distinction shows that we are dealing with an *explicit* duality, *i.e.*, one in which both members are capable of being treated descriptively on a par, whereas the pres-

[69] H. Helson, *op. cit.*, p. 352.

[70] See W. Kohler, in C. Murchison, ed., *Psychologies of 1930*, p. 149.

[71] J. Pikler, in *Zeitschrift für Psychologie*, (1), 106, 1928, p. 317.

[72] See G. P. Conger, *Epitomizations*, pp. 279 *f.*

[73] P. Janet, *De l'angoisse à l'extase*, 1926, p. 226; *cf.* M. F. Washburn, *op. cit.*, pp. 26, 56.

ent study is concerned rather with a considerable number of implicit dualities, where one member is treated descriptively but the other beyond the horizon is left indefinite and open. According to our view, developed in Chapter V, such biparental reproduction in the neuropsychological realm is not in any direct sense the basis of syllogistic reasoning, and so has not the importance for a theory of thinking that it may be mistakenly said to have.

(VIII) Still another kind of duality appears when, according to the epitomization hypothesis, patterns are (64.43) integrated into end-reaction complexes. Such patterns are commonly distinguished, in Sherrington's terms, as (64.431) precurrent and consummatory—the former leading up to or opening the way for the latter, and the latter yielding the final or end-reaction. The duality of precurrent and consummatory patterns involves the action of the distance receptors,[74] and is a special case of the more general duality of clonic and tonic patterns, respectively. The pattern of the consummatory reaction tends to persist while those of the precurrent reactions are run off. James says that physiologically considered we must suppose that a purpose means the persistent activity of certain rather definite brain processes throughout the whole course of thought.[75] Dewey says that the concluding term conserves within itself the meaning of the entire preparatory process.[76] According to Muller's constellation theory, the goal is merely a constant in the chaos of reproductive, or, as he might say, mnemic, elements or tendencies.[77]

(IX) Within an observed object, subordinate parts may be observed as if they were differentiated there (*cf.* 63.442) and sometimes dualities may appear as regards some of these differentiated parts and an undifferentiated ground, or as regards some of the parts and other parts.

This appears clearly in the *Gestalt* psychology, where the distinction between figure and ground may in complicated cases be within the figure. In a series of pairs of vertical parallel lines, the whole area in a group between the two members of a pair of parallels nearer to each other appears to be different from the surrounding paper, and also different from the area between two consecutive groups. Within a group there is a certain aspect of solidity;

[74] The part played by the distance receptors is considered in more detail in Chap. XX.

[75] W. James, *Psychology, Briefer Course,* 1892, p. 271.

[76] J. Dewey, *Experience and Nature,* p. 270.

[77] H. Helson, in *Am. Jour. Psy.,* 37, 1926, p. 53.

we might even say there is a something, whereas between groups and around the whole series we have emptiness and there is nothing.[78] In auditory perceptions, a pause in a series of rhythmic beats may appear as a *Gestalt*—a bounded silence—and may exchange its value as a *Gestalt* with the beats.[79] In general, analysis causes the members of a configuration to become themselves individual wholes, and the affective quality of the old configuration vanishes when the new configuration appears.[80] Sometimes the lack of differentiation obtains within the parts of the interior of the figure, as when, with continued discriminations, lines without breadth are said to be discriminated.[81] In other cases one may adopt a special attitude with reference to the field, selecting some of its members and more or less suppressing the rest.[82] Even portions of a line which is drawn at one stroke and, especially when seen at a distance, is apparently continuous may be distinguished from other portions of the line, although not very precisely. The parts of a surface like projections, etc., which are closely bounded by parts of the contour, tend to be regarded as independent of the other parts.[83]

The processes of perception, with all their dualities, are doubtless involved in thinking, but the process of thinking can not be described and perhaps can not be understood without processes of symbolization, culminating in language. Among readers and speakers, especially in academic circles, thinking in words tends to supersede any other forms and becomes so prominent that it necessitates a study of the relationships between it and language. This can best be accomplished by a survey of the development of language, which will occupy us in the next chapter.

[78] W. Kohler, in C. Murchison, ed., *Psychologies of 1925*, p. 170.
[79] H. Werner, *Zeitschrift fur Psychologie* (1), 82, 1919, pp. 203, 207.
[80] R. M. Ogden, *op. cit.*, p. 150.
[81] E. Rubin, *op. cit.*, pp. 193, 197 *f.*, 202.
[82] W. Kohler, *Gestalt Psychology*, p. 183.
[83] E. Rubin, *op. cit.*, pp. 162 *f.*, 181.

CHAPTER III

The Origin and Development of Language

"The hope of a successful theory of the evolutionary genesis of speech lies in casting the net widely enough, in bringing to bear upon the subject converging lines of evidence from widely different fields. And the test of any theory must lie in the light it sheds upon the many closely related problems."
—G. A. de Laguna, *Speech, Its Function and Development*, 1927, p. 8.

1. *Familiar in the literature on the origin of language are some theories which have come to be called classical.* These are the "bow-wow" theory, that language was originally imitative of natural sounds; the "pooh-pooh" theory, that it was originally interjectional; the "ding-dong" theory, that there was a mystical harmony between sound and sense, giving each substance its peculiar "ring"; and the "yo-he-ho" theory, that language arose from sounds associated with acts.[1] According to Pillsbury and Meader, each of the classical theories needs to be liberally interpreted.[2] As regards the first, Stebbing notes that inasmuch as most things have not a characteristic sound, a language based wholly on imitative sounds would not progress very far.[3] Imitation, according to Pillsbury and Meader, is the result of the process rather than its cause. The "pooh-pooh" theory lacks precision. In spite of Wundt's argument that each mental state may be assumed to evoke a specific instinctive response, *e.g.,* in the facial muscles, Pillsbury and Meader think that not much in the way of language, and especially designation of separate objects, can be traced to such emotional expressions. The "ding-dong" theory is, of course, too vague. As regards the fourth theory, probably some sounds do arise as by-products of activities, without contraction of the vocal organs,[4] but even if this is the case, it needs to be seen in another setting, if it is to help account for the origin of language.

2. *The trend of recent theories in science and philosophy has been toward the study of language as social.* Recent work has been

[1] O. Jesperson, *Language, Its Nature, Development, and Origin,* 1925, pp. 413 *ff*.
[2] W. B. Pillsbury and C. L.Meader, *Psychology of Language,* p. 113.
[3] L. S. Stebbing, *A Modern Introduction to Logic,* 1930, p. 11.
[4] W. B. Pillsbury and C. L. Meader, *op. cit.*, pp. 107, 113 *f.*, 117, 125.

so much influenced by the social sciences that all theories must now be estimated in the light of social situations. Language begins, of course, in the subhuman groups. Some social insects seem to be able to exchange a considerable range of information, though on principles which are quite different from those of articulate human speech.[5] The cries of animals more like man are familiar enough.

There is no society without individuals, and language may be studied in its individual aspects. For Lorimer, words have their origin in oral play. Symbolic activity arises in an autistic context, characterized by fluctual implicit processes.[6] But even rudimentary means of expressing emotions must be of survival value for social animals.[7] Language, like other perceptions or responses, selects and neglects, but the selection or discrimination has a social reference. What is discriminated in speech is that which demands indirect treatment through the instrumentality of others; it is the social object, common to both speaker and listener, which alone is capable of being named.[8]

3. *The trend of recent science and philosophy has brought general agreement that language is to be understood primarily in terms of activity.* The essential is that language shall be expressive, not that it shall be vocal. We find that the parts most directly employed in speech are used for respiration, mastication, deglutition, and excretion, and thus that articulatory movements doubtless are the outgrowth of movements attending the fundamental biological processes.[9] It is suggested, too, that in man the upright posture, releasing the forelimbs for the carrying of objects that need no longer be carried in the mouth, aided the development of language.[10] Still, language in its beginning does not need to consist of sounds made or shaped by the mouth. Any sense organ may serve in the development of at least a rudimentary language,[11] and any motor organ may be used at least in a gesture. The whirr of a partridge's wing may be as expressive as a cry.[12]

The central distinguishing feature of animal cries is their direct connection with specific type responses. A characteristic cry accom-

[5] A. H. Clark, *The New Evolution,* 1930, p. 8.

[6] *cf.* F. Lorimer, *Growth of Reason,* p. 132.

[7] W. B. Pillsbury and C. L. Meader, *op. cit.,* pp. 117 *ff.*

[8] G. A. de Laguna, *op. cit.,* p. 244.

[9] W. B. Pillsbury and C. L. Meader, *op. cit.,* p. vi.

[10] O. Jesperson, *op. cit.,* p. 413.

[11] *cf.* J. Vendryes, *Language: A Linguistic Introduction to History,* transl. P. Radin, 1925, p. 7.

[12] *cf.* W. B. Pillsbury and C. L. Meader, *op. cit.,* p. 120.

panies or forms an integral part of a specific attitude or activity.[13] In the human groups, language originally may have accompanied an activity as a kind of emotional overflow, an expression of excitement or intense feeling. But at any rate, before man has outgrown the primitive, language comes to function as a link in a concerted activity, a piece of human behavior.[14] Symbols are developed in actions and become stimuli for new actions.[15] Speech, says Mrs. de Laguna, must be envisaged as effecting something vital in the practical life of man, as performing some objective and observable function, before one can hope to discover factors which led to its development.[16] Piaget says that the process of learning an operation on the verbal plane reproduces the same incidents as had arisen when this operation was being learned on the plane of action. A process of shifting will take place from one apprenticeship to the other.[17] After language habits have become established, speech may indeed become a quasi-independent form of behavior, having no necessary or at least no apparent connection with what a man is doing, nor with his emotional states. But this sort of thing is not characteristic of the spontaneity and effusiveness of animal response, nor of the urgency of primitive human situations and the playfulness or precariousness of primitive or savage life.[18]

4. *It is obvious that language occurs typically as an accompaniment or a part of activities in end-reaction complexes, which also contain emotional elements.* The occurrence of end-reactions does not necessarily mean an initial purposiveness. It may even be that methods which at first merely happen successfully to indicate or direct activities, without any intent, afterwards come to be used with specific intent.[19] But whether simple end-reactions are to be understood as purposes or not, at any rate they do include a certain play of emotions and affective values in all the participating individuals. No theory of the development of language, or any other process of mind, can hope to be adequate, if it leaves language to develop in an emotional vacuum. This begins to appear even on objective experimental grounds; discovery of kinaesthetic innerva-

[13] G. A. de Laguna, *op. cit.*, p. 32.

[14] B. Malinowski, in C. K. Ogden and I. A. Richards, *The Meaning of Meaning*, 1923, p. 474.

[15] J. F. Markey, *The Symbolic Process . . .*, 1928, p. 161.

[16] G. A. de Laguna, *op. cit.*, p. 10.

[17] J. Piaget, *Judgment and Reasoning in the Child*, 1928, p. 214.

[18] *cf.* G. A. de Laguna, *op. cit.*, pp. 19 *f.*

[19] J. Dewey, *The Quest for Certainty*, 1930, p. 145.

tions from muscular tensions, for example in the hyperglossus nerve of the tongue, suggests the possibility that such innervations may be an essential link in the verbal processes, even when no audible sound is produced.[20] It is not necessary to distinguish, as Ogden and Richards do, between the emotive and the referential use of words.[21] At most the distinction is only relative. Every word is more or less emotive, and less or more referential or symbolic.

5. *When language is viewed as a social activity, it is seen to be involved in the early and inevitable differentiations which occur in social groups, subhuman as well as human.* Differentiation means division of labor; no one individual can do everything there is to be done. In animal groups, one of the common differentiations is that between sentinels or scouts and other members of a flock or herd. Sentinels give warning of approaching danger. Perhaps in the simplest cases, where the individuals of the group are not far apart, a mere gesture or abbreviation of some instinctive-emotional reaction will be sufficient to influence the behavior of those to be warned.[22] In such a gesture language, a present object is indicated by pointing, and the gesture is expected to be immediately intelligible. In many cases, however, the individuals are so far apart or the danger is so imminent that a gesture is insufficient, and gregarious animals sound the cry of alarm. Such a cry may be at first only the emotional overflow of fright in the individual uttering the cry, but by a process of natural conditioning this secondary response comes to serve in place of a primary stimulus for the other members of the group and to bring out from them the same type response as does perception of the danger itself. The function of the alarm cry is to adjust other individuals to some external condition. The cry of alarm does not excite a response to itself, or at least, if it does, the response is to itself only as a representative of a third thing, to which the stimulating cry stands in the peculiar relationship of sign to thing signified. The cry is a substitute for the situation perceived by the giver of the cry only so far as the perceived situation is like any other situation of the same sort;[23] but it seems most probable that a cry would not seem generalized if it were not regarded abstractly, and that in an actual situation a cry, with its intonation, if not its content, is concrete and specific.

[20] F. Lorimer, *Growth of Reason,* p. 67; *cf.* O. R. Langworthy, *Johns Hopkins Hospital Bulletin,* 35, 1924, p. 239.

[21] C. K. Ogden and I. A. Richards, *op. cit.,* p. 13; *cf.* p. 228.

[22] J. F. Dashiell, in *Psy. Rev.,* 32, 1925, p. 68.

[23] *cf.* G. A. de Laguna, *op. cit.,* pp. 32 *f.,* 256.

6. *The cry may be given by any member of a group, but sometimes it is the function of the leader.* The very important social differentiation of leader and follower has led to another analysis of the animal cry and to a very plausible theory of the nature of language, advanced by Pierre Janet.[24] At least partially, and very suggestively, he analyzes the situation which involves the cry of a leader attempting to direct the activities of the other members of a group.

According to Janet, the cry may be studied psychologically as the result of a conflict of tendencies in the leader. One tendency is for the leader to be a mere member of the group and do as the ordinary members do; the other tendency, which conflicts with this, is for him to act as leader and to be different from the others. The two tendencies conflict in a social situation, such as, for example, that of a pack of hunting dogs which has driven a quarry to cover. In simple cases, says Mrs. de Laguna, it might perhaps be sufficient for the leader himself to make a start in one direction or another; but if the case is not so simple, his ability to announce to his followers the course he is about to pursue,[25] or which they should pursue, becomes indispensable. So in the above case the leader, instead of closing in with the others and attacking the quarry, stops short, and, acting as leader, emits a cry, which is the signal to the other members of the pack to close in and attack. Another example, not here studied in detail, is where a mother hen, instead of picking up a grain of wheat for herself, clucks as the signal for the chicks to pick it up.

The cry thus represents a tendency which, inhibited in the leader, is left to be completed by the others. In such a situation, tendencies both dominant and inhibited are so numerous in leader and followers, that this view is not far from that of Mrs. de Laguna, who says that the primitive function of the predicative cry is perhaps more to inhibit an act already incipient or to encourage a response which is hesitating, than to initiate behavior.[26]

The cry is only a rudiment of language, but when it is studied in this way, it shows a relationship between language and action in the animal or primitive human group. This may be expressed by saying that *language is the result of an inhibited tendency to action in a social situation.*

[24] See G. P. Conger, *Course in Philosophy*, p. 463 *f.*
[25] G. A. de Laguna, *op. cit.*, p. 81.
[26] *ibid.*, pp. 77 *f.*

When we study such a situation in more detail, we find that the inhibition of such a tendency is only a part of what goes on, for, after all, there is a response by the leader. The response which the leader actually makes is the cry. This response need not be the first occurrence of the cry, which, as we said, may very well begin as a mere overflow response, without any necessary social significance. But in the example we are considering, as in the simple example of the cry of alarm, when the primary response is inhibited, the cry, originally loosely and even accidentally associated with it, is left as its representative. Hollingworth says that if a sensory item or image has been sufficiently well associated with a relational situation, there is no reason why the part should not function for or stand for the whole, and this is the basis for all representation.[27] Such substitutions are familiar enough in the work on conditioned reflexes. For Markey, there is no sharp dividing line between simple and complex conditioned responses.[28] A language habit is to be partly understood as an action stimulus which is a substitute for, or symbolic of, a bodily habit or process.

Now the cry as *uttered* combines something of the two tendencies conflicting in the leader; even the inhibited tendency to attack the quarry appears in a way in the resultant response. The attack is made, but it is made in accordance with the peculiar social status of the leader. The primary response of attack is inhibited in the leader, but by means of the cry, which probably was originally associated with it in a secondary way, the primary response is transferred to the followers, who do the actual attacking. Janet thinks that right here occurs the separation of the word from the act which it originally accompanied, since the word exists in the individual who gives the command, and the act in the one who obeys.[29]

For the followers, the leader's cry is a new stimulus, either first inducing them to be followers, or else at any rate reinforcing their primary tendencies to attack the quarry and at the same time inhibiting their other tendencies (*e.g.*, to scatter their efforts, or perhaps to play the leader, etc.), thus enabling the primary tendency to attack the game to prevail. At such rudimentary stages, the process may differ in leaders and followers. In the leader there are primarily inhibitions, with conflict of tendencies and a resultant which partakes of the nature of both tendencies; in the followers, while there

[27] H. L. Hollingworth, *Psychology of Thought,* 1926, p. 153.
[28] J. F. Markey, *The Symbolic Process . . . ,* 1928, p. 112.
[29] P. Janet, *De l'angoisse à l'extase,* pp. 219 *f.*

may be inhibition, the essential is rather reinforcement. But later these differences are equalized, as the followers develop capacity to respond to the leader, by answering cries.

7. *The basic features of animal cries are retained in primitive and even in developed human languages.* Lorimer says that symbolic activity never replaces the operation of presymbolic processes, but supplies structures in part new, and in part the explication of prior symbolic processes.[30]

Among the apes there is a considerable range of different sounds which are ordinarily used in the expression of the emotional attitudes, but authorities differ on the question whether these are connected with specific objects and situations so as to constitute a real language. Furness succeeded in teaching an orang-utan to say "papa" and "cup," and to use the words in connection with pleadings and spontaneous needs. It appears from the work of Yerkes that chimpanzees use approximately the same sounds to indicate the same emotions and specific conditions, but that this is altogether on the emotional level and can not be detached from the special setting in which the words developed, nor used as signs for objects and experiences not actually present.[31] It has sometimes in fact been held that the difference between animals and men in this respect is that men give to a sign an objective value, separating it from the object which it signifies.[32] Mrs. de Laguna thinks that speech became more flexible when the ancestral species came down from the trees, where the problem of shelter was worse, and where the young had to be guided and guarded more carefully.[33]

In the development of human language, there is likely to be more and more inhibition; the response is often noticeably delayed, apparently by inhibition and the substitution of symbols for acts and objects.[34] The rôle of inhibition has been noticed in the study of child symbolism. Lorimer says that the child, in the very beginning of his symbolic activity, hesitates in the presence of a familiar situation. Inhibitions of symbolic activity remain for a considerable period implicit and ineffective, but it is a great mistake to neglect their reality. The beginning of explicit symbolic reinforcement and the symbolic inhibition of tentative behavior is evidenced in the growth of the use of the affirmative "yes" and the negative "no."[35]

[30] F. Lorimer, *op. cit.*, pp. 151 *f.*

[31] W. B. Pillsbury and C. L. Meader, *op. cit.*, pp. 124 *f.*

[32] J. Vendryes, *op. cit.*, p. 11.

[33] G. A. de Laguna, *op. cit.*, pp. x, 62 *f.*

[34] J. F. Markey, *op. cit.*, p. 137.

[35] F. Lorimer, *op. cit.*, p. 134.

On the other hand, the process of conditioning is often furthered and the use of language facilitated by the fact that a child's babbling or cooing acts as a circular reflex between the sound of the syllable and the response of speaking it.[36]

In human as in subhuman groups, language develops in end-reaction complexes. By their substitution as symbols, words gradually become central in the organization of memories and the anticipations of future events. A symbolized intention may be labelled a purpose, and, if socially reinforced, an ideal, in contrast to the fluctual intentions and wishes of unverbalized personality organization.[37] But all this involves a great advance of the human over the subhuman groups, as is evident in the differentiation of the parts of speech.

8. *The most conspicuous difference between the language of animals and that of men is that the latter is articulated, and tends to take the form of the grammatical sentence with "parts of speech."* It appears that in this respect all human languages are essentially alike. Malinowsky says that in spite of great divergences, they show a certain fundamental agreement in structure and in means of grammatical expression. Language in its structure mirrors the real categories derived from the practical attitudes of the child and of primitive and natural man to the surrounding world. The fundamental outline of grammar is due mainly to the most primitive use of language, and the categories derived from that primitive use are identical for all languages.[38] We must, however, note the caution of Pillsbury and Meader, that if we attempt to distinguish the parts of speech on the basis of their form, we meet serious difficulty. There are, for example, languages which show no distinction between substantives and verbs, or between substantives and adjectives.[39] The proper distinctions seem to be not those of form, but of function.

The rival claims of the sentence and its components as to which is the primary unit of language are after a fashion adjusted by the theory of Vendryes, for whom the verbal image or the phonetic word, just because language is originally an action, possesses the value of a sentence. The various parts of speech are derived from them; thus the sentence, which may itself be a word, precedes the grammatical word.[40] Pillsbury and Meader maintain that from the point of view of the expressive movements involved, there is noth-

[36] J. F. Markey, *op. cit.*, p. 30.
[37] F. Lorimer, *op. cit.*, pp. 80, 82.
[38] B. Malinowski, *op. cit.*, pp. 496-8.
[39] W. B. Pillsbury and C. L. Meader, *op. cit.*, p. 270.
[40] H. Berr, in J. Vendryes, *op. cit.*, p. xi.

ing fragmentary about the one-word sentence; all the movements essential to the expression of an idea in a given situation are executed, and neither type of expression—the uttered word or the other expressive movements—performs its function more or less completely than the other.[41]

When we come to analyze the sentence-forms, we encounter the parts of speech, with the problem of their development and relationships. It has sometimes been held that they are not important; for example, in Chinese a word may be employed indifferently as one or another part of speech.[42] In accordance with his philosophy of language, Brunet holds that the classification by parts of speech should not be retained and that the methods of language study should be drawn up on a basis of ideas.[43] Pillsbury and Meader maintain that the parts of speech, like other grammatical forms, are abstractions of philosophers and grammarians; they are only statements about language, and not parts of language itself.

The authors last named seek to discern the states of mind on the basis of which the parts of speech were formulated; this gives what might be called a psychological theory—conceived, it might be added, in terms of individual psychology. We may, they say, designate a coherent and unified group of sensations, with their attendant feelings, as a substantive. If now we abstract an element or group of elements from this mass, and still conceive of it as characteristic of the mass, we shall have a quality. On the basis of sensations, we conceive ideas of relationship. In general, permanent qualities are expressed by adjectives, transitive qualities by verbs, and relationships by particles, such as adverbs, conjunctions, prepositions, numerals, and some pronouns.[44]

Washburn assumes that all the nameable relational or imageless processes are based on fusions of kinaesthetic excitations. The feeling of "but" can be traced to the excitation of certain incompatible movements.[45]

On the basis of Janet's theory, it is possible to find a more conspicuously social interpretation of the development of the parts of speech. For Janet, the original form and, one might say, matrix of language is the cry of command or the imperative. Language, as we

[41] W. B. Pillsbury and C. L. Meader, *op. cit.*, p. 256.

[42] W. D. Whitney, *The Life and Growth of Language*, 1875, p. 238; see J. Vendryes, *op. cit.*, pp. 117 *ff.*

[43] See C. K. Ogden and I. A. Richards, *op. cit.*, pp. 368 *f.*

[44] W. B. Pillsbury and C. L. Meader, *op. cit.*, pp. 263, 268 *f.*

[45] M. F. Washburn, *op. cit.*, pp. 199, 202 *f.*

saw, develops among the social tendencies, and the conduct of the man who speaks and of the man who is spoken to have developed out of acts of command and obedience which exist among the animals.[46]

It is true enough that, as Janet says, in such a situation the cry or word becomes in a sense separated from the act; the one belongs to the leader and the other to the followers.[46] But in another sense, the action and the object to be acted upon are still immediately present, both to leader and followers. Some of the parts of speech seem to originate here, while the need is for discrimination and analysis of situations actually present, before it is necessary to consider absent objects and actions distant in time. Other parts of speech seem to fit the other type of situation. Thus Mrs. de Laguna holds that the sentence evolves through the need for coordinated action beyond the limits of the common perceptually present situation, and the closely related need for coordinating the complex and varied behavior which depends upon an analysis of the situation.[47]

So long as all the objects referred to are present and the action referred to is going on, rudimentary cries are all that are necessary to direct the attention which is already partially fixed. Such rudimentary cries in the presence of objects correspond, for example, to what we know as demonstrative pronouns, and such cries in the course of actions going on probably correspond to imperatives and also to interjections. It is admitted that of all language sounds interjections are nearest to instinctive utterance, although of course the interjections of developed and especially of literary language are only superficially of instinctive nature.[48] With regard to their early development, Mrs. de Laguna thinks that inhibitory ejaculations come to qualify vaguely the object or situation to which they are applied, and hence are predicative in rudimentary fashion.[49] Such a view of interjections, in their natural setting, so to speak, renders very artificial the over-grammatical view of Vendryes, that the interjection is inherently different from other parts of speech and can not be put into the same classification.[50]

Probably sounds corresponding to the later adjectives and adverbs are also developed in the presence of objects and in the course of actions to which they refer. As group life develops more and more highly differentiated needs, it becomes necessary to specify

[46] P. Janet, *op. cit.*, pp. 217, 220.
[47] G. A. de Laguna, *op. cit.*, p. 94.
[48] E. Sapir, *Language, An Introduction to the Study of Speech*, 1921, p. 5.
[49] G. A. de Laguna, *op. cit.*, p. 102.
[50] J. Vendryes, *op. cit.*, p. 115.

actions and objects acted upon in more and more detail. This is an added reason for differentiations of parts of speech and nuances of meaning. Mrs. de Laguna says that if the object is present but not recognized, for example, as dangerous, the cry serves to call attention to a property of the object more than to the existence of the object itself.[51] Apart from the object, of course, the expression of a quality would be unmeaning.[52] For Malinowski, adverbs seem to mark the utilitarian characteristics of an object.[53] Again, Mrs. de Laguna, emphasizing the inhibitory function of early language, thinks that predication developed because of the need of an instrument for inhibition, the checking of certain parts of an activity. It may not be the act as a whole which needs to be checked. In this case, the inhibitory announcement must specify the feature of the situation, or the respect to which the qualification attaches and to which attention is called. In this way arises a predication more explicit than that of the interjection.[54] For us, it is still only a rudimentary predication, since predication comes to its full development only with the appearance of nouns and verbs. But such a theory of the origin of adjectives and adverbs is assuredly more natural than that of Pillsbury and Meader, who, from the point of view of the psychology above mentioned, say that just as a verbal idea is derived by abstraction from a substantive, so a modal adverb is the result of a second abstraction, this time from the already abstract verbal idea. They add, however, that the adverb represents a quality even more temporary than that of the verb.[55]

Sooner or later, by further processes of conditioning, cries come to refer not to objects that are immediately present, and to actions in which the members of the group are engaged, but to objects that are absent, and to actions that are distant in time, past or future. Thus the alarm cry of a sentinel, warning the other members of the group of a source of danger which they do not yet see or hear, elicits from them reactions appropriate for the actual perception of the object. Or, among primitive men, the cries accompanying warfare, when used retrospectively or prospectively, may call forth responses appropriate to actual warfare.

[51] G. A. de Laguna, *op. cit.*, p. 77.

[52] W. Wundt, *Elements of Folk Psychology,* transl. E. L. Schaub, 1916, p. 64.

[53] B. Malinowski, *op. cit.*, p. 505.

[54] G. A. de Laguna, *op. cit.*, pp. 78 *f.*, 101 *ff.*

[55] W. B. Pillsbury and C. L. Meader, *op. cit.*, p. 270.

This conditioning, whereby words come to be used in place of absent objects or distant acts, appears to be the key to the further development of the fully articulated sentence, with its differentiations of the principal parts of speech. Language thus comes to include nouns, or signs indicating objects, which if present are more clearly specified, and if absent are referred to as such. Among savages, according to Malinowski, a word signifying an important utensil is used in action not to comment on its nature or to reflect on its properties, but to make it appear, or to direct another man to its proper use. The primitive category of "isolation" of persons, animals, or things against an undifferentiated background requires articulate sounds to specify its various items. The class of words used for naming persons and personified things forms a primitive grammatical category of noun substantives. Thus this part of speech is seen to be rooted in active modes of behavior. For a child, too, the name of an object is the first means recurred to in order to attract, to materialize the thing.[56] The interpretation is more concrete than that of Mrs. de Laguna, that names come to denote objects because objects come to figure in more than one situation and in various situations play different rôles, but that by means of the name, whatever is individual and subjective gets weeded out.[57] This is true enough, but it marks the development of abstractions.

The tracing of nouns to objective situations which require varied actions makes natural the development of the various grammatical cases. Incidental to these developments is probably the appearance of prepositions, used to express relationships between things. On the other hand, there seems to be no rational basis for grammatical gender. In European languages it represents a mental attempt to classify exceedingly varied ideas expressed by nouns. The principle of classification doubtless corresponds to the conception of the world entertained by our remote ancestors, and mystic and religious motives contributed towards its stabilization.[58]

Where speech is associated with concerted behavior, the speaker stands in the foreground, but of course must continually refer to hearers, either present or absent. These facts afford the clue to the development of personal pronouns. These are short words, intimately connected with actions and with verbs, but similar in grammatical nature to nouns.[59]

[56] B. Malinowski, *op. cit.*, pp. 488, 502 *f.*

[57] G. A. de Laguna, *op. cit.*, pp. 94 *f.*

[58] J. Vendryes, *op. cit.*, pp. 92, 96.

[59] B. Malinowski, *op. cit.*, p. 505.

When used with nouns, demonstrative pronouns may degenerate into definite articles. The indefinite article is more abstract. In the Germanic and Romance languages, an indefinite article has been derived from a numerical noun signifying "oneness;" sometimes, too, the word for "man" has become a grammatical instrument serving to express the indefinite.[60]

When adjectives are used with nouns, the attribute or predicate is indicated by some form of the verb, "to be," used as a copula. Such words originally had a more concrete meaning.[61]

The part of speech which is most difficult to trace is the verb. Part of the difficulty is superficial, in the facts that many languages have no specific means for expressing a verb,[62] and that there are languages where noun and verb have no distinct forms; still, at least all developed languages are at one in distinguishing the substantive from the verbal sentence.[63]

Some of the theories about verbs trace them to situations where the object is present and some action or change is going on, and make the verbs refer specifically to such actions or changes. According to Malinowski, there is some reason to suppose that in primitive languages the separation of verbs and nouns reflected the distinction between the actions of the speaker and the objects which surrounded him. At a later stage this division in linguistic material was extensively used to mark the distinction between things and the qualities which belong to them.[64] With regard to the latter distinction, Pillsbury and Meader think it is superfluous to raise the question of priority as between noun and verb. Since the capacity to appreciate an object as moving and as at rest antedates the origin of speech, it follows that as soon as a special motor reaction develops to designate one as distinct from the other, both noun and verb develop simultaneously.[65] According to Wundt, one reason why the verb is so long obscured in language is that in primitive thought change and action are so long overshadowed by the concrete image.[66] Malinowski says that only late in a child's development does the child disentangle the changes in the surroundings from the objects which change.[67] Mrs. de Laguna holds that as the name comes to

[60] J. Vendryes, *op. cit.*, p. 165.

[61] W. B. Pillsbury and C. L. Meader, *op. cit.*, pp. 267 *ff*.

[62] W. Wundt, *op. cit.*, p. 72.

[63] J. Vendryes, *op. cit.*, p. 120.

[64] B. Malinowski, *op. cit.*, p. 407.

[65] W. B. Pillsbury and C. L. Meader, *op. cit.*, p. 282. F. Lorimer, *op. cit.*, p. 105, emphasizes action in his theory of "fluctual contexts" which characterize the verbal concepts.

[66] W. Wundt, *op. cit.*, p. 72.

[67] B. Malinowski, *op. cit.*, p. 504.

denote the object as such, in distinction from the acts which center in it and the qualities which it possesses, its use must be supplemented by that of a term capable of specifying the particular act demanded by the occasion—*i.e.,* by a verb or predicate adjective. Verbal terms do not at first denote acts as such, but acts performed on or in connection with specific concrete objects.[68] Thus a verb receives its meaning through participation in action.

It is doubtless true that many forms of the verb developed, like the imperative, in the presence of objects and in the course of actions which needed to be carefully specified in their details. But the "full-fledged" verb in the indicative mood, and with it the full-fledged declarative sentence, could hardly have developed except with reference to actions viewed more objectively. Possibly, as in the case of a chant accompanying a ceremony, the sentences were used in a quasi-objective description of the act, but by far the more usual procedure must have been to refer in independent fashion to actions past or future. The essential separation of the declarative sentence from any current action is reflected in the fact that this kind of sentence does not tend to elicit any single type of response.[69]

Various forms of the verb, representing differences in actions specified and in ways specified to perform them, may all be regarded as having differentiated out of a primitive matrix. Malinowski thinks that at the primitive stages of human speech there must have existed a real category into which entered all items of change capable of temporal modification, bearing the character of human mood and human will, and bound up with the personal action of man. The class of words used to denote items of this real category —the action word or verb—is capable in all languages of grammatical modifications expressing temporal relations, moods, or modes of utterance.[70] Thus we have various voices,[71] moods,[72] tenses,[73] persons,[74] and numbers,[75] as well as verbs used as auxiliaries and

[68] G. A. de Laguna, *op. cit.*, pp. 91, 98. [69] *ibid.*, p. 38.

[70] B. Malinowski, *op. cit.*, p. 505.

[71] See J. Vendryes, *op. cit.*, p. 102; G. A. de Laguna, *op. cit.*, pp. 105 *f.*; W. B. Pillsbury and C. L. Meader, *op cit.*, p. 201.

[72] See W. B. Pillsbury and C. L. Meader, *op. cit.*, pp. 279 *f.*, and F. Lorimer, *op. cit.*, p. 107. On the subjunctive, see W. G. Hale, in H. J. Rogers, ed., *International Congress of Arts and Sciences, St. Louis,* Vol. 3, 1906, p. 192. On the interrogative, see G. A. de Laguna, *op. cit.*, pp. 37, 106.

[73] W. B. Pillsbury and C. L. Meader, *op. cit.*, pp. 273, 290 *f.*; J. Vendryes, *op. cit.*, pp. 98 *f.*, 151.

[74] *cf.* G. A. de Laguna, *op. cit.*, pp. 277 *ff.*

[75] See W. B. Pillsbury and C. L. Meader, *op. cit.*, pp. 277, 291.

copulas.[75] But the distinguishing of such a category as that of Malinowski seems secondary and over-theoretical.

9. *Thus gestures and native vocal sounds are gradually replaced by signs, especially by the sounds of articulate language.* Language remains at best only a partial abstract of the concrete situation.[76] Its development is uneven and shows plainly the influence of emotive factors. It contains many elements of secondary value and importance.[77] But, after all, the developed sentence may be grasped as a unitary conception,[78] and the development of language is marked by a nervous organization, complex and flexible, which enables man to reassemble on a higher level the elements of simpler and now inadequate responses. To speak in complete sentences, says Mrs. de Laguna, is to act in a new way, which is far more independent than the utterance of the single sentence-word.[79] The change is so far reaching that it may even seem to belong in a new realm. Thus Piaget says that insofar as the child seeks to adapt himself to others, he creates between himself and them a new order of reality, a new plane of thought, where speech and argument will henceforth hold their sway, and upon which operations and relations, which till then have been the work of action alone, will now be handled by imagination and by words.[80] Whether or not they belong in a new realm, the relationships obtaining between parts of the sentence can be projected or objectified, and used in interpreting the relationships of objects to one another.[81] Thus the object referred to as acting may become the subject of a sentence, and other features of a situation may be reflected in other elements of the sentence.

10. *In its physical and mechanical aspects, language is marked by the discharge of nervous energy of the speaker into a muscle, with consequent contraction and some ensuing change in the environment.* Any muscle may be thus employed, but the muscles most often used are those of the arms and hands, in gestures, and, as we saw, those of the vocal apparatus in cries and articulate speech. The contraction of the muscle, which belongs in the biotic realm, sets up some change in the cosmogonic realm, or the environment—a change in the reflected light, in the case of the gesture, and a change in the air waves, in the case of the cry or speech. At least

[76] *ibid.*, p. 176.
[77] C. K. Ogden and I. A. Richards, *op. cit.*, pp. 197, 235.
[78] W. B. Pillsbury and C. L. Meader, *op. cit.*, p. 261.
[79] G. A. de Laguna, *op. cit.*, pp. 51, 277.
[80] J. Piaget, *Judgment and Reasoning in the Child*, p. 213.
[81] *cf.* H. L. Hollingworth, *Psychology of Thought*, pp. 154 *f.*

a portion of this environmental change is thereupon transmitted across the intermediate environment to the receptors, visual or auditory, of other members of the group. At least at the present stage of human development, other languages, so called, appear to be founded upon the auditory.[82] In the experience of the listeners, the physical change serves as a sign or symbol, the clue for nervous organizations, which are at least in some respects similar to those of the speaker.

These physical changes in the environment are essentially transitory; the sound of the voice, like music, fades upon the empty air. In the course of human development, however, it comes to be important that some physical changes shall be enregistered more permanently in the environment, so as to be available for signs or symbols over longer intervals of time or space. Thus we have the beginnings of recorded, and eventually of written language.

The starting point was quite likely in the practices of imitative magic, where control over the image of an object was held to give control over the real object in real life. Hence the carefully treasured sacred images, which so many primitive peoples regard as endowed with magical or miraculous powers, mark the beginnings of symbolism in religion. Immense progress, says Vendryes, was achieved when men learned to draw and to make the picture the emblem of the object.[83] No evidences of this stage of development are more amazing than the figures of animals which prehistoric men in southwestern Europe etched and painted on the walls of deep caves, where the figures would be secure and permanently available for use in their ceremonies. On the figures of some animals there appear to be certain signs, and in other caves on figures of animals of the same species there are similar signs—as if in addition to the picture, there were certain associated designations which were held to be peculiarly appropriate. It needs only a simple process of conditioning to make such signs stand for the painted figure and for the real animal. This avenue toward the development of writing, like so many other avenues which began to be opened by rather gifted prehistoric races, was apparently closed again in the general disaster which overwhelmed such cultures. The development of writing took the longer course and came by way of picture languages and hieroglyphics.

Picture writing, of course, tries by representing men and objects in various postures and relationships, to record and convey ideas of

[82] *cf.* J. Vendryes, *op. cit.*, p. 60.

[83] *ibid.*, p. 320.

action. It is vivid, but at best clumsy and incomplete, and just as, in the development of increasingly complex group life, spoken language requires sounds indicating increasingly precise modifications of actions, so written language seeks more and more refinement and precision. For refinement and precision, sound is more flexible than picture writing, and doubtless for ages in human history competes with it. But eventually, as in so many cases of cultural conflict, a compromise or combination is effected and, as Vendyres has it, the sign which at first represented the object becomes the sign of the sound representing the object, and we have phonetic writing. At an early stage, ideograms are used for the first syllables of the words they represent; from the syllabary of this stage the alphabet is said to develop when the need arises for indications of vowel signs. In the Semitic names of alphabet letters the names of objects are still discernible.[84] With ever more intricate developments and more precise modifications of actions and experiences, fresh combinations of sounds and corresponding letters are formed from the established roots, and written language evolves in all its combination of subtlety and awkwardness.

11. *It is obvious that language involves the social use of signs and symbols, but the theory of symbols exhibits a number of differing emphases.* Among recent writers, Ogden and Richards use the term "symbol" to designate words, arrangements of words, images, gestures, and such representations as drawings or mimetic sounds. These authors, as we saw, distinguish between such symbolic uses of words and what they call the emotive uses.[85] Whitehead makes symbolism virtually central in his epistemology. The human mind is functioning symbolically when some components of its experience elicit consciousness, beliefs, emotions, and usages respecting other components of its experience. For instance, projected sense data, in symbolic transference from the perceptive mode of presentational immediacy, serve in the analysis of the massive perception of causal efficacy into its components, with resultant locations in space. The development of symbols is of great importance for social stability.[86] Lorimer thinks that Whitehead's definition is either too loose or too narrow; his own definition is that a symbol is an item established in social conduct or reflective thinking as a functional substitute for

[84] J. Vendryes, *op. cit.*, pp. 147, 323, 327 *f.*

[85] C. K. Ogden and I. A. Richards, *op. cit.*, pp. 13, 30.

[86] A. N. Whitehead, *Symbolism, Its Meaning and Effect*, 1927, pp. 7 *f.*, 56, 66, 80.

certain other items in social or individual behavior. This definition is designed as an improvement also upon that of Markey, for whom gesture and sign situations form a basis for symbols, and a symbol may be defined as an act or object which is marked off by behavior as a substitute for a stimulus-act or -object and a response-act or -object, and which is also at the same time set off by behavior as different from them. With regard to Markey's view, Lorimer says that it confuses the words as symbols and the explicit recognition of the symbolic character of a word.[87]

Vendryes says that psychologically the original linguistic act consists in giving to a sign a symbolic value, and that when man has given to signs an objective value, in other words, has detached the sign from the thing signified, he is able to vary the sign or symbol indefinitely by convention.[88] This definition introduces the factors of objectivity and conventionality; in accordance with it, symbolism appears to include elements of a process of substitution, of separation, and of conventionalization. Markey brings out the fact that symbolic behavior is not limited to verbal language; when once symbols have arisen in behavior, practically any act or object may become symbolic in character.[89] Lewis and Langford notice that there will be, in the case of any symbol, only a small number of aspects selected to represent what is meant by the symbol, and an enormous variety of aspects which are to be neglected.[90] With increasing complexity of experience, symbols of symbols are developed.[91] The problem of symbols is closely allied with that of meanings, which will be considered in section 15.

12. *Thus the anticipatory bodily adjustment to the outcome of an act is replaced by the anticipatory sign or symbol, which tends to elicit a more or less concerted action of the group.* The sign, especially if it be a verbal response, constitutes a specific response to what is objective. Such a response, whether uttered or otherwise expressed, has grown up as an instrument for controlling response in others, and is to the hearer a conditional determinant of response.[92]

Thus language occurs in end-reaction or purposive complexes, with emotional accompaniments. In its rudimentary stages it indicates that the leader intends to act, or to have the followers act in

[87] F. Lorimer, *op. cit.*, pp. 53, 87; J. F. Markey, *op. cit.*, pp. 28, 161.

[88] J. Vendryes, *op. cit.*, pp. 11 *f.*

[89] J. F. Markey, *op. cit.*, p. 105.

[90] C. I. Lewis and C. H. Langford, *Symbolic Logic*, 1932, pp. 311 *f.*

[91] F. Lorimer, *op. cit.*, p. 121.

[92] G. A. de Laguna, *op. cit.*, p. 299.

more or less definitely specified ways. It need not be supposed that all rudimentary or primitive language was concerned with serious problems. Mrs. de Laguna says that adaptation of the play activity to the needs of social coordination was the essential agency in the process of speech.[93] Jesperson suggests that possibly all primitive speech was song.[94]

In the later stages of language, where sentences take the declarative form, the end or purpose of the speaker is frequently more intricate and remote, and what is said is likely to express more or less devious means of attaining it. Still, it is not too much to say that it is to the definite announcement of intention that primitive conversation naturally leads up,[95] and even highly developed conversation tends to prepare the way.

At any stage of development, the announced intention of the leader or speaker helps to organize the end-reactions and purposes of the followers or listeners. Between the individuals of a group there is a kind of induction of purposive patterns. This fact may serve to correlate all of what Ogden and Richards call the five functions of language—the symbolization of reference, the expression of an attitude to the listener, the expression of an attitude to a referent, the promotion of effects intended, and the support of reference.[96]

The words of a speaker tend to organize the experience of a listener in end-reactions, sentiments, values, etc., or, more frequently, to mesh into an organization already incipient in the listener. In fact, as Lorimer brings out, the relationship of symbol to context is typically an organic relationship, whereby the arousal of a context tends to evoke the symbol, and the symbol when enacted tends to arouse its implicit context in the behavior of the organism, especially its intellectual processes.[97] Purposes vaguely held, as it were, in solution tend to crystallize around words. Values tend to depend on propositions.[98]

In proportion as there is likemindedness, harmony, or understanding among the members of the group, the purposes of the hearers tend to correspond to, if they are not identical with, those of the speaker. Where a language is understood, the sound groups enunciated by the speaker awaken in the mind of the listeners cor-

[93] G. A. de Laguna, *op. cit.*, p. 72.
[94] O. Jesperson, *op. cit.*, p. 420.
[95] G. A. de Laguna, *op. cit.*, p. 305.
[96] C. K. Ogden and I. A. Richards, *op. cit.*, p. 360.
[97] F. Lorimer, *op. cit.*, p. 88.
[98] S. Alexander, *Space, Time, and Deity*, Vol. 2, p. 295.

responding groups of representations, each of which constitutes, grammatically speaking, a sentence.[99] If the understanding between speaker and listener is defective, either listening or speaking, or both, may result in clarification; as Woodbridge puts it, ideas gain in clearness by *dialectic,* by being "said through."[100] This process goes on until we may, with Sellars, assume that in the end the speaker and the listener have the same proposition in mind; that is, that they have logically identical ideas, although not psychologically identical ideas.[101]

13. *In the development of human groups, a factor of very great importance appears when the intention of a leader is announced in advance, in the form of a promise.* A promise is a social expression, usually in some form of language, of a purpose which has been formed by an individual leader or speaker, but not yet actually accomplished. This expression of the purpose leads the other members of the group to expect its accomplishment and to hold the individual responsible for it. A voluntary promise, too, involves an effort; the purpose is, we might say, brought to expression forcibly at a time when certain other tendencies might have repressed it. At later stages sometimes the whole complex of tendencies which we call the self may be cooperating in the expression. The purpose is brought to expression antecedently, before the end-reaction which it implies can actually be effected.

An individual who makes a promise typically says "I will (do thus and so)," and then proceeds to organize his future actions and experiences in accordance with the promise as thus made in advance. His future experiences may contain features very different from any which marked the original expression of the promise. But when a promise has once been made, it normally remains the essential organizing factor throughout all later experience until its fulfilment. In one sense a promise is superior to the reactions which fulfil it, for it dominates their organization, and they are committed to a course which it defines in advance. In another sense a promise is not to be compared to its fulfilment, for it is a mere word or two, whereas its fulfilment may take a lifetime.

The consequences of a promise are notable not only in the experience of the individual who makes it, but also in the experiences of other members of the group. The mature persons in any

[99] J. Vendryes, *op. cit.*, p. 64.
[100] F. J. E. Woodbridge, *The Realm of Mind,* 1926, p. 70.
[101] R. W. Sellars, *Essentials of Logic,* 1925, p. 78.

group do not "pay much attention" to the prattle of children. Legally, no one is obliged to "take the word" of a minor. But let the child, or the minor, become responsible—*i.e.*, capable of making a promise and being held to its fulfilment—as he does when he is initiated or comes of age, and the members of the group will be responsive. They will pay attention, or take the word of the individual, who thereupon comes to be recognized as a person. The new person, by expressing his ideas and purposes in accordance with his previously expressed or implied promises, will have more influence upon the end-reactions, purposes, sentiments, values, etc., of the other members of the group. Thus the ideas and purposes of an individual are gradually brought to conform to those of other individuals; they attain social currency and acceptance, and come to be approved as standard. And in this growth of responsibility and responsiveness, the individuals of a society mutually develop one another as persons.[102]

It must have been obvious already that several problems which we have considered involve not only language but thinking. Closely related to some of them are several others, which, besides being on the border line, involve intricate relationships between several epitomizing realms. Detailed discussion would be too long; with bare mention of some of the literature of the subjects, we shall append brief comments.

14. *Several attempts to account psychologically for the structure of the sentence, proposition, or judgment seem to need a broader basis in cosmic principles.* This applies to Washburn's analysis of the sentence in terms of movement systems;[103] to Hollingworth's attempt to identify parts of the sentence in terms of significance, etc.;[104] and to Lorimer's tensional nexus of intellectual activity between terms.[105] Some features of the broader basis will be indicated in subsequent sections.

15. *The much-discussed term "meaning" may be understood in a baffling variety of ways.* This appears from the exhaustive list of sixteen "meanings of meaning" given by Ogden and Richards,[106] and from the five classes of theories treated by Hollingworth, as well as from his own analysis.[107] Each analy-

[102] See G. P. Conger, *Course in Philosophy*, pp. 326 *ff.* On the promise, *cf.* P. Janet, *op. cit.*, pp. 221 *f.;* on the contract, *cf.* J. Dewey and J. H. Tufts, *Ethics*, 1908, p. 345.

[103] M. F. Washburn, *op. cit.*, pp. 175 *f.*

[104] H. L. Hollingworth, *op. cit.*, p. 289.

[105] F. Lorimer, *op. cit.*, p. 102.

[106] C. K. Ogden and I. A. Richards, *op. cit.*, pp. 306 *f.*

[107] H. L. Hollingworth, *op. cit.*, pp. 212 *f.*, 215, 289. *cf.* J. M. Baldwin, *op. cit.*, Vol. 1, p. 185, W. B. Pillsbury and C. L. Meader, *op. cit.*, p. 160.

sis and theory in its own way makes clear that "meaning" may have many meanings. For example, it may be understood as a psychological process, variously constituted of part processes or elements. The process, as a meaning, may refer to something positively, by inclusion, or negatively, by exclusion. It may refer to something subjective, but also, sooner or later, to something objective. Either or both of these referents can be called meanings, and can be either antecedent to the psychological process, as its context, or consequent upon it, as its outcome. The psychological process may be interpreted in terms predominantly individual or social. And of the subjective or the objective referents, either or both can be existent or subsistent, and they may be in complicated relationships.[108]

With such an appalling list of alternatives it is no wonder that many theories of meaning flourish. In fact, the various theories cover, or may be said to cover, the whole range of adjustments of a thought-pattern to the remainder of the cosmos, and so, eventually, the problem of truth. The questions are largely matters of descriptive detail which may vary with different meanings. Probably any complete case of a meaning involves them all.

16. *Language is vastly broadened and enriched by abstractions, generalizations, and metaphors.* The first two shade into one another and often are not carefully enough distinguished.[109] Some of their important features will be considered in Chapter X. The picturesque transferences of meaning common in metaphors[110] involve the same principles.

As development proceeds, various rather definite hierarchical organizations of experience appear as significant abstractions of abstractions.[111] But, in the words of Malinowski, as language and thought develop, the constant action of metaphor, of generalization, analogy, and abstraction build up links between the categories and obliterate the boundary lines, thus allowing words and roots to move freely over the whole field of language.[112] In the course of ages many metaphors have lost their freshness and vividness.[113]

According to the hypothesis of epitomization, the cosmic status of abstractions and generalizations is in principle like that of syllogisms, which is discussed in Chapter V, E (VII).

[108] See G. P. Conger, *Epitomizations*, index, "refraction."

[109] See G. A. de Laguna, *op. cit.*, pp. 94 *f.*; E. Rignano, *Psychology of Reasoning*, p. 111; H Berr, in J. Vendryes, *op. cit.*, p. xvii; C. K. Ogden and I. A. Richards, *op. cit.*, p. 342.

[110] C. K. Ogden and I. A. Richards, *op. cit.*, pp. 210, 342 *f.*

[111] F. Lorimer, *op. cit.*, p. 121.

[112] B. Malinowski, *op. cit.*, p. 509.

[113] *cf.* O. Jesperson, *op. cit.*, p. 432.

17. *In connection with the general position here taken, we should consider briefly the problem of the cosmic status of words and sentences.* We find, in a detailed investigation too long to include here, that words, which are accorded no special place as monads in the epitomization hypothesis, in their structures and changes exhibit most, if not all, of the monadic characteristics. The question then rises, to what level and realm they are to be ascribed. The distinctions made by Vendryes between verbal images, semantemes, morphemes, and phonemes[114] suggests that words may be studied from different points of view. They appear even more complicated and many sided when studied in the framework of the realms and levels of epitomization. Thus a word, or word in a sentence, involves social processes, as evidenced in the normative character of the science of grammar. It involves personal and valuational processes, as evidenced in facts of assent, assertion, denial, etc. It obviously involves the processes of neuropsychological patterns, with their stimuli, coordinations and responses. It involves physiological processes, since it is expressed or articulated through bodily mechanisms. It involves physical processes with its expression in sound waves or the more substantial media used in writing. It involves chemical processes, since the discharge of the mechanism uttering it is at least accompanied by the giving off or carbon dioxide. It has spatio-temporal properties in its length, accent, rhythm, etc. It has numerical properties in its grammatical number, and logical characteristics in its status in, or as, the syntactical framework of a proposition.

With all this variety of connections in various realms and levels acknowledged, it is useless to try to confine the monadic characteristics of a word or sentence to any one level. The process of refraction between realms[108] may cause additional confusions here, so that what belongs properly in one realm may appear only as refracted in the medium of some other. On the whole, it seems most adequate to say that words do not constitute a level[115] by themselves, but exhibit their monadic characteristics chiefly because of their status as pattern-reactions. But pattern-reactions do not occur in a vacuum; they are among the most highly coordinating processes of the cosmos, and are in intricate relationships with the monads of many other levels and realms.

114 J. Vendryes, *op. cit.*, pp. 20, 65, 74.
115 See G. P. Conger, *Epitomizations*, pp. 272 *f.*

CHAPTER IV

The Origin and Development of Thinking

"It has only been since the seventeenth century that the description and theory of thinking have ignored or underestimated the relevancy and significance of language, of grammar, and of rhetoric.

—M. J. Adler, *Dialectic,* 1927, p. 5.

Closely allied with the problems of language are those of the origin and development of thinking. Concerning the latter, we may say that there are six chief types of theory. In spite of their overlapping features, we shall consider (1) image theories of thinking; (2) motor theories; (3) theories of "pure thought;" (4) theories of thinking as conflicts, or the result of conflicts or inhibitions; (5) theories of thinking as derived from language; and (6) theories of thinking as involving meaning.

(1) According to Pillsbury and Meader, images, or the persistent remnants of sensations, visual, auditory, or kinaesthetic, are often in evidence as one thinks, and frequently seem to constitute thought.[1] Titchener held that the meaning of an idea or mental content is to be found in mental imagery of the concrete sort. Such ordinary images served to explain even ideas of relations, etc., which other investigators ascribed to processes other than imagery. Thus Titchener declared that "togetherness" was for him represented by the image of a rubber band.[2]

In some arguments against the image theory, it is admitted that images exist, but held they can not account for thinking. For example, it is said that images are usually concrete, and not general enough for thinking. The advocates of the image theory reply that thought is not necessarily general. Indeed, any general idea is, as a mental content, always particular. According to the meaning theory, considered below, thinking is general, not because of what it is, but because of what it does. Again, it is urged that the mental content of images may be vague, a confusion of bare impression, whereas thought should be definite. Against this objection, the

[1] W. B. Pillsbury and C. L. Meader, *The Psychology of Language,* 1928, pp. 152 *f.*

[2] *ibid.,* pp. 156, 160.

imagists reply that the vagueness of ordinary sensory content shows itself primarily in instances in which one has a preliminary awareness or plan that is gradually ordered by reference to definite notions. The rudimentary mental operations often lack clearness, but definite mental content gradually takes the place of the indefinite. Once more, it is said that ideas of relationships, such as are expressed by "but," "if," "and," "greater," "above," etc., are originally not represented in consciousness by any distinct images, although Titchener, as we saw, thought so, and everyone admits that images may be developed later to exemplify, if not to represent, such relational elements. These elements have been called feelings, or it has been asserted that they are elements of consciousness, which must be accepted as immediately given and added to other elements of consciousness. They have been accounted for in terms of "pure thought," having only slight connection with images. Once more, it has been held that thoughts are not to be identified with simple images; images when taken by themselves are treated inadequately, as if they had no meaning.[3]

Against the image theory, the most radical attack has come from Watson, but his position seems now rather isolated. Even in 1914, he admitted that the inclusion of images weakened the claim of the behaviorist, but held that it seemed "wisest to attack."[4] In 1919, Watson had found that he could get along without human images,[5] but in the meantime scores of investigators were studying them, and Watson's attack began to look like that of a motor-minded person whose visual imagery happened to be weak.[6]

Apart from the work of Watson, which is considered in (5) below, the imaginal theory of thinking has had to meet the difficulty that there is "imageless thought." According to Pillsbury and Meader, all careful observers agree that there are certain instances of thought without words or adequate images. Images may be present, but if they are present the thought fulfils exactly the same function as when images are absent.[7] Many conscious processes are not clearly reducible to sensations as elements, but there are various possible explanations for this. For instance, the imageless thinking is perhaps traceable to kinaesthetic or organic sensations or properties of sensations which are not recognized as such, because they

[3] *ibid.*, pp. 153-7.

[4] J. B. Watson, *Behavior*, 1914, pp. 16 *f.*

[5] J. B. Watson, *Psychology from the Standpoint of a Behaviorist*, 1919, p. viii.

[6] T. H. Pear, in *Brit. Jour. Psy.*, Gen. Sec., 11, 1921, pp. 75, 79.

[7] W. B. Pillsbury and C. L. Meader, *op. cit.*, p. 183.

are not referred to definite points in the body. Again, something may be blocking the associative processes; or, thinking may be especially rapid and easy because of condensations of images;[8] or an image may have been reduced in distinctness because a word, and finally a mere reference[9] or some element of implicit language,[10] has come to represent the object. All considerations make it probable that the so-called imageless thought is a development or degeneration from thinking in images, and that the images may in many cases be present but overlooked owing to their vagueness or to the interest in the meaning or the reference. Some of the advocates of the pure thought theories maintain that in most of the more important activities of mental life imageless thoughts are to be regarded as substitutes for images or words.[11]

Several theories of thinking may be described as on the border line between that of images and other views. Hunter concludes from his delayed reaction experiments that there is a type of response which he calls "sensory thought," involving the reinstatement of some recent sensory process but not requiring ideas or images. But a second type of thought, after twenty-five minutes or more delay, does require images.[12] There are various theories of substitution, other than those mentioned. Hollingworth holds that in theories of thinking our main concern is not with images but with certain substitutes for them.[13]

(2) The theories which trace thinking to images are often phrased in sensory terms but do not differ much in principle from the motor theories. According to Holt, thinking is a labile interplay of motor settings which goes on almost constantly and which differs from overt conduct in that the energy involved is too small to produce gross bodily movements.[14] Washburn thinks that possibly the original function of tentative movements was to try a movement before fully executing it, and out of this has grown the function of tentative kinaesthetic excitations in producing ideas. It is highly probable that the cortex is the organ for tentative movements. The peculiar motor reactions occasioned by differences in behavior of a move-

[8] M. F. Washburn, *Movement and Mental Imagery,* 1916, pp. 186-8, 194-7, 215 *f.*

[9] W. B. Pillsbury and C. L. Meader, *op. cit.,* p. 103.

[10] *cf.* T. H. Pear, *op. cit.,* p. 75.

[11] W. B. Pillsbury and C. L. Meader, *op. cit.,* pp. 163, 184.

[12] See J. F. Markey, *The Symbolic Process* . . . , 1928, pp. 107.

[13] H. L. Hollingworth, *The Psychology of Thought* . . . , 1926, pp. 57, 139.

[14] E. B. Holt, *The Freudian Wish* . . . , 1916, p. 94.

ment system are not further describable because they do not develop into a more widespread body of disturbances.[15] In "extreme" thought, says Hollingworth, sometimes the content that bears the meaning is some movement, incipient gesture, muscular strain, general body set or attitude,[16] It may be added that Lorimer says that we should avoid the crass assumption that thinking is an isolated verbal process. Thought, as implicit tensional behavior, and mind, as its immediate context, are prior to linguistic activity.[17]

(3) According to the "pure thought" view, thought is at least largely imageless in character, and is contrasted with imagery of the ordinary type, and with words. It can be indicated only in actual mental operations; it is not to be traced to any definable mental state; and it is described negatively. According to Bühler, imageless thoughts, as we saw, are substituted for images or words in most of the important activities of the mental life, so that, for example, decisions concerning the truth or falsity of complicated statements come to mind without imagery, or even a word. Buhler regards consciousness of rules, consciousness of relations, and intention as three types of pure thought.[18]

(4) In connection with the theories of conflicts, or inhibitions, the first point to note is that thinking involves a delay, or delayed response. Markey holds that thinking belongs in a process where in the field of excitation the ordinary routine and otherwise balanced stimulus-response to situations or wholes fails to function. With Köhler's apes, the act which enables the animal to solve a problem is made possible or at least facilitated by a pause.[19] According to Washburn, all thoughts and mental images, all the contents of consciousness, rest not simply on delayed full motor responses, externally visible, but on delays in the system of tentative movements. When the systems run smoothly, we have unconscious thought; when there are delays, we have sensations and images.[20]

Such a delay is invoked by Washburn to explain how movement systems are related and constituted when we use the subject-predicate form of sentence. Lorimer says that the organic foundation of

[15] M. F. Washburn, *op. cit.*, pp. 57, 60, 215.

[16] H. L. Hollingworth, *op. cit.*, p. 138.

[17] F. Lorimer, *Growth of Reason*, pp. 141, 149.

[18] W. B. Pillsbury and C. L. Meader, *op. cit.*, pp. 162 *f.*; see G. Murphy, *Historical Introduction to Modern Psychology*, 1929, pp. 238 *ff.*, and M. F. Washburn, *op. cit.*, Chap. 10.

[19] J. F. Markey, *The Symbolic Process* . . . , 1928, pp. 131, 133.

[20] M. F. Washburn, *op. cit.*, p. 58.

the logical principles of analysis and synthesis seem quite definitely to be in the delayed reaction involving the tensional adjustment of implicit response patterns in which patterns with certain features are rejected and other patterns are integrated (we should say, appropriated) in the final response.[21] We noted that, according to Washburn, imageless thoughts occur when something is blocking the associative processes.[15]

The views of the two authors last cited are enough to suggest that the delay characteristic of thinking is due to a process of conflict or inhibition.[22] This brings us to a group of theories familiar in connection with Dewey's analysis of the complete act of thinking into five steps, the first of which is a "felt difficulty."[23] Here thinking is traced to a forked road situation, one which presents a dilemma and proposes alternatives.

Lorimer says that unrelated or indifferent stimuli show a high tendency to be inhibitory to any specific conditioned reflex. There are strong antagonisms, too, between reaction systems like reaching and withdrawing, and in motor coordinations and visceral organizations we get more definite mutual inhibitions. The principle of inhibition does not apply in equal degree to the more intricate reaction patterns of compound stimuli, nor to the intricate implicit patterns of human intelligence. Human behavior shows a high degree of plasticity in its synthesis of various reaction patterns. At the same time, verbal activity lags behind other organization and is not forced by the environment.[24] Holt is certain that the so-called inhibition of ideas will offer no problem when we have really discovered the mechanism of motor inhibition.[25]

(5) Lorimer's statement leads from considerations of thinking in terms of inhibition to theories of thinking as derived from language. Janet, in connection with his theory of the origin of language, considered above, has helped to exhibit the connection. Let us suppose, he says in effect, that after the language habits of the pack of hunting dogs, used for illustration in Chapter III, have become well established, the pack finds itself in a more complex situation—for instance, where it is pursuing two quarries, and one takes to cover while the other keeps on. This means that the leader will have

[21] F. Lorimer, *op. cit.*, p. 153.
[22] *cf.* M. F. Washburn, *op. cit.*, p. 59.
[23] See G. P. Conger, *Epitomizations*, p. 279.
[24] F. Lorimer, *op. cit.*, pp. 131 *f.*, 155. *cf.* p. 122.
[25] E. B. Holt, *Animal Drive and the Learning Process*, Vol. 1, 1931, p. 189.

not merely a conflict of tendencies to *action,* but also a conflict of tendencies to *cry out*—for there will be two possible signals, and the leader will give one or the other. As he hesitates between them, each may be said to inhibit the other, and there is a kind of rehearsal of possible signals, a preliminary trying out, which according to this view is a rudiment of what later develops into thinking. This does not mean that animals reason, for other steps are necessary before anything more than this rudiment is possible. Some of these steps, such as the development of parts of speech, have for convenience been considered in the previous chapter. What has been added here is enough to show a relationship between primitive thinking and primitive language and action, which may be expressed by saying that whereas language is the result of an inhibited tendency to action, *"thinking is the result of an inhibited tendency to language."*[26] As Bekhterev puts it, "in those cases where we think by the agency of words, we deal with inhibited speech reflexes."[27]

Pillsbury and Meader indicate another possible line of development, assuming that speaking aloud is the simpler and more usual operation and that silence is imposed from secondary considerations, such as desiring not to warn an enemy. These considerations lead to an inhibition of the usual motor accompaniments, and one thinks without talking.[28] Piaget holds that there are two forms of thought, autistic and intelligent. The former and earlier is individual and uncommunicated, but the latter is socialized and guided by the increasing adaptation of individuals to one another. The mere fact of telling one's thought to others or keeping silence and telling it only to oneself must be of enormous importance to the fundamental structure and functioning of thought in general and of child logic in particular. While in autistic thought intellectual work is carried on by means of images and motor schemes, in logical thought word and concept replace these primitive instruments.[29] Lorimer says that reason is thought controlled by explicit statement rather than by merely intuitive sensory, motor, and visceral processes. He agrees with Piaget that unverbalized thought or inadequately verbalized thought is fluctual and syncretistic; it lacks clear distinction and logical organization.[30]

[26] See G. P. Conger, *Course in Philosophy,* p. 464.

[27] A. L. Schniermann, in C. Murchison, ed., *Psychologies of 1930,* p. 230.

[28] W. B. Pillsbury and C. L. Meader, *op. cit.,* p. 108.

[29] J. Piaget, *The Language and Thought of the Child,* 1926, pp. 44 *f.*; *The Child's Conception of Physical Causality,* 1930, p. 292.

[30] F. Lorimer, *op. cit.,* pp. 122, 129.

According to Mrs. de Laguna, too, it is in the development of conversation as a social enterprise that we shall find the clue to the development of the higher intellectual activities most characteristic of man. The form of conversation from which thought springs is the discussion, which has for its end agreement among the participants regarding some specific condition of common action. Thinking is the internalization of this form of conversation and its independent practising by the individual. It is originally and primarily a rehearsal; the course of overt action is traced out in advance, and the serial organization of behavior is preestablished by speech.[31]

No one has been more outspoken in his attempts to link thinking and language than has J. B. Watson. According to such a view, when we think, there is always something going on in our body which as uniquely defines our thinking as does our consciousness of thinking, and this something is especially distinguished by being in normal persons chiefly compounded of incipient speech expressions or impressions.[32] Strictly speaking, according to Watson, the behaviorist has never held the view that thinking is merely the action of language mechanisms; any statement of his which has been construed to this effect was due, he says, to a loose way of writing, an overemphasis "indulged in for the sake of sharpness of presentation before elementary students." The more adequate statement is that a whole man thinks with his whole body in each and every part. We ought to make "thinking" cover generally all implicit language activity and other activity substitutable for language activity. Thinking is the subvocal use of any language or related material. It may be a mere unwinding of vocal habits, as in day dreaming, or a solving of problems which are not new but which need trial verbal behavior, or, again, a solving of new problems.

Thinking in the narrower sense where learning is involved is a trial and error process wholly similar to manual trial and error. Just as any adjustment is completed when a response effects a satisfactory change in the stimulus, so the thinking adjustment is completed

[31] G. A. de Laguna, *Speech, Its Function and Development*, 1927, pp. 286, 301, 307 *f.*, 352 *f.* For theories concerning the neural mechanisms involved, see J. F. Markey, *op. cit.*, pp. 15 *f.*; J. F. Dashiell, in *Psy. Rev.*, 32, 1925, pp. 61 *f.*, 69; J. Vendryes, *Language, A Linguistic Introduction to History*, p. 64; E. Sapir, *Language, An Introduction to the Study of Speech*, 1921, p. 17; E. J. Kempf, *The Autonomic Functions and the Personality*, 1921, p. 23; W. B. Pillsbury and C. L. Meader, *op. cit.*, pp. 101 *f.*

[32] G. H. Thomson, in *Brit. Jour. Psy.*, Gen Sec., 11, 1921, p. 69, agrees in part with Watson.

when the final word grouping or the overt bodily reaction which comes as the end-result of the process of thinking makes the initial stimulus to thinking inoperative or inert.[33]

Experimental work has sometimes seemed to favor Watson's view and other views related to it. Objective statement is evidently helpful in thinking; Lehmann has shown that persons can readily discriminate only as many shades of grey as they have names for.[34] By the aid of special apparatus, some other connections have appeared. The automatograph of Jastrow is said to show that ideas are a motor phenomenon.[35] Lashley found that overt but whispered repetition of a sentence produces a tracing on a drum wholly similar, except for amplitude, to that obtained when he told the subject to think the same thing without making overt movements. If the experiment was performed after other work had intervened, the tracings changed, but evidently because the motor set had changed.[36] According to Thorson, a careful instrumental study of tongue movements during implicit verbal activity indicated the presence of slight movements of the tongue when words were silently thought, immediately after they had been spoken aloud. But the study also indicated that there are variations between individuals in the extent of tongue movements during verbal thought, and, what is more important, that sometimes there is absence of tongue movements during silent verbal thought.[37]

The last point is strengthened by the fact, brought out by Lorimer, that neither surgical removal of the larynx nor removal of the tongue produces aphasia in thinking. This seems to indicate that the cortical processes involved in verbal thought become at least potentially independent of muscular adjustments. Undoubtedly minute verbal processes are focal in most adult human thought, but probably the verbal forms afford a structure which is ordinarily slurred over and syncopated in mature reflection.[38] And Lashley, in a paper later than the work referred to above, says that while attempts to demonstrate implicit movements in the verbal mechanisms during thinking have shown movements, the movements are irregular, by no means always present, and they do not correspond

[33] J. B. Watson, in *ibid.*, pp. 87-90, 104.

[34] B. B. Bogoslovsky, *The Technique of Controversy . . .* , 1928, p. 137.

[35] J. F. Dashiell, *Psy. Rev.*, 32, 1925, p. 70. See J. Jastrow, *Fact and Fable in Psychology*, 1900, pp. 309 *f.*

[36] J. B. Watson, *op. cit.*, p. 96.

[37] A. M. Thorson, in *Jour. Exp. Psy.*, 8, 1925, pp. 1-32.

[38] F. Lorimer, *op. cit.*, p. 68.

in rhythm or pattern to those of overt speech. They appear to be rather the results of chance irradiations to verbal mechanisms from centrally maintained processes.[39]

The experimental results unfavorable to the view that thinking is altogether to be accounted for by language habits are strengthened by other considerations. Thus against the argument that we can not think anything without having a word present itself, it may be argued first that this is due to acquired habits and associations, and again that the very suggestion that we shall think without words works negatively and tends to make words occur to us.[40] G. H. Thomson warns against the statement that thinking is a language *habit*, because thinking ceases precisely as habitual responses crystallize.[41] But this process, after all, may be only one habit replacing another.

Among the views which in one way and another combine features of these theories and mediate between them may be mentioned those of Lorimer,[42] Rignano,[43] Malinowski,[44] Berr,[45] Hollingworth,[46] and Dewey.[47] Pillsbury and Meader maintain that language and thought are to a large extent interdependent and in a certain sense are in part identical. Most thinking is in words, but thought without words is by no means infrequent. Thus thought may precede language in definite and detailed imagery and then be translated into language as a separate process. There may be mixtures or alternations of the two kinds of thinking; thus there may be a general idea of what is to be said in the vaguest, most symbolic terms, and this may be developed in words directly. Probably one thinks in images only when dealing with material that is not easily represented in words, as in musical compositions; but all this varies in different individuals.[48]

39 K. S. Lashley, in C. Murchison, ed., *Foundations of Experimental Psychology*, 1929, p. 550.

40 W. B. Pillsbury and C. L. Meader, *op. cit.*, pp. 159 *f.*

41 G. H. Thomson, *op. cit.*, p. 67.

42 F. Lorimer, *op. cit.*, pp. 68, 73, 86, 149.

43 E. Rignano, *op. cit.*, p. 114.

44 B. Malinowski, in C. K. Ogden and I. A. Richards, *The Meaning of Meaning*, 1923, p. 498.

45 H. Berr, foreword, in J. Vendryes, *op. cit.*, p. xi; Vendryes, *ibid.*, p. 65.

46 H. L. Hollingworth, *op. cit.*, p. 47.

47 See J. Dewey, *Jour. Phil.*, 19, 1922, p. 565; *Experience and Nature*, 1925, p. 166 *ff.*

48 W. B. Pillsbury and C. L. Meader, *op. cit.*, pp. 4, 99 *f.*, 160.

(6) The last group of theories to be considered are those which make thought essentially involve "meaning." Bartlett and Smith criticize Watson's view to the effect that thought is identical with its expression. In order to have thought, we must have not merely a conditioned reflex but a later associative connection of the word when learned with the bodily habits connected with the objects for which the word stands. Watson, however, never tells what this associative connection is. Bartlett and Smith go on to say that the thinking response occurs not to a physical stimulus alone or to a combination of physical stimuli, but to some characteristic of a presented object or situation which is common to that and to many other objects or situations. It is the response to a universal quality or relation which gives us the peculiar characteristic of thinking. At least, the effort to discover a general quality is the main point.[49] Watson's reply is to the effect that if we exhaust the concept of action we have exhausted the concept of meaning.[50] Others in various ways have supported the theory of thinking as essentially involving meaning. No attempt need be made here to distinguish the various meanings of meaning (see Chapter III, 15). It is obvious that any or all of them may be invoked in such theories of thought. Dewey maintains that discourse changed dumb creatures into thinking and knowing animals and created the realm of meaning.[51]

According to Pillsbury and Meader, each view of thought resolves itself into the problem. How can one mental state mean something which it is not? If we can solve the problem of meaning, we can solve the problem of thought. Thinking may go on in words or in images equally well, but that which makes both images and words capable of carrying thought is not their peculiar structure but their reference, not to other words and images, but to types, to organized experiences, and their representatives. So in thinking one is concerned with result and meaning rather than with imagery, and it is the meaning of mental processes, not their structure, which is important.[52]

Having examined these six theories of the origin and nature of thinking, we must now come back to the fact noted above, that they

[49] F. C. Bartlett and E. M. Smith, in *Brit. Jour. Psy.*, Gen. Sec., 11, 1921, pp. 56, 59. *cf.* pp. 66 *f.*

[50] J. B. Watson, in *ibid.*, p. 103.

[51] J. Dewey, *Experience and Nature*, pp. 166-9.

[52] W. B. Pillsbury and C. L. Meader, *op. cit.*, pp. 165 *f.*, 182, 184 *f.*

often overlap. It is possible to make the "pure thought" theory an exception, but for the others we noted how Janet's theory serves as a link between that of inhibition and that of language habits. The language habit theory, again, is not very different from that of images, because words, after all, are merely visual or auditory or kinaesthetic images.[58] And it is easy to see that any thinking in some sense involves meaning, and probably also involves movements or vestiges of movements. The question, as so often when rival theories are concerned, seems to be a question of emphasis. And interesting as the question is, it is not necessary for the present argument that it should be decided here. All that we need to do, having canvassed the field of current theories, is to show how they all exhibit some kinds of duality, and thus raise for us the problem of the dualities of thinking.

If thinking is a play of images and images are persistent remnants of sensations or perceptions, we are justified in examining images in order to detect in them the dualities characteristic of sensations or perceptions. Motor theories of thinking suggest the dualities characteristic of responses. If thinking involves conflicts, certain dualities obviously should be its most conspicuous features. If thinking is derived from language and language involves auditory and other perceptions and images, thinking is doubtless marked by the dualities of perceptions and images, as these are indicated above. A thinking which involves meanings which are in any sense objective must be marked at least by dualities of selection and neglect. In short, only the theories of pure thought offer possibilities of evading dualities, but these theories, perhaps for this very reason, are themselves not clear nor consistent enough to be preferred to the others.

[58] *ibid.*, p. 152.

CHAPTER V

Some Dualities Involved in Thinking

"The concept of a noumenon is . . . merely *limitative* and intended to keep the claims of sensibility within proper bounds, therefore of negative use only. But it is not a mere arbitrary fiction, but closely connected with the limitation of sensibility, though incapable of adding anything positive to the sphere of the senses . . . . Our understanding . . . immediately proceeds to prescribe limits to itself by admitting that it can not know these noumena by means of the categories, but can only think of them under the name of something unknown."

—I. Kant, *Critique of Pure Reason,* transl. F. M. Muller, 1896, pp. 208 *f.*

We shall now examine the structures and processes of thinking, in order if possible to detect the dualities which in Chapter II we found to be involved in perception. It is not possible to draw the dividing line with precision, but in this chapter we shall be concerned rather with the psychology of thinking, leaving problems of epistemology and logic mainly for Chapter VI. We hold that, whatever be the origin of thinking, as discussed in Chapter IV, its structures and processes are those of monads which, like perceptions, belong to the level of patterns. In this chapter, for facility of reference, two numberings are used—the roman numerals of Chapter II and the arabic decimal numbers of the epitomization hypothesis. We begin also the use of daggers (†) to indicate some "horizon" problems discussed in more detail in the later chapters of this book.[1]

(1) In Chapter II we said that perceptions, regarded as patterns, are (64.11) relatively individuated. The same thing appears to be true for thought-patterns, or patterns involved in thinking. Putting it first in terms of consciousness, we find that, according to Rignano, the connection of ideas peculiar to a state of attention is

[1] Daggers (†) used in the text mean merely that discussions of horizon principles might well start from the terms thus indicated. Some of the terms in the passages are used by their authors without proper regard for these principles, but others are quite in accord with them. A few of the terms are not discussed in detail in this book, but synonyms and other similar terms are. Only comparatively few of the terms which might provoke discussion are indicated. If our general thesis, that all thinking involves horizons, is correct, it would be permissible for every word of the text to carry a dagger.

produced in part when all foreign sensations or sensory evocations which are not inhibited (64.111) physiologically, are (64.112) excluded from consciousness.[2]

For Köhler, thought-patterns or configurations may be said to have individuations corresponding to that of their objects. The appearance of complex unities which possess specific structures in the physical world and which are copied in perception leads us to think that even the higher thought processes are accompanied by configurational processes within the nervous system. Configurational principles enable us to achieve this correlation.[3] If Köhler is right, it will not matter whether, in questions of structure, we consider the individuation of thought-patterns or of the objects to which the patterns have reference, as the relationships in the two cases will correspond.

Some at least of these patterns, or their objects, may stand in (64.1131) that dyadic relationship to which the term "polarity" is often applied. As in the case of perceptions, however, we find the notion of polarity often confused with other notions. According to Bogoslovsky's interpretation of the principle of polarity, any cognitive activity is based on contrast and differentiation. In general, no unit of thought has any meaning without its opposite. Units of thought are always in pairs; there is no *A* without its non-*A*.[4] Lorimer thinks that there is some confusion in Bogoslovsky's statement, which might be improved by saying that every concept or judgment or unit of thought involves the emergence of one particular implicit behavior pattern or idea against physiological inertia, interference, and inhibition, which frequently include one specific outstanding alternative pattern or logical opposite. The symbolic statement or explication of these alternative ideas as "opposite concepts" clarifies the issues involved in the judgment process.[5]

It may be noted that polarity is here interpreted in terms of conflict between patterns or possibly even processes of reproduction (see VII, below).

(II) In accordance with the integrative action of the nervous system, there are quite evidently various degrees of complexity of organization, both (64.13) in thought-patterns and in objects thought about.

[2] E. Rignano, *Psychology of Reasoning*, p. 66.
[3] H. Helson, in *Am. Jour. Psy.*, 36, 1925, p. 367.
[4] B. B. Bogoslovsky, *Technique of Controversy* . . . , pp. 14, 17, 117 *f.*, 129.
[5] F. Lorimer, *Growth of Reason*, p. 156.

(III) It appears that, like perceptual patterns, thinking patterns exhibit (64.20) selective interactions, and that our thinking is a process characterized throughout by selection and its correlative neglect. Some of the classical philosophers saw this clearly. Kant's view of the limitations of reason is usually stated in terms of the Lockian assumption and phenomenalism, to the effect that we are limited to the data supplied by our senses. But Kant, as appears from the passage quoted at the head of this chapter, held that this phenomenalism carried with it a distinction between the content of reason and its conditions, and if it made possible definite statements in the first field, made necessary indefinite statements in the second. This latter point is Kant's most significant contribution to epistemology—namely, that there is a thoroughgoing distinction in our world between the content of our reason, which Kant called phenomena, and the conditions of our reason, which he called noumena, and the two can not be treated in the same way. In the later portions of the *Critique of Pure Reason* the contrast is somewhat softened by the doctrine of constitutive and regulative concepts, but still the distinction is important.[6]

Baldwin holds that for the derivation of a meaning, it is only necessary that there should be some degree of dislocation or differentiation of the habitual processes involved. Even the feeling of limitation or restriction would indicate such a dislocation. Any germ of positive exclusion of content from the grasp of cognition, whereby positive opposition might spring up, would be by a process of so changing the body of the determining conditions that some new shading of meaning would be due to the limitation.[7]

Among recent writers, R. M. Ogden says that in the development of reasoning the first stage is the dichotomous separation of any given thing or event from anything else. We call this process particularization, since the thing or event under consideration is specified. This specified particular is in contradictory opposition to all the rest of existence.[8] Bridgman says we never have perfectly clear-cut knowledge of anything, but all our experience is surrounded by a twilight zone, a penumbra of uncertainty, into which we have not yet penetrated.[9] Wittgenstein once attempts to indicate a limit, not to thinking but to the expression of thought. He says that philosophy

[6] See G. P. Conger, *Course in Philosophy,* p. 467, n. 1. The views of Sir William Hamilton are described in the same note.

[7] J. M. Baldwin, *Thought and Things . . . ,* 1906, Vol. 1, p. 184 *f.*

[8] R. M. Ogden, *Psychology and Education,* 1926, p. 313.

[9] P. W. Bridgman, *The Logic of Modern Physics,* 1927, p. 33.

should limit the unthinkable from within, through the thinkable. It will mean the unspeakable by clearly displaying the speakable.[10]

Whitehead says that there is a limitation which breaks off the finite† concept from the higher grades of illimit*able*† complexity, although for him this breaking off from an actual illimitability is what in any occasion marks off that which is termed mental from that which belongs to the physical event to which the mental functioning is referred.[11] For Cohen, always there remains the beyond, the unexplained, the contingent.[12] Sometimes, after the limitations of perception have been recognized, it is held that thinking is not subject to them and is able to overcome them. For example, Mrs. Swabey says that in a sense the only answer to the denial of the self-transcendant power of thought is simply to point out that the removal of the prerogative would destroy all that makes thinking possible.† Even the most elementary concept can hardly be evolved without making use of it.[13]

According to Broad, we can remove in thought certain limitations† which seem to depend on certain *de facto* limitations of the powers of sensing and remembering. Thus we can think of sense history as stretching back indefinitely† into the past, although in fact we can only remember a certain distance back. We can remove in thought the limitation of a finite† specious present. We can regard as merely contingent† the fact that only a very thin slab can ever be sensed at once and that the whole† history is a series of such slabs. We can regard the whole history as a continuous† four-dimensional strand. We can, *e.g.*, imagine the spatial limits† of our visual fields indefinitely extended. And we can remove the limitation which is imposed by the fact that we can not see all around us at once.[14]

Whitehead says that in older theories the thought-object was conceived as possessing a simplicity not belonging to the material universe† as a whole.† But the modern thought object has the complexity of the whole material universe.[15] Whitehead, as we saw, holds that we have some knowledge of what is beyond discriminating perception; this he holds to be the basis of the scientific doc-

[10] L. Wittgenstein, *Tractatus Logico-Philosophicus*, 1922, pp. 27, 77. This work will hereafter be cited as *Tractatus*.

[11] A. N. Whitehead, *Science and the Modern World*, 1925, p. 239.

[12] M. R. Cohen, *Reason and Nature*, 1931, p. 135.

[13] M. C. Swabey, *Logic and Nature*, 1930, p. 369.

[14] C. D. Broad, *Scientific Thought*, 1923, p. 459.

[15] A. N. Whitehead, *The Aims of Education*, 1929, pp. 224 *f.*

trine of externality, involving the "constants of externality." These refer to perceptions, but involve thinking about perceptions. Whitehead regards perceptions as notably selective, but makes a number of affirmative statements about the neglected region. Of the constants, "which," "what," and "how" invite specification of qualities and discrimination amid alternative entities. "When," "where," and "whither" refer to the spatio-temporal relation of a part† to a whole†, within which in some sense the perceived part is located. In detail, the first constant involves the belief that what has been apprehended as a continuum† is a potentially† definite† complex of entities for knowledge. The third refers to the fact that an event as apprehended is related to a complete† whole of nature. The last three convey the very essence of externality. Without them our perceptual experience appears as a disconnected dream. They embody the reference of an event to a definite—an absolute†—spatio-temporal position within a definite whole of nature, which whole is defined and limited† by the actual† circumstances of the perception.[16]

In another place, however, he says that if anything is out of relationship, then there is complete ignorance as to it. Either we know something of the remote occasion by the cognition which is itself an element of the immediate occasion, or we know nothing.†[17]

Sometimes one finds views less positive. According to Eaton, the fact that the existent is capable of being consistently presented and represented is the limiting condition of the notion of reality on which the definition of truth rests, but it need not be a condition of a more extended reality.[18] These views can be best discussed in our Chapter VI (III), below. The question whether processes of selection are to be ascribed to *objects* of thought will occupy us in Chapter VI, E (I and III).

We hold that in thinking, as in perception, the selected object frequently appears in an otherwise undifferentiated background. But, again as in the case of perceptions, sometimes the selection is made with reference to one object of thought as contrasted with other similar or cognate objects. Thus, Hollingworth, taking account of more of the factors involved, traces reasoning to the operation of an instigating, or stimulating, and a specifying, or controlling, detail, and says that the specifying detail may act as a selective agency,

[16] A. N. Whitehead, *Principles of Natural Knowledge*, pp. 72 *ff*.
[17] *idem*, *Science and the Modern World*, 1925, pp. 36 *f*.
[18] R. M. Eaton, *Symbolism and Truth*, 1925, p. 194.

selecting from random consequents those that are relevant to it.[19]

(IV) The theories to the effect that thinking is or involves processes of interaction with cognate monads, especially (64.23139, etc.) conflicts or inhibitions, obviously make it exhibit dualities. The importance of inhibition for thought processes is brought out by Lorimer, who says that the use of the inhibitory gesture ("No") and the accessory gesture ("Yes") is later transferred to control of symbolic responses, and that ultimately negation would seem to be rooted in inhibition.[20] In a case of what Hollingworth calls "complete inhibition," the contrasted idea, which he calls the "associate," may not even occur. He calls this "rejection," but it is in accordance with our terminology better regarded as (64.232391) preemption.[21] Rignano maintains that the connection of ideas peculiar to a state of attention is produced not merely by the exclusion mentioned above, but also by (64.232392) physiological inhibition of all other possible affective tendencies which would recall unconscious sensory evocations or other sensations.[22] Janet says that antecedent tendencies, expressed in verbal form, become motives or arguments, and the force of these tendencies (64.232393) increases or diminishes that of the formula under discussion. The conflict of these tendencies constitutes deliberation, when it issues in a volition, or reasoning, when it issues in a belief.[23]

(V) In recent years Lévy-Bruhl has made familiar the view that in primitive mentality, concepts are not as definite and clean cut as with us. Prelogical mentality, he says, although not completely different from ours, does not bind itself down to avoiding contradiction†; it is indifferent to contradiction. Among primitive peoples, individual mentality does not differ so much from ours, but in its collective representations, or group ideas, the primitive mind, in place of our law of contradiction, uses, if anything, the "law of participation." In the midst of the confusion of mystic participations and exclusions, the impressions which the individual has of himself, whether living or dead, and of the group to which he belongs, have only a far off resemblance to ideas or concepts. They are felt or lived, rather than thought. Neither their content nor their connections are strictly submitted to the law of contradiction.

19 H. L. Hollingworth, *Psychology of Thought*, pp. 252 *ff.*, 274 *ff.*
20 F. Lorimer, *op. cit.*, pp. 135, 152.
21 H. L. Hollingworth, *op. cit.*, p. 275.
22 E. Rignano, *op. cit.*, p. 66.
23 P. Janet, *De l'angoisse à l'extase*, 1926, pp. 225 *f.*

Consequently neither the personal ego nor the social group nor the surrounding world, seen and unseen, appears to be yet definite in the collective representations, as they seem to be as soon as our conceptual thought tries to grasp them.[24]

Even in later stages of thinking, concepts are not always clean cut. According to Baldwin, there is a shading out of a context of what is used into a margin or penumbra of what is there and not used, but there is a rapid shifting of the nucleus of determination here and there over the threads of the context, and this rapidly develops into definite selections and exclusions.[25] R. M. Ogden says that in the intellectual world of reasoning, the definition of ideas as concepts takes the same course and follows the same law of precision that guides the refinement of action in the acquisition of skill and the definition of the objective in perception. The stages of reasoning—particularization, gradation, and articulation—would apply as well to stages of learning to perceive.[26]

The sharpening of concepts seems to depend very largely upon symbolization, especially symbolization in words. Even the most vivid concrete concepts are likely to come trailing clouds of vagueness, but the symbol, which only represents the concept and probably can represent only a part of it, must be definite enough for social use. The increasingly differentiated demands of social life, too, make for increased definiteness and precision of symbols. By virtue of linguistic accomplishment, a unit of experience is, so to speak, tied up in a bundle and labelled with a name, and, as we saw in Chapter IV, thinking tends increasingly to take the verbal form.

Sometimes the roots of logical and grammatical forms can be discerned beneath the symbolism. According to Lorimer, the symbolic statement or explication of such alternative ideas as "opposite concepts" clarifies the issues involved in the judgment process. The structures of implication, negation, and contrary alternatives seem quite clearly to be grounded in presymbolic functions, and receive a new organization in symbolic processes.[27]

If it be true that physical objects with their structures have configurational correlates in the nervous system, it is quite possible that thought-patterns have actual physical boundaries, but the

[24] L. Lévy-Bruhl, *How Natives Think*, transl. L. A. Clare, 1926, pp. 79, 106. *Primitive Mentality*, transl. L. A. Clare, 1923, p. 447.

[25] J. M. Baldwin, *op. cit.*, Vol. 1, p. 185 and n. 1.

[26] R. M. Ogden, *op. cit.*, p. 315 *f.*

[27] F. Lorimer, *op. cit.*, pp. 153, 156.

sharpening of a concept, whether by verbal or other means, seems to reveal little or nothing corresponding to the meniscus of perception or the contour of a *Gestalt*. To be sure, if we think of any object, we can usually think of the boundary of the object if it has one, or we can supply a boundary for it, but this seems to be secondary and unimportant. We must remember, however, that the perceived object itself does not always have a definite boundary, and that at least in some cases the concept is the result of two abbreviating symbolizations, one in language and one in thinking. Apparently the boundary condition is one which is most likely to be disregarded when the symbol is evolved. Perceived objects may have contours, but concepts seem at least speedily to lose them and words as symbols of concepts seem not to have them at all. Thus Cohen, from the point of view of his principle of polarity, pictures a "twilight zone" or region about the point of equilibrium of opposite tendencies. Such zones, he says, are always present in the realm of physical or psychical existence, but in the realm of formal or strictly logical concepts there can be no twilight zones.[28]

Physiologically there is a suggestion in the fact that boundary conditions are peripheral, and that in neuropsychological patterns they would tend to be localized in (64.2543) the receptors and effectors. This is not as fantastic as it seems. At least on the effector or motor side, certain distinctive conditions are evident in the fact that we "let the voice fall" at the end of a spoken declarative sentence, and also in any corresponding relief from tension which may be said even vaguely to round off each of our ideas and make it relatively distinct from others. In most written languages, such "breaks" are indicated by punctuation signs; the deeper significance of these usually escapes notice. This has some bearing upon problems of identity†, contradiction†, and excluded middle†, which, according to Lorimer, might be described as laws of the transition from the presymbolic to the symbolic structure of intellectual activity.[29]

(VI) If there are boundaries between a concept and its context or milieu, they may be said frequently to shift, in (64.27) a process analogous to growth or depletion. Hibben has put the matter metaphorically in saying that the life of every individual may be appropriately represented by an inner circle of knowledge, placed within

[28] M. R. Cohen, in *Jour. Phil.*, 24, 1927, p. 679.
[29] F. Lorimer, *op. cit.*, p. 153.

a vast outer circle of the unknown. Growth, progress, attainment, all are possible only when there is an ever-increasing expansion of this inner circle, transcending its own limits and appropriating more and more of the outlying region within the area of its comprehension and appreciation.[30] We shall see in Chapter VI that the process of growth or depletion is involved in several important methods of dealing with the dualities of thinking.

(VIII) Probably at least some cases of suggestions issuing from Dewey's "felt difficulties" are to be regarded as products of (64.33) a process similar to biparental reproduction.[31] Washburn traces certain characteristics of creative thought to inhibitions of two problem systems, tentative hypotheses, or suggestions, at common motor outlets. At length a direction of motor discharge is found which is common to them all, and a solution is reached which leaves no part of the problem out of account.[32] Something of this sort appears to be indicated by Janet, when he says that after a conflict of tendencies one often notes a pause, a stopping as if a certain amount of work were necessary to conciliate the opposed tendencies, to invent a new formula which synthesizes them. The new formula is not the reproduction of one of the preceding. It is in a way new, since it unites all preceding formulas.[33]

There have been some attempts to explain syllogistic reasoning in terms of processes characteristic of nervous patterns. According to William James, psychologically as a rule, *P*, the conclusion's predicate, overshadows the process from the start. We are seeking *P*, or something like *P*. Washburn says that this means that *P* acts together with the subject as a problem idea. We have then in operation two systems of tentative movements, those connected with the subject and those connected with the predicate. The outcome of the reasoning is the setting into action of movement systems that are connected with both these systems. Any other movement systems will be inhibited by the persistent recurrence of these two as the problem idea.[34]

For us, the question here is one of individuation and proper identification of the elements concerned. Subject and predicate

[30] J. G. Hibben, *A Defence of Prejudice*, 1911, p. 26.
[31] See G. P. Conger, *Epitomizations*, p. 279.
[32] M. F. Washburn, *Movement and Mental Imagery*, pp. 177-83.
[33] P. Janet, *op. cit.*, p. 226.
[34] W. James, *Principles of Psychology*, Vol. 2, p. 338; M. F. Washburn, *op. cit.*, p. 177.

are parts of one idea, or thought-pattern, and the two systems of tentative movements most conspicuously involved are those of the major and minor premises. The predicate of the major premise is more important for the conclusion than any other term.[35]

In attempting to account for the judgment "Today's weather is cold," Washburn says that old associative dispositions do not really connect "today's weather" with "cold"; they only connect "weather" with "cold." If the movement systems of the subject, 'today's weather" are active only as part of the problem, they are soon inhibited by a return to the full problem. But "today's weather" is a movement system much more complicated than weather in general. Not until it proves to have associative dispositions in common with "cold" will the delay and constant reference back to the problem cease.[36]

Here we should say that the mere predicative judgment, "Today's weather is cold," while it would not occur without some experience of weather and of cold, does not directly raise or involve the question of weather in general, and that any inhibition between the two kinds of judgment is not important for the present problem. For the present problem, a better example is the relationship of "Today's weather is cold" with "Cold weather is healthful." Hollingworth associates with each of his four features of meaning (see Chapter III, 14) a function in the proposition or in the syllogism. What he calls the "significant," *i.e.*, the instigating event, corresponds to what logic calls the subject of the proposition. What he calls the "signified" corresponds to the middle term. By virtue of a redintegrative sequence, the present "significant" functions in a way indistinguishable from the way in which the "signified" total antecedent event would function if repeated. The "signifying" corresponds to the copula of logic and association of psychology, and the "significance" corresponds to the logical predicate.[37] This account also seems defective, particularly because it does not furnish a sufficiently clear analysis of the propositions which serve as premises. For instance, the middle term must be at least once either a subject or a predicate, but then its relationship to the other subject and predicate is a problem.[38]

In the whole problem of syllogistic reasoning, there is much confusion concerning the relationships of neurological, psychological, grammatical, and logical structures, processes, and

[35] See G. P. Conger, *Epitomizations*, pp. 277 *ff.*, 440 *ff.*
[36] M. F. Washburn, *op. cit.*, p. 178.
[37] H. L. Hollingworth, *op. cit.*, p. 289. [38] See G. P. Conger, *op. cit.*, p. 447.

categories. According to the hypothesis of epitomization,[39] the logical constitutes a prior realm, as do the biotic and the cosmogonic, and the relationships between neuropsychological monads and logical monads are to be read off in the relationships of the neuropsychological realm to the other prior realms. The relationships important for us are six.

In the first place, the neuropsychological is contained within the logical, with the other four realms intermediate in the order indicated in Chapter I. Secondly, each of the later realms, in general, *conforms to* the structures and processes of the earlier realms which contain it; there is no violation of the principles operative in the earlier realms, except such as may be involved in differences of level and realm. Thus there is nothing contrary to logic in the physical cosmos or in the nervous system, except as these later realms exhibit characteristic developments which the logical as such does not. This relationship of conformity is necessarily vague and sometimes not very conspicuous, but it is a basic feature of the order of the cosmos.

Thirdly, in its own structures and processes the neuropsychological *epitomizes* the logical, as does the biotic and cosmogonic; this statement is the summing up of the relationships exhibited in detail throughout *A World of Epitomizations*. It is there argued that the structural relationships of the parts of the syllogism are cosmic in their nature, and that they have not merely logical, but also metaphysical character and status.[40] In particular, at this point, the operations of the syllogism, while they may involve inhibitions between conflicting tendencies, and even the development of a joint result compounded from both, should not be identified with such inhibitions or compoundings. Both syllogisms and neuropsychological patterns exhibit parallelisms to biparental reproduction, but the syllogism with its processes in which two premises produce a conclusion belongs in one realm, the logical, whereas conflicting tendencies or patterns which, under certain conditions, produce a new one belong in a later realm, the neuropsychological. The conflicting tendencies and their resultants are no more to be identified with the reproductive processes in bodies of propositions, which processes we call syllogisms, than they are with the biparental reproductive processes in multicellular organisms, or any other parallel process in any other realm.

In the fourth place, the neuropsychological monads (64.2111, etc.) *interact with* the prior logical monads, as they do with the biotic and cosmogonic. This means that we should regard our apprehension and treatment of terms-in-propositions and

[39] *cf., ibid.*, p. 278.

[40] *ibid.*, pp. 440 *ff.*

syllogisms as having to do with objectively real monads, monads in their own right, as objectively real as atoms or elephants, although, as we shall see, our interactions with logical monads are complicated by some peculiar conditions. There is no important or primary conflict of tendencies involved in thinking of a syllogism, except as we think of the syllogism in spite of a tendency to think of something else, some non-syllogism, in place of it. With or without any such primary conflict, we may think *of* a conflict of tendencies, or of the interactions of two parents, or of two premises in a syllogism, and may even in thought, by a kind of mimicry, take the part first of one and then of the other. In accordance with this relationship of interaction, a formal, subsistent syllogism tends to be *thought about.*

In the fifth place, a peculiar condition which complicates these interactions with the logical monads is that the neuropsychological does not act upon the cosmogonic directly, but through the biotic, *i.e.*, in any given case through the sense organs, etc., of the physiological organism. This relationship is called *refraction,* and as applied to the logical, means that we can not directly perceive the monads of the logical realm as they are in pure logic, in and by themselves, but only as they are refracted, especially through the cosmogonic and biotic. So it is only under conditions imposed by the intervening realms, that the logical is recognized and dealt with by our minds. We do not, for example, perceive relations, but only things with relationships, or related things. Our conception of terms is affected by our necessary intermediate interactions with things signified by the only terms we know. Again, we do not directly perceive syllogisms in their pure subsistent state, but only as they are entangled or embodied in facts and socially symbolized statements like "All men are mortal." (The grammatical categories which have been developed in various societies and languages sometimes distort the logical forms, but in no very widespread or important way.) This refraction is the result of the predicament of the neuropsychological realm as the inmost of the six in the transitive relationship of container and contained. In accordance with this principle of refraction, the syllogism thought about is never purely formal and subsistent, but always has some material or symbolic content.

In the sixth place, as another peculiar condition, logical monads form parts of the structure of monads of all later realms, including the neuropsychological, and as parts of those structures (64.24., etc.) *persist* in the interactions characteristic of them as logical monads. Now the neuropsychological monads,

in this case patterns, may be studied in terms of any of their constituent monads from prior realms. There are, for example, structures and processes typically physiological, or biotic, as is indicated by the effects of fatigue on neuropsychological processes. Other constituent structures and persistent processes are physicochemical; still others, geometric-kinematical, as is indicated by the importance of spatio-temporal localizations and motion; others are numerical, appearing in the numerical properties of things; and still others are logical. The persistence of the logical monads in the neuropsychological realm gives us the logical properties of thinking, just as the persistence of the cosmogonic monads in the neuropsychological gives us the chemical feature of thinking. But these logical properties are not the syllogisms of the classroom any more than the chemical features are the atoms of the laboratory. The syllogistic processes which persist in this way are in the *thinking*; they are not the syllogisms which are thought about. The reproductive (syllogistic) processes of bodies of propositions in the thinking do not affect its content, any more than the reproductive processes of constituent molecules or cells do. In accordance with this relationship of persistence, the processes of thinking show the laws of logic, just as naturally as they show the laws of chemistry and physiology, but none of this is apparent except indirectly, as it is read back into our heads, so to speak, from what we detect in other connections outside.

Taking all these considerations together, it appears that in the process of syllogistic reasoning, much more of the machinery of the universe is involved than is commonly realized. This is really why syllogisms are known to us as valid or not. We shall see in Chapter XIII that the problem of validity in logic is the problem of the fitness of our judgments, made in the form of propositions, to represent the conditions of the real universe, built up in its successively epitomizing realms. The problem is not solved by reading syllogisms in terms merely of nervous reflexes or patterns.

(VIII) Thought-patterns typically (64.43) enter into larger structures, in which the dominance of purposes is evident. To the discussion of this point in *A World of Epitomizations*[41] and in Chapter III, 4, we add only brief mention of the views of two other writers.

Rignano's theory of attention as the result of conflict of affective tendencies is really a theory of a larger organization of patterns in time. The fact that one and the same primary affectivity is held a

[41] *ibid.*, pp. 290 *f.*

long while in suspense by the secondary gives a continuity of action to a single evocative (*i.e.,* stimulating) process and thus succeeds in maintaining the connection of ideas. At the same time it causes the evocative process which serves as a basis for the intellectual process pursued, and is provoked especially by the primary activity, to remain none the less continuously under the effective control of the secondary affectivity. For instance, the fear of falling into error acts as a secondary affectivity, with the result that a thinker pays attention to what he thinks. Thus the secondary activity is necessary in order to keep the whole reasoning process in continuous and actual correspondence with the real situation.[42] Hollingworth agrees with Rignano, in that reasoning is made to involve two cues, those of propulsion and direction, or the instigating and specifying details. He says that the aim, bodily posture, or direction of attention prescribes the general field in which coordination will lie. Reasoning as a process not only involves at least duality and perhaps multiplicity of clues, but also requires the joint operation of at least one enduring clue and a certain number of transitory ones. This need for endurance is one reason why motives are so commonly constituted by kinaesthetic processes—attitudinal, postural, and motor events, or by autonomic events such as organic and intraorganic processes and conditions. For this reason a motive is sometimes thought to be subjective, psychic or occult.[43]

We shall see in Chapter XX that recognition of end-reaction complexes and purposive organizations is important, but the duality of ends and means, or consummatory and precurrent reactions, is more often explicit than implicit.

(IX) If we assume that in a thought-pattern nervous configurations are at least in general like those of other monadic structures, we may expect to find in them (63.442) some differentiations of parts. From the side of processes, this appears with characteristics recalling selection and neglect, when Bogoslovsky develops what he calls the "law of the partial† functioning of concepts." Concepts, when they act as specific items presented to be incorporated into the general framework of our experience, function only partially and by a certain aspect, not as wholes.† In no case are we capable of thinking, for example, of education, in such a way as to take into

[42] E. Rignano, *op. cit.,* pp. 64, 94 *f.*
[43] H. L. Hollingworth, *op. cit.,* pp. 250 *f.,* 266, 270.

consideration and pay attention to all aspects of the concept at the same time.[44]

Others have emphasized differentiations as perceived or thought of among the *objects* of thinking rather than in the processes of thinking. As in the case of perception, the discerned or differentiated parts may be regarded as relative to an undifferentiated background. There seem sometimes to be successive instances of selection and neglect, with a kind of "zoning" effect.[45]

[44] B. B. Bogoslovsky, *op. cit.*, pp. 121-5.

[45] *cf.* B. Bosanquet, *Logic, or The Morphology of Knowledge*, 1911, Vol. 2, pp. 144 *f.*

CHAPTER VI

Dealing with Dualities

"The visible is set in the invisible, and in the end what is unseen decides what happens in the seen; the tangible rests precariously upon the untouched and ungrasped. . . . We may term the way in which our ancestors dealt with the contrast superstition, but the contrast is no superstition. It is a primary datum in any experience."

—J. Dewey, *Experience and Nature,* 1925, pp. 43 *f.*

A. From all that appears in the preceding chapters, perception, language, and thinking are characterized by dualities. There are other dualities in addition, but the nine which we have examined seem to be the most important in revealing the limitations or horizons of thought. Most of them are involved in each of a long list of time-honored problems of philosophy. In succeeding chapters we shall take up a number of these problems in detail. They will include, in a field which roughly may be called that of logic, identity and difference; negatives; contradictories; inductions, generalizations, and universals; the logical universe, or universe(s) of discourse; "nothing"; and validity, truth, and probability. Under the heading of mathematics, we shall consider continuity, limits, and infinites. In a field which is roughly that of ontology, we shall discuss beginnings and endings; order and disorder; whatness and thatness; actuality, possibility, impossibility, and necessity; being, non-being, and becoming; monism and contrasted views; totality, or wholes, and parts; and absolutes and relatives. As problems of cosmology, we shall study space-time, and the physical universe. And finally, returning to some problems of psychology, we shall offer some remarks on freedom, value, and intuition. The problems are frequently so interlocked that precise division is impossible. Some of the more important cross-references are indicated by daggers (†).

We find that methods of dealing with the dualities persist as we pass from one problem to another, and failure to recognize the "horizons of thought" and to deal adequately with them is at the root of much confusion and error.

It may be noted that the nine dualities we have studied in Chapters II and V are all grounded in the structures and processes of the cosmos, as these are discussed in the epitomization hypothesis, and so have the perhaps formidable backing of a system of metaphysics. In the chapters now to follow, we shall try to show, from this point of view, the defects in some other current methods and answers to the traditional problems, before indicating a treatment of them in accordance with the dualities we have recognized. The methods and answers which we take to be insufficient seem to occur in three main groups, which will be indicated here and in later chapters in divisions captioned by the letters B, C, and D. These will be preceded in each chapter by a division captioned by the letter A, devoted to introductory definitions, etc. In a final division of each chapter, captioned by the letter E, we shall discuss methods of dealing with the given problem in accordance with the dualities which, grounded in the nature of perception and of thinking, as well as in the metaphysical principles of epitomization, exhibit horizons of our thought. We shall have repeatedly to see that the various dualities and methods of dealing with them are, like the problems to which they are applied, interlocked, but the complexity of a field of problems is no excuse for not attempting some analyses.

B. The first group of inadequate or questionable treatments approaches the traditional problems by way of certain psychological and epistemological theories. We shall mention four subclasses, which for convenience in this and the following chapters are indicated by Greek letters.

(α) The first subclass comprises attitudes and methods by which it is sought to dismiss the problems by adopting or affecting skepticism concerning them. Without taking up this question in detail, we shall try to show by our treatment as a whole that there are answers which are at least somewhat better.

(β) Next we find discussions which lose sight of the issue in a more or less faulty or fragmentary account of the psychological structures and processes involved. In these connections we shall try to show that the psychological structures and processes themselves, especially when studied as in Chapters II and V, exhibit the dualities we are investigating, and so if offered as answers to problems of horizons still involve the problems.

(γ) Again, we find discussions in which the ground is shifted to some phase of the historic controversy between the idealisms and

the realisms, as when by a bifurcation of nature a break is made between the subjective and the objective, and they are treated as in some respects essentially different. Much current emphasis on intensional and extensional logic has affiliations here. According to current usage, terms in intension name something logically conceivable, and in extension denote something which exists.[1] Any such dichotomy of subjective and objective, according to our view in Chapter I, is futile and leads nowhere. The fact is that any discussion presupposes and consists of interactions between the subjective and the objective, with the latter either existent or subsistent. But even if the dichotomy is allowed, there is selection-neglect, or "implicit duality," with horizon effects, *both* in the subjective and the objective, and, if any one cares to insist on it, between the two. The answers to the problems with which we are now concerned must be sought elsewhere than in such a division.

(δ) In this group of psychological and epistemological treatments of the traditional problems we shall mention discussions which more or less uncritically and questionably appeal to intuition as against intelligence. Especially at the present time these views require careful examination; they are discussed in Chapter XXI.

C. The second defective way of treating the problems which we are investigating consists in discussing a given problem, for example, identity, in terms of another problem, for example, infinity, without seeing that the answer in terms of the second topic really involves the same formal principles and horizons, in short the same problems of duality, under another name. Of the countless examples available in the works of both classical and contemporary philosophers we shall cite comparatively few, and confine them for the most part to the more recent books. We shall contend that it is no thoroughgoing answer to one of these horizon problems to restate it in terms of another. This is often hardly more than translation; it is a kind of glorified "passing the buck," and there is an amazing amount of it in contemporary discussion. Sometimes four or five problems are found entangled in an all but impenetrable mass of confusions. The problems most directly concerned in the examples which we shall give in Division C of each of the following

[1] C. I. Lewis and C. H. Langford, *Symbolic Logic*, p. 68; but *cf.* D. F. Swenson, in *Jour. Phil.*, 29, 1932, p. 521, n. 4. J. N. Keynes, in his *Studies and Exercises in Formal Logic*, 1906, pp. 26 *ff.*, distinguishes between conventional, subjective, and objective intension, and between objective and subjective extension.

chapters are indicated by the letters a, b, c, etc., corresponding to the order in which these problems are discussed in this book. Other problems which involve horizons, if they are mentioned in a given treatment, but not discussed in that connection, are marked with daggers (†). It is maintained at the end of Chapter XIII that such horizon concepts are inevitably involved and properly employed in our own discussion; these instances are sometimes marked by double daggers (‡).

D. The third class of mistaken ways of dealing with the problems involving dualities is very common, and is often found in connection with the others. It consists in making unwarranted descriptive statements concerning the problems we have mentioned, forgetting that these problems carry references which point beyond the horizons but do not point to anything definitely descriptive there. The mistake is like running a football out of bounds and expecting to continue the game.

These descriptive statements fall into four principal subclasses, which are given arabic numerals: (1) affirmative; (2) negative, usually negatives of denial; (3) alternative, the "either-or" type of description; (4) antinomial, the "both-and" type. Once in a great while there is some point in including (5) a nihilistic, or neither-nor type. The last two are discussed only in some of the problems where they are of considerable importance. Unwarranted affirmative and negative statements are likely to be reinforced by attitudes of dogmatism. Sometimes, by a turn of expression or emphasis, affirmative statements may become negative, and *vice versa,* like the shiftings of figure and ground in the "staircase" and similar optical illusions.

It may be well to repeat that the three groups of methods of dealing with problems involving horizons are not mutually exclusive. In fact, they could not be. Skepticism is always possible, and is protean in its forms. Anything which is said to be known can always be treated in terms of the knowing. Statements couched in terms of psychology or epistemology must frequently be in marked affirmative or negative terms. The same is true of statements which raise other problems involving horizons, and for that matter, even of treatments of any given problem which are phrased more directly in its own terms and which in a given case may be regarded as adequate.

E. How can we, then, think adequately in problems involving "horizons"? The answer is that we must think according to the dualities of perception and thinking as they have been exhibited in Chapters II and V. These will now be taken up in detail. As in Chapters II and V, so here and in section E of each of the following chapters, they will be indicated by roman numerals I-IX.

(I) From our study of perception and thinking, it appears that patterns are individuated relatively (64.111) to a milieu, (64.112) to prior neuropsychological monads, and (64.113) to cognate patterns, and furthermore that such patterns (64.20) interact selectively with objects in the environment and with objects of thought. If patterns are to be treated adequately, this distinction between individuation and interaction must be observed. Some questions, however, arise from the "predicament" of our thinking-patterns in the midst of their objects. The predicament leads to what may appear to be an inconsistency in the arrangement of our materials. In chapters where we consider the process of perception and thinking, the individuation, indicated by (I), is that of the thinking-pattern, and the selective interaction, indicated by (III), is the selection of an object by the perceptual or thinking-pattern. But in chapters where we consider objects of thought, existent or subsistent, the individuation, indicated by (I), is the relative individuation of the object, *as discerned by our selective perception or thinking,* and the selective interaction, indicated by (III), is, in accordance with our realistic presuppositions, a process exhibited by the object itself.

The difference here is not as great nor as important as it appears to be. After all, the individuation of a perceptual or thought-pattern, is, like that of any so-called object, discerned in thinking about it, and the selective interaction of perceptions and thoughts is also discerned by thinking selectively about them. The question is really that of the relationship of our knowledge of perceptions, thoughts, etc., to our knowledge of other objects. Even if there is an immediate quality about our perceptions of our own thoughts, it has to be mediated in discussion, and the most nearly adequate arrangement we can make is to let a discerned relatively individuated perception or thought stand for the discerning, and the selected interactions stand for the selecting.

Turning for a moment to the objects of thought, we find that any given object may also be (00.113) relatively differentiated with

respect to other objects, which, although once neglected, may come to be recognized in the once undifferentiated milieu. If the objects thus selected are monads, they need not all belong to the same level or realm. Thus, in one way or another, the field of selection comes to include a number of objects in a group, with external relationships and often with marked internal relationships. Prominent among such cognate monads, and sometimes between cognate objects, is (00.1131) the dyadic relationship. This is perhaps the most potent of all metaphysical dualities, but is too explicit to be of much concern to us in the present discussion.

(II) Perceptual and thought-patterns may be (00.13, etc.) either simple or complex, and the individuation of some of these in relationship to some of the others may exhibit dualities. The same difference of level, or sublevel, appears in some objects of thought; it is especially marked in the theory of types, which we discuss at E (II) of Chapter X.

(III) Considering thinking in interaction with its objects, we find that the relative individuation of an object of perception or an object or topic of thought is discerned in a process of selection by a pattern, and that any selection automatically involves a correlative neglect. The selection, with its correlative neglect, is essentially momentary, if not instantaneous; it is a snapshot, not a time exposure. At a later moment or instant, as we shall see, that which was formerly selected and that which was then neglected may have quite different status. Once a given selection is made, the given object of perception or thinking is individuated relatively to a milieu which for the time being, except for qualifications to be later considered, is neglected and left undifferentiated, unexplored. This amounts to saying that our thinking is implicitly dual thinking. It goes on within a horizon. At any given moment, it deals with a "something" in the midst of a "setting." It is our contention throughout this book that *at the given moment we must think of the "something" and the "setting" in ways essentially different.* Thinking deals with a content in the midst of conditions, and the conditions can not be treated as we treat the content. I think of any given selected object of thought, say *A,* by reason of the fact that I am able to think of its neglected background or setting, *non-A*. Usually, while I am *able* to think of *non-A,* I do not "think it out," and my thinking is thus implicitly dual rather than explicitly dual. If I think "*A and non-A,*" I have an explicit duality.

The two members of an implicit duality (or, for that matter, of an explicit duality) can not be dealt with at the same time in the same way. The selected *A* at the given moment can be indicated as such, and can be thought of descriptively or analytically. But, while the neglected *non-A* at the given moment can be indicated, it can be indicated only by reference and contrast to *A,* and whatever else can be done with reference to it, it can not be described nor analyzed. If description or analysis is attempted, the attention shifts from the originally selected *A* to some portion of the originally neglected *non-A,* and the description or analysis proceeds with a different content, which we may now call A_2, but with the formal conditions persisting in a new *non-A,* which we may now call *non-*A_2. This *non-*A_2 at the new moment is neglected, outstanding, and unexplored. If, again, we reduce some of *non-*A_2 to description, calling it A_3, it will be conditioned by an outstanding *non-*A_3, and so on. Thus any alleged reduction to description of a region *non-A* is only reduction of a part† and can not justly claim to be reduction of the whole† of *non-A,* nor description of the whole of it.

Let us now attempt to give names to these procedures. We find that we may call our thought of the "something" either *denotative,* or *connotative,* or both; the two procedures may overlap, or partially coincide. The two words are derived from a root meaning "to mark." Denotation is a marking down, or marking out, or marking off; connotation, literally, a marking together. At any given moment any object which we know denotatively is marked off more or less as a whole, and often for further treatment, while that which we know connotatively has its parts marked, or better, interrelated, more or less consistently, in our descriptions and analyses. Anything which is known is known at least denotatively; it is known denotatively if it is discussed at all. Denotation, as we use the term, thus does not necessarily imply that connotation goes along with it; denotation may be mere demonstrative indication, pointing, or reference, to one individual or more than one, or to a class of individuals. A proper name, if purely demonstrative, may be denotative; in proportion as it is descriptive, it is connotative.

The use of the terms "connotative" and "denotative" has varied somewhat in the works of different writers. According to Mill, the distinction between connotative and denotative is bound up with that between subjects and attributes. A non-connotative term is said to be one which signifies a subject only (for example, John) or an

attribute only (for example, whiteness), while a connotative term is said to denote a subject and imply an attribute.[2] According to Keynes, by connotation we mean the attributes on account of which we place any individual in a class or call it by a name. By denotation we mean the individuals which possess these attributes and which are therefore placed in the class and called by the name.[3]

Sometimes the adjectival or attributive element in connotation is thought of in quasi-Platonic fashion. Sellars, failing to make the distinction which we make in Chapter X between a universal and a generalization, holds that connotation by itself presents us with abstract terms. For example, the connotation of the class "animals" is "animality." Logic calls such abstract terms universals. Sellars defines that which is universal in the sense that it is separable from particular cases and repeatable. The connotation of a class is discoverable in the individuals as known. This rose, for example, must have the characteristics of all roses. A connotation is a complex of universals—in the case of a rose, color, shape of leaves, thorniness of stem, etc.[4]

Creighton expresses the usual view which associates denotation and connotation with the extension and intension of terms. A term is used in extension, or denotation, when it refers to things or names them, and in intension, or connotation, when it describes the qualities or attributes of things. But the distinction should not be regarded as absolute. Every term denotes some object or group of objects more or less directly and at the same time connotes or signifies certain qualities or attributes. Sometimes the one purpose, sometimes the other, is predominant.[5]

Very important for us is the fact that both denotation and connotation refer to objects which are comparatively familiar and accessible, without taking into account the fact that such objects, whether denoted or connoted or both, are all selected in the midst of a setting or condition which must be dealt with at the same time, but dealt with differently. Our accompanying and essentially different thought of the setting, or condition—our non-descriptive, non-analytical, merely vague indication that there is a background in relationship to the "something"—we may call "enotative,"[6] from the

[2] J. S. Mill, *System of Logic,* 1904, p. 34.
[3] J. N. Keynes, *op. cit.,* pp. 17, 24.
[4] R. W. Sellars, *The Essentials of Logic,* 1925, pp. 68 *f.*
[5] J. E. Creighton, *An Introductory Logic,* 1932, pp. 69-72.
[6] See G. P. Conger, *Course in Philosophy,* p. 468, n. 1.

same root as above, but with the prefix indicating a marking or thinking which deals with something outside the denotative-connotative field. Enotative means virtually the same as "transcendent."

The question may be raised as to how we know the enotative even well enough to mention it. The process seems to be analogous to what goes on in perception. Just as in a perceived geometrical figure, the contour of the figure may in a way be used to refer to the ground, so there may be a kind of effect upon what we call the enotative, exercised from within a selected, connotatively known region.[7]

It may be objected that such recognition of the unknown enotative is merely giving a name to ignorance.[8] The proper reply seems to be that this is precisely what should be done first with ignorance; it should be at least given a name to indicate it, whether it later proves amenable to other methods of treatment or not. In fact, the classical fallacy of "argument from ignorance" has its roots here; an *argument* from ignorance is an attempt to make some positive statement where the only proper procedure is to leave the question open, unexplored, or as we should say, enotative.

We speak then of denotation as indicating selected objects for analysis and description, of connotation as affording such analysis or description by exhibiting the relationships of the parts of the selected objects, and of enotation as our reference to the neglected milieu. Thus far, however, one might suppose that our connotative thinking were complete within its own limits, whereas of course the fact is that even there it is fragmentary. We select the tree and neglect the "non-tree" and describe the tree as green and tall and old, but we do not make anywhere near all possible descriptive statements about it. Even in the connotative field, then, there is at least some necessary neglect, and we need a name to give to this ignorance, too. In accordance with the other three terms, we propose to call such thinking *innotative,* with the prefix carrying the suggestion both of a negative, with its failure to mark or note, and of a failure which lies *within* the field already marked out. Such a distinction between innotation and enotation corresponds to the familiar distinction between internal and external relationships, and it is as hard in the one case as in the other to mark out any precise dividing line between them. In fact, enotation usually involves

[7] *cf.* L. Wittgenstein, *Tractatus,* p. 77.

[8] *cf.* A. N. Whitehead, *Science and the Modern World,* 1925, p. 36.

innotation; the two together may be indicated by the term *anotation.* We shall speak of enotation, enotative concepts, enotative reference, enotative regions, the enotatively known, etc., with similar variants for innotation.

Objects of perception or thought, or both, exhibit the relationships corresponding to denotation, connotation, enotation, and innotation. These objects are known by us only as our thinking selects them; we may say that their relative individuation corresponds to the selective interactions of our patterns with them. But, in accordance with our realistic presuppositions, we ascribe to them, in general, not merely such relative individuation, but also selective interactions on their own account, as if they were independent of our thoughts. The interactions of minds, which are characterized by enotation, etc., are distinctive of the neuropsychological levels. It is difficult at first to think of an identity, or a universe, or an infinite appropriating additional elements or engaging in other interactions. We must remember that no problem which we are about to examine necessarily involves all forms of duality in important ways, and that the duality involved in selective interactions may for some problems not be prominent. In cases where it comes up, it does not mean that the objects which exhibit it are monads, but only that they are enough like monads to exhibit selective interactions. But on fundamental grounds, we must suppose, although we can not prove, that objects of thought selectively interact with other objects of thought, and that even the high abstractions of logic and mathematics exhibit this characteristic—as long, that is, as their abstractions continue to be an adequate interpretation of the world. If any abstraction about infinites, for example, leaves a universe or an infinite permanently high and dry, out of interactions with some other reality, then we must reject the abstraction as artificial and distorting, and substitute for it some abstraction which remains more true to the nature of things.

Let us repeat that in succeeding chapters which deal with objects of thought, their properties of individuation, etc., are discussed under the numeral (I) and their selective interactions under (III).

(IV) The interactions of one pattern with another, and quite similarly, the interactions of one object of thought with other cognate objects of thought, involve conspicuous instances of duality. Among them the most important for us are those describable in terms of (00.232391) preemptions, because preemption usually

means that one object so crowds or supersedes another that the second is left out of account. Hence the duality is implicit; one of the objects is left enotative.

This, for example, is the case in reasoning which is expressed in axioms or more or less arbitrary assumptions. Such axioms or assumptions are not argued about—not because no argument can be made, but because the axioms, etc., are allowed to preempt the field and crowd out any opposed axioms or assumptions. Assumptions help to account for the unwarranted affirmative or negative statements considered in D, above, but we are here concerned with assumptions which are more evident. An initial assumption once made may impose a restriction upon future statements throughout the whole course of a process of reasoning. So an initial preemption may be reinforced by a series of considerations which follow upon it (see 00.231393 below) or an initial preemption may develop further in a process of growth (see 00.27, below).

Some of the most important preemptions are those of habit and memory, which tend to control future actions in accordance with past experiences.

Processes of (00.231392, etc.) conflict between two objects of thought obviously exhibit dualities, but the dualities are often explicit. Sometimes one member, however, may be left enotative. Cohen says that the principle of polarity makes us search for that which prevents operative causes from producing greater effects than they do.[9] Processes of (00.231393, etc.) reinforcements and (00.231394) interchanges also exhibit explicit dualities. In the latter, particularly, some structures and processes may often be left innotative.

(v) We noted that, unlike perceptions, thoughts often seem at least to be marked by no boundaries more definite than those which obtain between two concepts, *A* and *non-A*. It appears at first sight as if boundary conditions, often apparent in perception, have been left behind in further symbolization. We noted, in Chapter V, however, that certain vague characteristics of patterns, whereby there seem to be tensional changes at their inception or discharge, may mark just such variations in them as could correspond to boundary conditions. This, as we said, accords with Cohen's theory of twilight zones in psychical and physical existences.[10] If there are these or any other boundaries between thinking-patterns or con-

[9] M. R. Cohen, in *Jour. Phil.*, 24, 1927, p. 678. [10] *ibid.*, pp. 677 *ff.*

cepts, they are obviously ordinarily left out of account; as regards any given *A*, they are enotative, and as regards any given *A* and *non-A*, they are innotative.

We shall see that the matter of possible boundaries varies in different problems. The most important case occurs when, carried into problems of logic, what we have said supports the view, held by other writers for other reasons, that the laws of contradiction and excluded middle are too rigid.

(VI) It seems plausible to suppose that an enotative region might be progressively reduced to a connotative region by a process of "biting into" it and growing at its expense. To employ this method of dealing with the duality would not be quite the same as making unwarranted affirmative statements about the enotative region; the statements which are made as a result of a process analogous to growth tend to acquire a certain momentum and even to carry a kind of practical and dynamic justification with them as they go. Undeniably they afford a kind of confidence in the procedures which are or have been going on.

In this way, a perceptual or thought-pattern or an object *A*, selected at a given moment, changes its content with respect to its *non-A*, including or excluding parts of what was originally left enotative or innotative. Ordinarily nothing very definite is said about boundaries, unless reference is made to the first or last terms of a series. Some writers, relying in one way or another upon the growth or extension of present experience, have approached the enotative with a good deal of confidence. In fact, this approach in terms of a process of growth, or becoming, offers some of the most promising ways of dealing with enotatives. Past experience as it accumulates is sometimes thought to afford guidance for the future†. Köhler finds a felt definite direction traceable to certain physiological states beyond a restricted field of experience. Though not experienced actually, those physiological states seem to be definite enough, for in general our reading and talking go the right way, sufficiently determined by those "states beyond."[11] The words "in general" and "sufficiently" here are loose enough to make room for the margin of uncertainty, the enotative reference, which is for us essential in such guidance.

In employing this procedure, a good deal of importance may be attached to potentialities†, as in theories of "constructibility," or to

[11] W. Köhler, *Gestalt Psychology*, pp. 270 *f*.

"freedom from contradiction." These contemporary discussions raise questions which we must consider in detail later; we shall find that the principal question involved is whether these doctrines make adequate provision for possibilities which carry open enotative reference.

Sometimes the process of change is formulated in a law of the formation of a series, or in a so-called law of nature. Thus Whitehead says that unoccupied events possess a definite† character expressive of the reign of law in the creative advance of nature, *i.e.*, in the passage of events.[12] But the so-called "laws" here carry their own enotative reference; if the laws were properly and completely stated, this would be revealed. They are relative to past events, not absolute† as regards future events.

Sometimes enumeration or other study of individual cases or instances leads to estimates of increases or decreases in probability and to certain attitudes of greater or less assurance, expectancy, etc., with regard to the enotative. The difficulty here is not merely the enotative reference to an unknown future, but also the enotative or innotative reference to an imperfectly known past. Stated in numerical terms, we do not know precisely either the numerator or the denominator of the fraction expressing the probability. If the experience of the past has been so long continued and unbroken that assurance about the future markedly predominates, even though we can not express the probability precisely, we may adopt its suggestion with regard to the future, on the ground that the probability exceeds reasonable doubt. Then the question is, in view of possible if not actual opponents, what is to be defined and dignified as reasonable, and what is not.

Sometimes the process of change is treated as if it approached, and even as if it actually reached a limit† of increase or decrease, although when examined closely, it is found that often the description of the process in the region of the alleged limit has to be left innotative. Again, the process seems in puzzling fashion to alternate, reaching a limit, or at least a halting place, and then, with a fresh start, continuing beyond it to a new halting place, as in the boys' game of leapfrog. In such a procedure it is hard to know where to stop; the winner, if there is one, is the party who stays in the game longest. An example of leapfrog procedure, so simple that it appears quite absurd, would be the attempt to describe the great-

[12] A. N. Whitehead, *Principles of Natural Knowledge*, p. 97.

est cardinal number as an even number. Any "greatest" number *specified* as even may at once be rendered odd by the addition of 1, but this, in turn, by a new leap, can be rendered even by another addition of 1, and so on. This is simple enough; and yet the "diagonal procedure"[13] is another example of the same inadequately interpreted method.

(VII) The process analogous to biparental reproduction, where it occurs in neuropsychology, or in the syllogisms of pure logic, is usually explicitly rather than implicitly dual and need not directly concern us. If, however, its subordinate processes were taken into consideration, there might be ways of accounting for the propositions of the form "either-or," "both-and," and "neither-nor."[14]

(VIII) Thought-patterns may be integrated into end-reaction complexes and other more inclusive monads, but objects of thought are not ordinarily integrated into anything other than objects of thought. Thus they remain at the same level, amounting if to anything only to organizations of different degrees of complexity. This is another way of saying that while thought-patterns are monads with the monadic characteristics, the objects of thought are not necessarily monads.[15]

(IX) Especially as objects of thought become more complex, they exhibit differentiations of constituent objects. The process of differentiation here is like that of individuation; in fact, it is frequently that of individuation, seen from the point of view of more inclusive rather than less inclusive monads. Differentiation, too, is relative, and within the complex objects of thought some parts may thus be left out of account, or innotative.

Sometimes matters are left innotative on account of ignorance rather than any difficulty more nearly fundamental. Eaton, in discussing the distinction between the so-called analytical and synthetic judgments, says that whether the predicate of a judgment adds something to what is meant by the subject depends on what one means by the subject. Ignorance is the great restricter of meanings and ignorance renders external to objects certain properties, which on different definitions of these objects become internal to them.[16]

[13] For the diagonal procedure, see A. Fraenkel, *Einleitung in die Mengenlehre,* 1928, pp. 46, 49, 55; *cf.* pp. 31, 197.

[14] See G. P. Conger, *Epitomizations*, pp. 444 *ff*.

[15] *ibid.*, p. 17.

[16] R. M. Eaton, *Symbolism and Truth,* 1925, p. 131.

Innotation is often combined with enotation. Wittengenstein says that contradiction vanishes, so to speak, outside, and tautology inside all propositions. Contradiction is the external limit of the propositions, tautology their substanceless center.[17]

Within a group, some relationships of constituent members are usually left innotative, although often without clear recognition of the process involved. Eaton maintains that in a symbolic group the quality, relation, or operation is not related to its terms, for this would involve an infinite† series of relations. He can only say that the terms enter into the peculiar unity† which makes them a group or a fact. If the unity of a fact depended on the knowledge of such an endless series of relations, the fact would not be presented as a whole.† The unity is given immediately.[18] Bertrand Russell holds that when a relation obtains between two terms, the relations of the relation to the terms and of these relations to the relation and the terms, and so on *ad infinitum*†, though all implied by the proposition affirming the original relation, form no part of the meaning of this proposition.[19] According to our view, both Eaton and Russell here seek, if they do not find, justifiable ways of disregarding the "endless" series of relations. It is better, in accordance with our general principles, merely to recognize once for all the fact that the relations of relations, etc., are left innotative.

[17] L. Wittgenstein, *Tractatus*, p. 111.

[18] R. M. Eaton, *op. cit.*, p. 79.

[19] B. Russell, *The Principles of Mathematics*, Vol. 1, 1903, p. 51. This book will hereafter be cited as *Principles*.

PLAN OF CHAPTERS VII-XX, INCLUSIVE

A. INTRODUCTION, DEFINITIONS, ETC.

B. SOLUTIONS IN TERMS OF PSYCHOLOGY AND EPISTEMOLOGY
- (α) SKEPTICISM
- (β) PSYCHOLOGISMS
- (γ) REALISM-IDEALISM
- (δ) INTUITIONISM

C. SOLUTIONS IN TERMS OF OTHER HORIZON PROBLEMS
- (a) IDENTITY, DIFFERENCE, ETC.
- (b) NEGATIVES
- (c) CONTRADICTORIES
- (d) INDUCTION, GENERALIZATION, UNIVERSALS
- (e) THE LOGICAL UNIVERSE
- (f) "NOTHING"
- (g) VALIDITY, TRUTH, PROBABILITY
- (h) LIMITS
- (i) CONTINUITY
- (j) INFINITES
- (k) BEGINNING-ENDING
- (l) ORDER, DISORDER
- (m) WHATNESS, THATNESS
- (n) ACTUALITY, POSSIBILITY, NECESSITY
- (o) BEING, NON-BEING
- (p) UNITY, PLURALITY, ETC.
- (q) WHOLES (TOTALITIES), PARTS
- (r) ABSOLUTES, RELATIVES
- (s) SPACE-TIME
- (t) THE PHYSICAL UNIVERSE

D. SOLUTIONS IN TERMS OF VARIOUS UNWARRANTED AND INADEQUATE STATEMENTS
- (1) AFFIRMATIVE STATEMENTS
- (2) NEGATIVE STATEMENTS
- (3) ALTERNATIVE STATEMENTS
- (4) ANTINOMIAL STATEMENTS
- (5) NIHILISTIC STATEMENTS

E. SOLUTIONS IN TERMS OF SOME MONADIC CHARACTERISTICS WITH ENOTATIVE AND INNOTATIVE REFERENCES
- (I) RELATIVE INDIVIDUATION
- (II) DIFFERENCES OF SUB-LEVEL
- (III) SELECTIVE INTERACACTIONS
- (IV) PREEMPTION, CONFLICT, ETC.
- (V) BOUNDARY CONDITIONS
- (VI) PROCESSES ANALOGOUS TO GROWTH
- (VII) REPRODUCTION, ETC.
- (VIII) INTEGRATION
- (IX) DIFFERENTIATIONS

PART TWO

Some Problems of Logic

"We shall be better and braver and less helpless if we think that we ought to inquire than we should have been if we indulged in the idle fancy that there was no knowing and no use in seeking to know what we do not know."

—Plato, *Meno,* 86 (translation by Jowett).

CHAPTER VII

Identity and Difference

"We step and do not step into the same rivers."
—Heraclitean fragment: J. Burnet, *Early Greek Philosophy*, 1920, p. 139.

A. We begin our study of some problems of logic with the one which is formulated in the first of the classical laws, identity. Along with it we shall consider its correlative, difference, and, since for our purposes they are in one way or another closely related, we add sameness, otherness, definition, equality, inequality, equivalence, and inequivalence. Identities and differences suggest questions of individuality, entity, independence, and uniqueness.

B. The law of identity is often studied in connection with some characteristics of the process of knowing.

(α) Sometimes there is a kind of marginal skepticism. According to Eaton, it is possible† that reality can not be trimmed to the neat dimensions of thought. There may be nooks and corners of the real hidden from the knowledge which demands identity and diversity of its objects. Eaton, however, thinks that the law of identity is self-affirming, and that even if it is denied, it is assumed.[1]

(β) Of course the judgment of identity involves psychological processes, and attempts are sometimes made to solve the problem in psychological terms. Bradley took resemblance or similarity or likeness, in the strict sense, to be the perception of the more or less unspecified identity or sameness of two distinct things, implying the distinct consciousness that the two things are two and different.[2] According to Adler, an "identical" verbal term may be rendered diverse by the definitive force of its propositional context.[3] Eaton thinks that the judgment "*A* is identical with *B*" tells us that a single fact permits different analyses.[4] The point essential for us is that the identity carries the suggestion of difference left innotative. Again, for Eaton, the judgment of self-identity, "*A* is *A*," informs

[1] R. M. Eaton, *Symbolism and Truth*, pp. 206, 208.
[2] F. H. Bradley, *Appearance and Reality*, 1893, pp. 592 *f*.
[3] M. J. Adler, *Dialectic*, p. 195. [4] R. M. Eaton, *op. cit.*, p. 54.

us that something is singled out, fastened on by the attention, and referred to through symbols.[5] This suggests selection and neglect.

The judgment "*A* is *A*," when analyzed, is seen to involve the psychology of recognition. Baldwin holds that consciousness, starting with "this and not other," then identifies "this" and "the same." When a meaning of recurrent sameness has arisen, there is a longitudinal before-and-after reading of meanings.[6] For us, any such case of identity involves an undescribed difference, *e.g.*, in the time elapsing between cognition and recognition. This may be suggested even by Leibnitz's identity of indiscernibles, which, according to Baldwin, means in principle that in the absence of discern*ible*† difference, two or more objects are judged to be one and the same in recurrent experience.[7]

Eaton's treatment of Leibnitz is not so much psychological as logical. He thinks that the postulate† of the identity of indiscernibles rests upon the view that only that which is distinguished by predicates is distinct.[8] Predicates, then, are marks of difference; and identity, for us, means disregard of some predicates, which are left enotative or innotative.

(γ) The problem of identity is treated in terms appropriate to the controversy between idealism and realism, when Eaton says that the law of identity, along with that of contradiction, if taken as a statement of general intention in the use of symbols, is *a priori* in the sense that it is a rule laid down by mind for its own guidance. According to a more realistic interpretation, Eaton goes on to say, the two laws are construed not merely as general principles of symbolism, but as general truths about objects. They express the minimal conditions of the being of objects which are presented or represented in empirical and rational knowledge.[9] For Lovejoy, it is necessary to know or to postulate certain propositions about the class† *cognoscenda* before we can compare it with the class *data,* to ascertain whether or not the two satisfy our criteria of identity.[10]

It is to be noted that in these treatments, identity, whether in mind or world, is not made to preclude difference. This is easy to see, for example, in the Boole-Schröder algebra, where $A = B$ refers to class extension. "Man" = "featherless biped," because,

[5] R. M. Eaton, *op. cit.*, p. 44.

[6] J. M. Baldwin, *Thought and Things . . .* , Vol. 1, pp. 192, 194.

[7] *ibid.*, Vol. 2, p. 402. *cf.* R. Carnap's "*Grundrelation,*" in *Der logische Aufbau der Welt*, 1928, p. 110.

[8] R. M. Eaton, *op. cit.*, p. 71.

[9] *ibid.*, pp. 206-8.

[10] A. O. Lovejoy, *The Revolt Against Dualism*, 1930, p. 16.

though the two terms ordinarily have different connotations, they denote the same class† of objects.[11]

To us the different connotations are matters of experience rather than of the difference between intension and extension, or mind and world. According to E. Meyerson, identity is traced to mind's operations upon reals which are different; but we should say that in mind and world alike the horizon principles hold.

(δ) The intuitional and near-intuitional theories of knowledge have also been invoked. William James regarded Bergson as his deliverer from false theories of identity, and from trying to discover some mode of conceiving the behavior of reality which should leave no discrepancy between it and the accepted laws of the logic of identity.[12] W. E. Johnson says that the conceptions of identity and otherness are two independent indefin*ables*†, the understanding of which is required in order intelligently to accept the truth of any proposition which explicitly or implicitly involves the notions of identity or otherness. The indefinable means what is understood so directly and universally that it would be mere intellectual dishonesty to ask for further definition.[13]

C. We find some attempts to answer the problem of identity in terms of other problems, which also carry and often conceal enotative or innotative references.

(a) According to Lewis and Langford, it is both possible and advantageous to define the relation of logical equivalence, though the definition does not allow us to dispense with the primitive idea† of equivalence, since that is the defining relation.[14]

(b) It is obvious that identity can be treated in terms of negatives by denying to it the qualities of difference, and *vice versa.*

(c, d) Ramsey maintains that *Principia's* definition of identity does not define the meaning for which the symbol for identity is actually used. The definition makes it self-contradictory for *two* things to have all their elementary properties in common. Yet for Ramsey this is perfectly† possible†, even if in fact it never† happens.[15] We should say that it is possible because of enotation or innotation, or both.

[11] C. I. Lewis and C. H. Langford, *Symbolic Logic,* p. 28.
[12] W. James, *A Pluralistic Universe,* 1916, pp. 214 *f.*
[13] W. E. Johnson, *Logic,* Part 1, 1921, pp. 105 *f.*, 195 *f.*
[14] C. I. Lewis and C. H. Langford, *op. cit.,* p. 124.
[15] F. P. Ramsey, *The Foundations of Mathematics . . . ,* ed. R. B. Braithwaite, 1931, p. 31.

(e) Broad says that the laws of identity, contradiction, and excluded middle apply to a fixed universe of discourse.[16] But for us a fixed universe of discourse is relative to a non-fixed, which is left enotative.

(f) The problem of identity is given a curious twist in the statement of Bogoslovsky, that nothing is identical, even with itself.[17] But the statement is too strong; it asserts too much about the relationship of two terms, both of which carry enotative and innotative reference.

(h) The notion of limits is suggested when Whitehead regards identity as a limiting case of equivalence.[18] But the notion of limits, as we shall see, has its own difficulties.

(i) According to Sheldon, one and the same event can be occurring at continuously successive moments.[19] This avowedly introduces difference into the sameness, but, besides, makes it necessary that one of the two shall be treated enotatively or innotatively.

(j) Appeal is sometimes made to infinites. Eaton says that Leibnitz's postulate of the identity of indiscern*ibles*† rests upon the doctrine that an infinite number of universals† would determine the individual uniquely. Again, Leibnitz postulates that the indeterminate quality of our knowledge would disappear in an infinite knowledge; but Eaton properly says that this leaves finite knowledge no better off.[20]

Sheldon maintains that, in the case of an apple and its qualities, sameness runs undiminished through all the infinite list of qualities, whatever their difference.[21] But the doubtful assistance to be derived from considerations of infinites is shown when Bradley says first, that it might be said that an individual which is finite or relative† turns out in the end† to be no† individual, and that individual and finite are inseparable characters; and then says that it might be said that the individual is finite and there can† not be an absolute† individual.[22]

[16] C. D. Broad, *Scientific Thought,* p. 83.

[17] B. B. Bogoslovsky, *Technique of Controversy,* p. 34.

[18] A. N. Whitehead, *A Treatise on Universal Algebra,* Vol. 1, 1898, pp. 5 *f.* This book will hereafter be cited as *Universal Algebra.*

[19] W. H. Sheldon, *Strife of Systems,* p. 479. In contrast to this, E. W. Hall, in *Monist,* 41, 1932, pp. 538 *ff.,* contends, in effect, that an identity need not exhibit continuity. We should reconcile both views in terms of innotation.

[20] R. M. Eaton, *op. cit.,* pp. 72, 74.

[21] W. H. Sheldon, *op. cit.,* pp. 462 *f.*

[22] F. H. Bradley, *The Principles of Logic,* Vol. 1, 1922, p. 190.

In his doctrine of the various descriptions of substances, McTaggart runs into a whole cluster of notions which should be left enotative or innotative. For him, an "exclusive description" applies to only one substance, and a "sufficient description" is one which is entirely in terms of characteristics, containing no undescribed substances. A given substance, *A*, if exclusively but not sufficiently described, must be dissimilar to all† other substances. The possibility† of this depends upon the existence† of *B*, and the existence of *B* depends on its dissimilarity to all other substances. And this depends on the existence of *C*, and this on its dissimilarity to all other substances, and so on. If this series is infinite†, it is vicious. For, starting from the existence of *A*, each earlier term requires all the later terms, and therefore requires that the series should be completed†, which it can† not be. If, therefore, the series is infinite, *A* can not be dissimilar to all other substances, and can not exist.[23] For us, the thing which makes a description appear exclusive or sufficient is that somewhere the undescribed substances are left enotative or innotative, as if the description were complete. Even if they are allowed to start in an "infinite regress," this regress also must somewhere be left the same way.

(m) Ramsey thinks we can know that there are two indistinguish*able*† things, without knowing what they are.[24] But for us the weight of his statement belongs better on the adjectival ending expressing possibility than on the suggestion of valid knowledge of indistinguishables.

(n) In the same passage, Ramsey says that we can not know any two particular indistinguish*able* things; to give the two things different names implies that they have the different properties of having those names. Yet we can perfectly well consider the possibility. At another place, he says that two propositional symbols are regarded as instances of the same proposition, when they express agreement and disagreement with the same sets of truth possibilities of atomic propositions. The appeal to possibility is somewhat more involved when he says that whether there are indefin*able* classes or not is an empirical question; both possibilities are perfectly conceiv*able*.[25] We shall see later that possibility always involves an open enotative reference. What Leibnitz or any one following him actually discerns in a given case is identity, inasmuch

[23] J. E. McTaggart, *The Nature of Existence*, Vol. 1, 1921, pp. 102, 104, 108.
[24] F. P. Ramsey, *op. cit.*, p. 31. [25] *ibid.*, pp. 9, 22, 31.

as, or insofar as, differences are *undiscerned*. To say that they are indiscern*ible* is to make a negative statement about a possibility that, strictly speaking, ought to be left enotative. Tautologies, defined as agreements with all† possible truth-functions of arguments,[26] are discussed in Chapter XIII.

(p, q, r) In strictly monistic systems, identity and difference are regarded as partial aspects, modes, or qualities of the whole or the absolute reality†.

(r) The attempt is often made to strengthen the notion of identity by attaching to it the adjective "absolute" or "ultimate,"† but the effect of this is merely to shroud the difficulty in those terms, which should be examined for their own enotative or innotative references.

(s) Alexander traces the category of identity to the fact that to occupy space-time is to occupy it.[27] For us, when we consider psychology, it is plain that in order to express the judgment of identity, "*A* is *A*," in language, or formulate it in thought, or even to perceive the objects which it relates, time and space are required, and the second *A* is distinct from the first in time and space. Even if the act of recognition obliterates the separateness, by allowing one *A* to fuse with the other, cognition is not recognition. One *A* is cognized, the other is recognized. One has occurred first and the other second, and the difference is left innotative.

D. (1, 2) In the views considered above there are several examples of affirmative statements. We saw that Leibnitz's identity of indiscernibles amounts to a negative statement, in a problem which properly should be left only enotative or innotative.

(4) If we say "*A* is identical with *B*, and *A* is not identical with *B*," we have the antinomial method of dealing with enotative or innotative references. This may be expressed by saying "*A* is the same as *B*, and *A* is different from *B*," and either of the two antinomies may be generalized in the statement that things are the same and also different. When interpreted by Hegel, this is enough to yield the identity of opposites; it is only a step from the Kantian antinomy to the Hegelian antithesis and synthesis. Sheldon makes much of the point that difference does not exclude sameness; that sameness and difference exclude each other is the purest dogma and the root cause of strife among the philosophical systems. The really

[26] F. P. Ramsey, *op. cit.*, p. 9.
[27] S. Alexander, *Space, Time, and Deity*, Vol. 1, p. 205.

fruitful principle for understanding reality is sameness-in-difference. Any two things may be identical and yet different. This allows a variety of manipulations. For example, a thing or datum remains the same, while its qualities are different, and any two things may be identical and yet different. We can† say *A* is *A* throughout all changes—the principle of externality; or we can say *A* is *B*—the principle of internality; or, finally, we can say both are true.[28]

According to the epitomization hypothesis, each relatively individuated monad is continually appropriating other prior or cognate monads. If this process is kept in view, there need be no difficulty in saying that *A* is *B*, where the predicate is different from the subject. We might say, too, that by (00.23134) combinations of different appropriations of cognate monads, *A* is *B*, and *A* is *A* at the same time. The judgment of identity is, for us, a matter of degree, with something left enotative or innotative; Sheldon, writing of sameness, seems to call it once relative and once absolute.[29]

Sheldon, as we shall see in our study of monism, holds before us an "either-or" choice, but eventually adopts the "both-and" view. As an example of what can be done with the "both-and" interpretation of identity and difference, suppose, says Sheldon, the simplest possible dyad—any two things which possess both sameness and difference. Call them *A* and *B*. Then *B*, being the same as *A*, must have the relation to *B* which *A* has, to wit, difference. *B* is therefore different from *B*. This of course does not destroy the identity of *B*, as sameness and difference are not mutually destructive. This second *B* should be called by a new name *C* to distinguish it. Now *C*, being the same as *B*, must be, as *B* is, different from itself, so we have *D*. The series is indefinitely† long.[30]

In all such cases, according to our view, neither member of the antinomy can quite exclude the other; the two, in fact, must somehow get on together, and they can do so only when one member is neglected and left enotative. The principle which, for us, is real productive duality, *i.e.*, (00.431) the integration of monads in the dyadic relationship at one level into monads of a new level, is discussed under E (VIII).

E. Our point is that the legitimate ways of dealing with the problems of identity and difference must recognize the enotative and innotative references involved.

[28] W. H. Sheldon, *op. cit.*, pp. 456, 462, 474 *f.* [29] *ibid.*, pp. 474, 495.
[30] *ibid.*, p. 509.

(I) An identity, or identical thing, is not merely an individual, or individual thing; it is an individual maintaining itself, maintaining such a degree of constancy in its interactions that it is not supplanted or disintegrated. Even as individuation is relative, so identity is a matter of degree. Any identity or any identical thing, is (00.111) relatively individuated in the midst of a milieu which primarily is left enotative, and properly referred to as non-identical (by the negative of suspension†), rather than not-identical (by the negative of exclusion†) or not identical (by the negative of denial†). The non-identical *may*† be identical, and it may not. Similarly, non-difference may be different, and it may not. Any milieu which is not-identical may be recognized or referred to as other, and it may include (00.112, 00.113, etc.) other monads, terms, or objects.

Bradley holds that to say that the subject of a proposition is the same implies and may† be meant to convey the truth that the attributes differ.[31] We should say that if an object is identical with itself or with another object, there are still differences of times or aspects, and when the object is taken to be identical, such differences are left enotative or innotative. Again, if an object is said to be other than its former self or different from itself, its identity is left enotative or innotative. We shall see, in III, below, that what has been said about identity and difference applies also to equality and inequality. Where there are other objects, etc., or where an identical object is taken at successive times, there will be resemblances and differences. Resemblances and differences are most clearly seen as properties of classes† of objects. All† objects belong at least in the class called the universe† of discourse; this statement, in the plural, is enough to give them at once minimum difference and minimum resemblance, and to show that resemblance and difference are not inconsistent with each other. In fact, they coexist. When objects are said to resemble one another, differences are left enotative (or innotative), and when differences are asserted, resemblances are left.

(II) In any complete discussion of the subject, provision has to be made for some complications which are especially confusing because differences of sublevel or type are here so likely to breed dualities. We must be prepared to meet supposedly identical identities, as, in the supposedly self-evident axiom† about things equal to the same thing, we are supposed to see equivalent equalities.

[31] F. H. Bradley, *op. cit.*, Vol. I, p. 27.

What has been said above about enotative or innotative differences lurking in our identities may be construed in terms of different identities, but at the risk of over-confident affirmative descriptive statements.

Problems can be constructed about identical differences and different differences. Baldwin speaks of the recognition of the same as "not-other."[32] This recognition means that the same and the other differ, and thus involves a second degree of difference. We have first $A\ R_1\ A$ (the same) and $A\ R_1\ B$ (the other), then $A\ R_1\ A \quad R_2 \quad A\ R_1\ A$ (the same and the same) and $A\ R_1\ A \quad R_2 \quad A\ R_1\ B$ (the same and the other). Baldwin is concerned with the last of the four expressions.

Russell says that against the notion of specific differences it may be urged that if differences differ, their differences from each other must also differ, and thus we are led into an endless† process. He seeks to escape by a logical or epistemological turn. He says that an endless process is not to be objected to unless it arises in the analysis of the actual meaning of a proposition; here the process is one of implication rather than analysis, and so it is harmless.[33] According to our view, in all such intricate and subtle problems, it is useless to try to exhaust the possibilities† of further complication, and we might as well be content, early as late, to leave the question of the differences of differences enotative or innotative.

Especially complicated is a statement of Russell about identities of differences. He says that it seems plain that even if differences did differ they would have something in common.[33] This is quite in accord with our general argument. But, says Russell, the most general way in which two terms can have something in common is by both having a given relation to a given term.[33] Let us suppose, then, that if differences are to have something in common, they must both have a given relation to a given term.

Now, Russell goes on, if no two pairs of terms can have the same relation, it follows that no two terms can have anything in common, and different differences will not be in any definable sense instances of difference.[33] The question about sameness or difference of terms is here made over into a question about the sameness and difference of relations, and a negative statement made to the effect that the relations can not be the same. He overlooks the fact that they, too,

[32] J. M. Baldwin, *op. cit.,* Vol. 1, pp. 193 *f.*
[33] B. Russell, *Principles,* pp. 50 *ff.*

must be in some respects the same, in order to be discussed at all. Again, he need not attempt to define different differences, nor to describe denotatively or connotatively the respect in which different differences should have something in common; this respect, if there be such, should be left enotative or innotative. In the face of his supposed difficulties, Russell is forced to conclude that the relation affirmed in the proposition "*A* differs from *B*" is a general relation of difference and is precisely and numerically the difference between *C* and *D* in the proposition "*C* differs from *D*."[33] But this statement is not merely insecurely grounded, as above; it obscures for Russell the fact that what is said about differences does not necessarily apply without qualification to differences of differences, which may belong to another type.

Ramsey thinks that of two things, *A* and *B*, there is nothing† self-contradictory† in *A* having any† self-consistent set of elementary properties, nor in *B* having this set, nor therefore, obviously, in both *A* and *B* having them, nor therefore, in *A* and *B* having all† their elementary properties in common. There are, however, further possibilities.† Two things might have all their elementary properties in common without being identical in other properties. Ramsey says that, apart from the axiom† of reducibility, they might differ in functions of a higher order, and that we ought not to define identity as agreement in respect of all predicative functions, because two things can† clearly agree as regards all atomic functions, and therefore they can clearly agree as regards all predicative functions, and still they are, after all, two things and not, as the proposed definition of identity would involve, one thing.[34] In other words, the difference of order or type here, taken together with enotative or innotative references, makes it possible to regard the two terms in either way.

(III) In the judgment of identity, both subject and predicate are relatively individuated with respect to an enotative milieu of possibilities, relations, and other terms, and the subject is to be regarded as selectively appropriating a predicate.

Several writers have noted that if we say that *A* is identical with *B*, the statement is open to exceptions. For Bradley, equation is an indirect way of stating difference. Whenever we write the equality sign there must be a difference, or we should be unable to distin-

[34] F. P. Ramsey, *op. cit.*, pp. 30 *f.*, 50.

guish the terms with which we deal.[35] Whitehead says that the proposition asserted by the equation $A = A'$ consists of two elements, a truism and a paradox. The truism is the partial identity of both A and A', their common A-ness. The paradox is the distinction between A and A', so that A is one thing and A' is another thing; and these things, being different, must have in some relation diverse properties.[36]

When a judgment of identity is, like other judgments, interpreted in terms of selective appropriation, we mean, by saying that A is identical with B, that one of these has appropriated at least so many of the relationships of the other that the remaining relationships of both may be neglected, innotatively or enotatively, and one term may be regarded as having supplanted the other. This is closely allied with the view of Montague, according to whom all propositional relations may be resolved into the basic relation of identity, but the judgment of identity involves identity of denotation with diversity of connotation.[37] We should say identity of denotation and perhaps even of connotation, with possible diversity of innotation and enotation.

The judgment of self-identity, "A is (identical with) A," may mean, then, that one A appropriates so many possibilities, relations, etc., of another that it comes to supplant it; or, it may be understood negatively†, as meaning that A appropriates something not specified but rejects it, and remains A, as before. The fact that whatever is appropriated is not specified, but is left enotative, does not mean that the appropriation and rejection are not selective. They are selective at least with respect to *non-A*.

The fact that all appropriations and rejections are "selective-neglective" explains how identity or difference is frequently affirmed with such confidence and apparent adequacy. When either is explicitly affirmed, the other is left implicit; or, in other words, when one is denotative or connotative or both, the other is enotative or innotative or both. Enotation and innotation alike mean not that identity or difference is denied, but that the question is left open and unexplored.

It is clear from the statements of Whitehead, cited above, that equality, like identity, carries an enotative or innotative reference.

[35] F. H. Bradley, *op. cit.*, Vol. 1, p. 27.

[36] A. N. Whitehead, *op. cit.*, pp. 6, 7. *cf.* C. I. Lewis and C. H. Langford, *op. cit.*, pp. 189, 285, n. 2.

[37] W. P. Montague, *The Ways of Knowing*, 1925, pp. 78 *ff*.

An apparent exception occurs in mathematical logic, where equality is defined as mutual implication.[38] If A implies B, and B implies A, then $A = B$. That is to say, if each of two propositions is deducible from the other, they are equivalent. But here again, even more than in the simpler cases, "equivalent" means equivalent in the respects indicated or chosen, and thus equivalent in a restricted sense. It is a case where an abstract symbolism can be manipulated until it loses touch with the world from which it was derived.

(IV) The affirmation of identity is sometimes made as an assumption. Eaton calls the identity of indiscernibles a postulate,[39] and Johnson thinks that it has no logical justification.[40] According to Lewis and Langford, any mathematician would feel free to take the relation of identity and its properties for granted, and would use the sign = as a shorthand expression of that relation.[41]

The method of assumption may be used in the problem of difference. Ramsey says that by adopting a convention proposed by Wittgenstein we can eliminate the sign of identity, or equality, from any proposition and replace it by a convention that different signs must have different meanings.[42] Any such assumption or convention has the effect of a preemption and has a certain strength, but its strength is lessened by the fact that the opposite assumption might have been made. Either assumption, as a matter of fact, is a selection of one alternative with corresponding neglect of the other. That Wittgenstein's proposed convention does not help much is evident when Ramsey points out that it puts us in a hopeless position as regards classes, because, having eliminated "=," we can not use "$x = y$" as a propositional function in defining finite classes.[42]

(V) Bogoslovsky says that definition is always a separation of what belongs to a concept from what does not. In other words, it is the establishment of a definite barrier around the dominion of a concept, so that it may not illegitimately claim a larger territory than is given to it by its definite meaning.[43] Mention of a "barrier" here is, of course, a figure of speech; but it is possible to argue that indications of a boundary between identity and difference are still

[38] C. I. Lewis, *A Survey of Symbolic Logic,* 1918, p. 122. This book will hereafter be cited as *Survey*.

[39] R. M. Eaton, *op. cit.,* p. 71.

[40] W. E. Johnson, *op. cit.,* Part I, p. 194.

[41] C. I. Lewis and C. H. Langford, *op. cit.,* p. 116.

[42] F. P. Ramsey, *op. cit.,* pp. 31, 49.

[43] B. B. Bogoslovsky, *op. cit.,* pp. 7 *f.*

more subtly reflected in language and thinking. When we come to think of it, we do not say "identity difference;" we say "identity *and* difference," or "identity, difference" (with the comma), or in some other way we indicate a zone or a moment of transition or conjunction between the two.

(VI) It appears that a way of dealing adequately with the doctrine of identity is to consider the process of identi*fication,* and to interpret the judgment of identity as a process of appropriation, fusion, or identification which takes place in time. We can then say, for instance, that in the judgment "*A* is *A*," the first *A* and the second *A* fuse and thereafter are one, and that the reverse process characterizes judgments of difference.

Something like this is the view of Sheldon, that the little bit of externality, identified with the primitive monad of the original single datum, implies a further bit, and this another, and thenceforth passes out to any required degree. For Sheldon, sameness and difference coexist and do not exclude one another, but, more than this, they furnish a creative principle and show that real duality is productive. Given any one fact, we are at liberty to fasten on it in its isolation, or to consider its relation to the rest of the world, and we may consider either the distinctness of these aspects or their identity. If we choose to regard the identity, then *A,* the thing in itself, is *B,* the thing as related to other things. Now from their difference, *A* is in some relation to *B, i.e., A* is *RB*. But by sameness *B* is *A*. Hence *B* is *RB,* and the second *B* is different from the first. In this way novelty arises, and the twofoldness of a single object is directly seen to operate, to create another object. The same principle can be applied to the problem of causation, to the genesis of classes† and universals† and to certain practical issues.[44]

We should make a distinction between "*A* is *B*" and "*A R B,*" calling the first an incorporative and the second (usually) a non-incorporative appropriation. This distinction makes it unnecessary to reduce relational to predicative judgments.[45] We should not say *A* is *RB*; it is misleading to say that the composition of the candle flame is the relationship it bears to the effect of the flame,[46] and to say that it *is* in that relationship is more directly expressed by *A R B*. Furthermore, our recognition of the process of growth as frequently leading to reproductions makes Sheldon's derivation of

[44] W. H. Sheldon, *op. cit.*, pp. 494 *ff.*, 507 *ff.*, 513 *ff.*

[45] *cf. ibid.*, pp. 53 *ff.*

[46] *ibid.*, p. 495.

B as *RB* unnecessary. A more productive duality, for us, is afforded by (00.33) the "biparental" type of reproduction, or (00.431) dyadic integration.

In ways somewhat less involved, the principle of growth may be appealed to in order to save the process of definition from some of its classical rigidity. Bogoslovsky says that modern dynamic thinking can not avoid opposition to every limitation† upon the possible constant growth of its concepts, and is therefore rather hostile to definitions.[43] This is another way of saying that identities can† not be established beyond the possibility† of future† revision.

A rather neat way of accommodating several related problems here is to regard resemblances (or differences) as progressively increasing in a process akin to growth, and regard identity and otherness as (00.271) the limits of that process. The only *caveat* here is that such a limit, if treated adequately, must be treated enotatively and innotatively.

The fact that individuations, resemblances, and identities are relative means that the concept of independence must be left enotative or innotative, and that anything "new," like anything different, is in some sense relative to that which is already in the denotative-connotative field.

(VIII) We saw above that the doctrine of "identity of opposites" leads to doubtful results. The fruitful results sometimes attributed to it really come from the integration of monads which are of the same level and which yet differ among themselves, especially in their dyadic relationship. In this way (00.431) individuals of one level give rise to new individuals of a later level; it is clear that resemblances and differences are both involved in the process, although either may be left enotative or innotative.

(IX) In the problem of identity and difference it is difficult to separate enotation from innotation; since both these references are indefinite, we can not expect to distinguish precisely between them. The problem is like that of external and internal relations. It would ordinarily be said that except in limiting† cases, like the universe† and nothing†, any given relation may be either external or internal, according to the inclusiveness of the entity considered. The point for us here is that once any entity has been regarded as an identity, that given entity is to be understood not merely with reference to its milieu, which is left enotative, but also with reference to inner, subordinate differences of qualities, aspects, number, attributes,

etc., which are left innotative. On the other hand, if difference is recognized, resemblance or identity is left innotative.

That differences are left innotative appears in connection with Russell's view that we have to admit non-symmetrical relations of terms to themselves; there may† in fact be two different relations which are each other's converses, and each of which holds between a term and itself.[47] This would indicate that Lewis and Langford's primitive idea of "self-consistency" is subject to similar analysis.[48] One may continue such explicit analysis to any length, but sooner or later one will recognize the innotation.

It may be added that our views of enotation and innotation find support in E. W. Hall's work interpreting external and internal relations by what he calls "focalized identity."[49]

[47] B. Russell, *op. cit.*, pp. 96 *f.*

[48] C. I. Lewis and C. H. Langford, *op. cit.*, p. 123; *cf.* p. 159.

[49] E. W. Hall, in *Monist*, 43, 1933, p. 213.

CHAPTER VIII

NEGATIVES

"Our whole system rests upon the proper interpretation of the negative; the most shadowy of all concepts, perhaps, and the least likely to attract attention."
—W. H. Sheldon, *Strife of Systems and Productive Duality*, 1918, p. 525.

A. Before considering the other fundamental laws of the classical logic it will be convenient to study negatives, which the laws of contradiction and excluded middle involve.

In a notable attempt at definitions in this field, Sheldon minimizes the ordinary meaning of negation as denial, and emphasizes its meaning of "otherness." To say "white is not sweet" may mean only to signify the duality† of these two.[1]

With regard to any given *A*, we shall distinguish between several kinds of negation. First, Sheldon's "negative of otherness" may for our purposes be largely disregarded, since the duality is explicit rather than implicit; when he says "red is not blue," meaning "red is other† than blue," he has both red and blue in the connotative field. Second, we shall distinguish what we call a "negative of suspension," indicated by *non-A*, which leaves open and undecided the question whether *A* is *B*, for example, or not. Next, we shall distinguish the negative of exclusion, indicated by *not-A*, definitive as regards the fact that† exclusion has taken place, but enotative as regards what† the excluded is; and finally we shall distinguish the negative of denial, "*A* is not *B*," denotative-connotative as regards the subject, but enotative as regards the predicate. The negative of exclusion, *not-A*, is treated from another point of view in Chapter IX, on contradiction. Of the two, negation emphasizes the exclusion of *not-A* from *A*, while contradiction emphasizes the distinctness of meaning between *A* and *not-A*;[2] but the two are so intricately involved that precise distinction is impossible. We shall consider the universal negative, "No *A* is *B*" in connection with induction, in Chapter X, and the interpretation of a negative where $\blacksquare p$ signifies "*p* is false," in connection with truth and validity, in Chapter XIII.

[1] W. H. Sheldon, *Strife of Systems*, pp. 471, 523 *f*.
[2] *cf*. R. M. Eaton, *Symbolism and Truth*, p. 205.

B. (α) It would be most unusual to find any one skeptical about negatives, for skepticism itself usually takes a negative form.

(β) In the case of negatives of denial, considerable difficulty is encountered in the fact that assertion is often confused with affirmation, and that the opposite of both of these may be called denial. For us, assertion is a psychological and social process, the expression of a person's belief. Affirmation is a logical process, and indicates that a term exhibits the characteristic of appropriation so that a proposition has a positive quality. Denial is properly also a logical process, the opposite of affirmation, and indicates that a term exhibits the characteristic of rejection, so that a proposition has a negative quality. In the sense just used, either an affirmation or denial may be asserted and believed. The opposite of assertion is not denial, but is refusal to assert. This is in line with Eaton's view that disbelief is belief in the falsity of a proposition; it is the belief in a negation, not the negation of a belief. The tendency to belief is always present in understanding but forks toward the two alternatives, one of which is disbelief.[3]

The main distinctions above are those between the psychological and logical processes. As regards affirmation and denial, the process of logical obversion tends to turn the edge of the distinction, although obversion itself is not free from difficulties. Negatives are rooted more directly in psychological processes by other writers. Sheldon says that we do not deny the actuality of what is beyond the fringe of vision. We ignore it, we exclude it from our sight, but there is objectively no exclusion. Narrowness of a field of attention is not of a denying sort, but is just an ignoring. To ignore is far from denying.[4] Bosanquet remarked that he could not believe that the consciousness of a positive world could in fact exist for an appreciable time without the development of negation.[5] According to Eaton, every proposition tends to suggest its negative.[6] For Lorimer, negation is apparently rooted in inhibition; the early inhibitory gesture "No" is later transferred to the control of symbolic responses.[7] It will appear as we proceed that our survey of the dualities of thinking would support most of the points made by these writers.

[3] *ibid.*, pp. 186 *f.*
[4] W. H. Sheldon, *op. cit.*, pp. 475 *f.*
[5] B. Bosanquet, *Logic*, 1911, Vol. 1, p. 280. [6] R. M. Eaton, *op. cit.*, p. 186.
[7] F. Lorimer, *Growth of Reason*, pp. 135, 152.

(γ) The interpretation of negation is sometimes made to involve theories of the relationships of our minds and the world.

For Eaton, a negative must be such that it can be entertained for thought, without any consideration whatsoever of any object that it might mean or any other proposition might mean. In fact, negation belongs to propositions, not things. Logical opposition is not perceived, for what is meant by the logical opposites can† not be in the same† universe†. If perceptions seem to be inconsistent, the inconsistency is in the concepts.[8] Bogoslovsky points out that it is always possible† to find a contrasting concept to any notion merely by adding a "non-" to it, even if no real contrast can be found in first-hand experience.[9] Still, according to Eaton, negatives must have a point of attachment to existing objects. Distinctness of meaning is the symbolic counterpart of diversity or otherness of objects. A negative proposition means something distinct from that which is meant by the positive.[10]

With the realistic presuppositions, we should expect that there is at least some ground in the objective world for negatives of suspension, exclusion, and denial. We should say that perception and thinking operate within the limitations of selection-neglect, so that, even in the case of a universe† of discourse, there is exclusion from the limited† denotative-connotative field which we happen to know or think about at a given moment. Sheldon, as we saw, says that in our ignoring of what is beyond the fringe of vision we exclude it from our sight, but there is objectively no exclusion.[11] Bergson thinks that negation is a judgment of a judgment, not of an object.[12] But for realists the structures, processes, qualities, etc., underlying suspension and exclusion can scarcely depend upon the work of our minds, and may be understood to characterize the world, whether minds are in it or not. Whitehead admits exclusion in nature but thinks of it more positively. Every event is known as being related to other events which it does not include, but exclusion is as positive a relation as inclusion. There are no negative† events in nature, and exclusion is not the mere negative of inclusion.[13] This positive character of exclusion may be granted, so long as it is seen that to

[8] R. M. Eaton, *op. cit.*, pp. 199-203.
[9] B. B. Bogoslovsky, *Technique of Controversy,* p. 143.
[10] R. M. Eaton, *op. cit.*, pp. 186, 203.
[11] W. H. Sheldon, *op. cit.*, p. 476.
[12] H. Bergson, *Creative Evolution,* transl. A. Mitchell, 1911, p. 288.
[13] A. N. Whitehead, *Concept of Nature,* pp. 186 *f.*

admit that† exclusion takes place is not to describe what† is excluded.

(δ) The mystics have their own ways of treating negations. Bennett maintains that even when it is inarticulate, total† working is positive, for it is a fruitful source of negations and exclusions. The man knows what he does not want. This is one of the reasons why it is natural to deny the very existence of whole† knowledge. Yet negation may be other than "mere"; it may be a symptom of a hold upon that which one denies.[14] We shall argue in Chapter XXI that it is doubtful if intuition ever includes the excluded, although intuition may engender an attitude toward the excluded which makes the excluded insignificant.

C. (a) We saw that Sheldon makes very important for his whole theory his analysis of negation into *otherness* rather than removal of what is negated. To negate one thing without denying it is to present another. Otherness is the original of negation. White is not sweet, he says; but what is evident here is that white is other than sweet, not that white refuses to be identified† in any way with sweet. White has to sweet a certain relation called "otherness." Why should it not have that relation called identity?† The word "not" means now the relationship of otherness between terms, now the denial of a suggested judgment.[15] For us, according to the epitomization hypothesis, the individuation of any monad is relative to its milieu and to other monads, and the same applies to any thing. Other monads or things may be what are involved in Sheldon's "otherness," but otherness may also characterize the undifferentiated milieu. Compared to a given monad or thing, *A,* the milieu and other monads or things may be regarded as negative. But if so, the negative should properly be left enotative, and, if otherness must be expressed negatively, it is better to use *non-A*. When the word "not" is used by Sheldon to mean the relationship of otherness, this is ambiguous; it is a negative statement about a term which should be left enotative. But it is better to avoid the negative altogether. If statements of otherness are wanted, they can be made directly and affirmatively; white is other than sweet. By avoiding the indeterminateness of the negative, we remove what seems to be a faulty support of Sheldon's theory of sameness in difference.

[14] C. A. Bennett, *A Philosophical Study of Mysticism,* 1923, p. 98.
[15] W. H. Sheldon, *op. cit.,* pp. 471 *f.,* 524.

Baldwin applies the term "negative opposition" to the mode in which all shades of otherness or difference are made meaning and intent.[16] We should say that this is legitimate enough, and is different from interpreting the word "not" ambiguously in such a way that sometimes, instead of exclusion or denial, it means otherness, either of an indeterminate milieu or of determinate things. The negative opposition, however, may be explicit, and hence not of much importance for us here.

(c) Negatives are frequently interpreted by the aid of contradictories, but our discussion of the relationships of the two is best deferred until the next chapter.

(e, f) A negative is defined in terms of nothing and everything, or zero and the universe, when it is held that in the application of the classical algebra of logic to classes, and specifically, to the class A, $-A$ is a class such that $A \times -A = 0$, and $A + -A = 1$.[17]

The first statement becomes a matter for discussion when we find, as in Chapter XII, that "nothing" should be left enotative or innotative, and may be interpreted as an indeterminate boundary. The second statement raises the question of the admission of an excluded negative within the universe which, after all, is selected with respect to a non-universe left enotative. Urban declares that it is only in a limited† universe of discourse that negation is significant;[18] for us, any universe of discourse is selected and limited by the selection, and a negation is significant only as it is left enotative or innotative with respect to such a universe.

(f) A negative is defined with the help of "nothing" or zero when, in Lewis's postulates of the Boole-Schröder algebra, it is stated that for every element A there is an element $-A$, such that if $x \times -A = 0$, then $x \times A = x$; and if $y \times A = y$, and $y \times -A = y$, then $y = 0$.[19] But we shall see in Chapter XII that 0 is best treated enotatively or innotatively. In the above expressions, for example, it may be interpreted either as a region utterly excluded, or as an indeterminate boundary between A and *not-A*.

(h) On the Hegelian view, negation appears as a kind of limit upon affirmation. Boundless affirmation, said Stirling, is dead, dull, unconscious nonentity.† A limit is necessary† to the reign of extension, and negation is necessary to that of affirmation.[20] This would

[16] J. M. Baldwin, *Thought and Things . . .*, Vol. 1, p. 183 n. 1.
[17] C. I. Lewis, *Survey*, p. 185.
[18] W. M. Urban, *The Intelligible World . . .*, 1929, p. 151.
[19] C. I. Lewis, *Survey*, p. 119.
[20] *cf.* J. Stirling, *The Secret of Hegel . . .*, 1865, Vol. 2, pp. 93 *ff.*

be an adequate statement if the limit or necessity were left with enotative reference, but the Hegelians have not been content to leave their antithesis uncircumscribed. More often, by appealing to a supposed higher identity† of opposites, they have insisted upon describing their negatives by methods and even in terms which virtually were those of their affirmatives.

(j) Eaton describes a negative by an appeal to infinity, but the very amplitude of his description suggests enotative reference. He says that the "infinity" of the negative allows for the totally† unforeseen and unforesee*able*†—for contingencies† which none† of the principles of science and common sense embrace. When one believes a negative he can find in his belief a margin of preparation for the wholly† unexpected, which arises from the ambiguity of the negative.[21] The inexhaustible qualities of an infinite left enotative afford the ground for the so-called "infinite judgment," where a denial, though unquestion*able,* is unmeaning, as when we say "Virtue is not square."[22]

(n) Bergson holds that to deny always consists in presenting in an abridged form a system of two affirmations; the one determinate, which applies to a certain possible; the other indeterminate†, referring to the unknown or indifferent reality† which supplants this possibility.[23] He makes both affirmation and denial refer to possibles. We should say that a more definite treatment would make affirmation refer to something (at least affirmed as) actual and leave the corresponding denial with an enotative reference. As contrasted with the actual, possibility may also be enotative, and denial and possibility may share this reference and appear to have something in common.

(o) A peculiar twist is given to the interpretation of negatives, when, according to Lewis and Langford, it sometimes happens that when a negative function is interpreted, the resulting proposition is true because there do not exist things sufficient to fulfil its hypothesis. When this is the case, they say the function is satisfied vacuously. A negative proposition will be true†, if and only if nothing† makes it false. It does not require anything to make it true.[24] This interpretation of negatives and appeal to non-being involves certain "peculiar propositions," discussed in Chapter XIII, D (1).

[21] R. M. Eaton, *op. cit.,* pp. 217 *f.*
[22] B. Bosanquet, *op. cit.,* 1911, Vol. 1, p. 282. [23] H. Bergson, *op. cit.,* p. 293.
[24] C. I. Lewis and C. H. Langford, *Symbolic Logic,* pp. 347 *f.*

(q) The problem of negation is dealt with in terms of totality by Bradley who says that in asserting "this" you in effect deny that it is "that" and you thus affirm a universe† in which are two differences†, each one of which you find excludes the other. And disjunction within a whole is the one way in which and by which in the end negation becomes intelligible. Negation everywhere has a ground, not on one side but on both. There is a positive character on account of which "this" excludes "that."[25] In other words, for Bradley the whole† is, so to speak, to be stretched to include the negative. But this, for us, means that the negative which should be neglected is treated in the same way as the positive which is selected. Negation of course has a ground, but it is a ground which should be left enotative. Bradley seeks to banish disjunction and contrast by calling such a process impossible†. But there can easily be disjunction within a whole where one member of the disjunction is left innotative, and both "negation" and "impossible" are capable of just such reference.

(s) Bergson brings in time when he says that a negation is but the half of an intellectual act, of which the other half is understood or rather put off into an indefinite† future.[26] Be this as it may, the future, as his adjective indicates, is enotative.

D. (1) Many affirmative statements about negative terms, etc., are found in attempts at definitions of them. Some of these involve contradictories, and may be found at the corresponding place in Chapter IX. According to the view developed in Chapter VII, there are no perfect definitions, in the sense of complete identities† between terms. Every definition, whatever it attempts to define, carries an enotative or innotative reference, or both. When it attempts to define a negative by making affirmative statements concerning it, there is a new uncertainty; a negative is always indeterminate, and to define it is to attempt to put upon it some limitation† inappropriate to it. The same is true even if it is not definition which is attempted, but assignment of some descriptive quality, property, or attribute.

Sometimes affirmative statements about negatives seem to follow very closely from the nature of the negative of exclusion. They are similar to the *Gestalt* psychology's positive statements concerning the ground with reference to the figure, and recall Whitehead's

[25] F. H. Bradley, *Principles of Logic,* Vol. 2, pp. 662, 664.
[26] H. Bergson, *op. cit.,* p. 287.

view that exclusion is as positive a relation as inclusion.[27] Thus, a double negative sign may be made to replace the sign of a positive term, as in the Boole-Schröder elementary theorem, $\blacksquare(\blacksquare A) = A$.[28] According to this theorem, the negative of $\blacksquare A$ is A again. As it stands, it may be interpreted as merely another way of stating the relationship of exclusion between A and $\blacksquare A$. But recognition of *non-A* as, strictly speaking, the enotative negative of suspension leaves this ambiguous. That which is not $\blacksquare A$ may be *non-A*, and not necessarily the original A. The theorem is used as one of the grounds for another, which we now consider.

This is A implies $B = \blacksquare B$ implies $\blacksquare A$; a relationship affirmed of positive terms is reversed and affirmed of corresponding negative terms. The relationships are easy to see when diagrammed spatially; it may be recalled that the Boole-Schroder algebra was developed primarily for classes taken in extension. If the region A is contained in the region B, then the portion of the plane not in B is contained in the portion of the plane not in A.[28] In the spatial interpretation, what is $\blacksquare A$ is apparently greater than what is $\blacksquare B$, and may be said to include it. But the plane in question should not be circumscribed; to do so would mean that we take only a particular circumscribed portion of $\blacksquare A$ or $\blacksquare B$, whereas both $\blacksquare A$ *and* $\blacksquare B$, being negative, may be thought of as extending to infinite† distances and at least as bringing up, although they do not settle, the uncertainties which pertain to infinites. In other words, negatives, like infinites, must be left enotative. Moreover, $\blacksquare A$ can not be unambiguously described. It may be B, or $\blacksquare B$, or perhaps even *non-B*. The interpretation for propositions will concern us in Chapter XIII. When Lewis proves that for every element A, there is one and only one element having the properties postulated of $\blacksquare A$,[28] the proof is made to rest upon the selection of any† two elements $\blacksquare A_1$ and $\blacksquare A_2$; and the theorems that $\blacksquare(\blacksquare A) = A$, and $(A = B) = (\blacksquare A = \blacksquare B)$. In the Lewis and Langford *Symbolic Logic,* the result is reached from these two theorems, together with $(A = \blacksquare B) = (\blacksquare A = B)$.[29]

Whitehead in his discussion of "supplementary elements" first says that an element B will be called supplementary to an element A if both $A + B = 1$ and $AB = 0$; then, that it will be proved that only one element supplementary to a given element can exist, and

[27] A. N. Whitehead, *Concept of Nature,* p. 186.
[28] C. I. Lewis, *Survey,* p. 123 *f.*
[29] C. I. Lewis and C. H. Langford, *op. cit.,* pp. 31 *f.*

assumed that one such element does exist; and then, that if A denotes a given element, $\blacksquare A$ will denote the supplementary element.[80] But the description of $\blacksquare A$ is built up, within 1, by affirmative statements about the positive term B. Again, the theorem $(A = B) = (\blacksquare A = \blacksquare B)$[28] or that negatives of equals are equals, in the light of our study of equality†, as well as of negatives, must be said to disregard its enotative or innotative references.

According to McTaggart, the non-possession of a quality gives the existent a nature besides its existence, and has a positive side. If a thing is not square, it is possessed of the quality of not-squareness.[81] This procedure, placing the negative, with the hyphen, in the predicate of a proposition, renders the proposition affirmative merely in form, but still essentially indeterminate in content.

The processes of conversion and obversion in ordinary logic follow the same principle of substituting negative for positive terms; but the processes presuppose universals† and particulars, and will be dealt with in connection with induction.

(2) The problems requiring enotative-innotative reference are interlocked, and, while they agree in general in the feature we are discussing, differ among themselves in various ways. Methods of dealing with one problem may not be important in the case of another. Sometimes two methods which in one problem are distinct in another problem coincide; an example of this is afforded by negative statements about negatives, which are virtually equivalent† to differences of sublevel or type among negatives. But because the differences of type are hard to recognize among the indeterminate negatives, the statements involved are best considered here.

Negative statements about negatives are doubly indeterminate, unless it be granted that they merely restate the relationships of exclusion or denial. The theorem $\blacksquare(\blacksquare A) = A$ can, as we said, be accepted in this sense; "that which is excluded by $\blacksquare A$ is A." But this takes for granted that from knowing A we know $\blacksquare A$ definitely enough to know what the latter excludes; and, furthermore, it takes no account of what we call *non-A*. Other negative statements about negatives may be treated similarly.

E. (1) Interpretation of the relative individuation of any object easily involves the use of negative terms. This is illustrated

[80] A. N. Whitehead, *Universal Algebra,* p. 36.
[81] J. E. McTaggart, *op. cit.,* Vol. 1, pp. 60, 63.

by Baldwin's theory of "limitation," where, as we might say, the object is selected in its otherwise undifferentiated milieu, and "privation," where it is selected in relationships to other cognate objects in the milieu. Thus he regards "limitation" as the root meaning of all negation. In the earliest mode, limitation is the beginning of difference† as positive incompatibility, and the limit set by the indeterminate content is the beginning, the *fons et origo* of all grounds for later negative meanings as of what is not natural, customary, and to be expected. The later meaning of not belonging together takes its rise in simple limitation and indeterminate otherness. Because of the need for discrimination, the unity of simply being an object passes over into psychic discrimination and (00.112, 00.113, etc.) individuation as between objects. The meaning is "this thing and not that." This quasi-logical bare cognition of difference is preparatory to the judgment of distinction or relation.[32]

The distinction we would make in regard to negatives is not, as for Sheldon, between otherness and exclusion or denial. The distinction is properly fourfold. There are negatives of (1) otherness, which is best expressed positively and left out of account; (2) suspension; (3) exclusion; and (4) denial. The negative of suspension is *non-A,* while that of exclusion is *not-A*. Both of these refer in different ways to *A's* enotative milieu. The negative of denial, "*A* is not *B*," means that *A* rejects *B,* which *B* is thereupon not further distinguished, except as belonging to *not-A* rather than to *non-A*. Between cognate monads, or things like the objects discussed by Baldwin, there are relationships of exclusion, and what is affirmed of one may be denied of another. But these relationships, when they are discussed, are rendered explicit; within the enotative milieu of the first monad or thing, a second monad or thing is distinguished and, at least for the moment, treated connotatively. This, however, does not by any means exhaust the enotative milieu, nor answer its problem.

(11) We saw that where there are negative statements about negatives, they may be said to illustrate differences of sublevel, type, or order of negations. But all negatives are indeterminate; differences between them can not be determined, so these differences of level or sublevel, from their very vagueness, tend to collapse into mere restatement of conditions of exclusion. Hence statements

[32] J. M. Baldwin, *op. cit.,* Vol. 1, pp. 161, 183 n. 1, 199 n. 1.

of this kind are best reduced to the type of $\blacksquare(\blacksquare A) = A$, as discussed in D (2), above.

(III) The interactions of terms, etc., give rise to negatives of denial. A negative of denial, typified by "*A* is not *B*," according to our view is a rejection, by a subject-term *A*, of some relation or attribute *B* which is incompatible with *A* and so is relegated to the enotative. That denial is essentially rejection has been seen by a number of writers. Bradley says that in the negative judgment where yellow, for example, is denied, the positive relation of yellow to the tree must precede the exclusion of that relation.[33] According to Bosanquet, negation in its primary shape is the exclusion of a suggested qualification.[34] Baldwin, too, says that in its import negation is of the nature of an unacceptable or rejected suggestion of a positive.[35] That denial as rejection involves the relegation to an outer region is at least suggested when Wittgenstein says that the denying proposition determines a logical place with the help of the logical place of the proposition denied, by saying that it lies outside the latter place.[36] According to Eaton, negation is essentially unspecific in its reference, and to make it specific is to rob it of its negativity. Negative judgments, no matter what their grounds, remain evasive to the end.[37]

(IV) In Lewis's version of the Boole-Schröder algebra, the definition of $\blacksquare A$ is one of the postulates.[38] In general, if any definite meaning, value, or content is given to a negative, it will be likely to be by way of assumption, and the opposite assumption might have been made.

(V) Our interpretation of negatives makes possible provision for a kind of intermediate or boundary region as regards *A* and $\blacksquare A$. The fundamental dichotomy (reflected in the process of selection-neglect) is a dichotomy of *A* and *non-A*. This *non-A* is a negative of suspension, not forthwith to be identified with *not-A* (or $\blacksquare A$), the negative of exclusion. The *A* is selected with reference to a neglected *non-A*, which may or may not contain more *A* than that originally selected. It should be made clear that the presence of further *A* in *non-A* is not affirmed, nor is it denied. The question is

[33] F. H. Bradley, *op. cit.*, Vol. 1, p. 115.
[34] B. Bosanquet, *Logic*, 1911, Vol. 1, p. 281.
[35] J. M. Baldwin, *op. cit.*, Vol. 2, p. 220.
[36] L. Wittgenstein, *Tractatus*, p. 75.
[37] R. M. Eaton, *op. cit.*, pp. 199, 218.
[38] C. I. Lewis, *Survey*, p. 119.

suspended, left open. In a spatial interpretation, the ***non-A*** may lie between *A* and the excluded ***not-A***. This rather loose possibility† is somewhat tightened up in the consideration of the laws of contradiction and excluded middle, considered in Chapter IX, and even more so in the process of induction with its organization of universals, considered in Chapter X.

(VI) Where there is a given object, *A*, and its negative of exclusion, ■*A*, the field of the negative is, as we have said, left enotative. But within that enotative field further objects may sometimes be discriminated as present and positive, although other than the original object. In Baldwin's terminology, as we saw, this means a passing from limitation, which is indeterminate with no other object discriminated, to privation, which is still somewhat indeterminate, but where other objects begin to be discriminated. Now privation, in turn, leads to what Baldwin calls selection and exclusion, which he regards as determinate. For us, this amounts to saying that in limitation the other objects are enotative; in privation, they are denotative; in what Baldwin calls selection and exclusion, they become denotative or connotative. With regard to the stage last mentioned, Baldwin says that the advance is one from privative or indeterminate negation to exclusive or determinate negation; the germinal duality of meaning involves the taking on of a determinate character. Baldwin's example concerns perceptions more directly, but is applicable to thinking. He says that when all the objects on a table are familiar and of a type well known, we can say either "this one among those," or "(all)† these and among them this one." In the latter case, the original negative meaning as applied to the other things seems to have disappeared altogether, so far as the act of cognition is concerned; the privation of the indeterminate has become not the exclusion but the inclusion of the determinate. This leads Baldwin to say that limitative and privative meanings should perhaps not be called negative. They seem capable of progression into positive inclusion as well as exclusion, but without doubt they are rudimentary forms of opposition.[39] The point is that Baldwin's exclusion differs from that which we have associated with the negative of exclusion, in that the objects excluded remain discerned within the denotatively or connotatively known field. They are appropriated, but non-incorporatively, by the original object. This means merely that the original denotatively or connotatively known

[39] J. M. Baldwin, *op. cit.*, Vol. 1, pp. 185 n. 1, 191, 193.

field is enlarged by the inclusion of new objects. It is a process of growth; but the appropriation within the connotative field does not exhaust the enotative.

(VIII) Recent Russian psychology has found useful the view of Engels, that in formal logic *A* excludes *not-A,* but dialectic logic gives quite another meaning to negation. "Negation of negation" does not undo negation; it transcends it. Hegelian "internegation" is never exclusive, though the processes may limit† each other. Negation does not mean simply "no —"; it is not a declaration of non-existence†, nor of arbitrary destruction of something. The "first negation" must be constructed so that the second remains possible.† For each series of things there is a peculiar species of negation which makes development possible; this applies also to ideas.[40] We should say that this species of negation appears to be peculiar because, when a given negative ■*A* is so designated, it is negative merely with respect to the original *A*. Its positive qualities, which are the real secret of Hegelian synthesis, are left enotative or innotative.

(IX) Differentiation of parts in any object of thought is likely to lead to otherness of parts and to negatives properly left innotative. Sheldon's negative of otherness may mark innotation within the connotative field. Baldwin notes that we may have contents set up within one sphere of control, but individuated further as distinct classes, subclasses, or individuals. Here there is the possibility of alternative determination, of selective moulding of material, giving rise to exclusion and contradiction†.[41]

He also says that the negative meaning attaching to the case of least-togetherness-in-fact, though the terms be in thought reinstated together, is that of mutual exclusion. The essential thing is the mutual exclusion of the contents held together. It illustrates opposition by exclusion or division.[42] The interpretation here might be clarified by saying that the components of the system, while together in fact, leave their mutual exclusions innotative. Innotation is also suggested when Baldwin says that we may have elements which are fugitive or uncontrolled—an item which is so far detached and unrelated that it has no further development in connection with anything. Or, again, we may have relatively irrelevant worlds of fact which do not intrude on each other; the relations of their con-

[40] See K. N. Kornilov, in C. Murchison, ed., *Psychologies of 1930*, p. 258.

[41] J. M. Baldwin, *op. cit.*, Vol. 2, p. 326. [42] *ibid.*, Vol. 1, p. 197.

tents are unintelligible and meaningless.[43] Here we observe his words, "*so far* detached" and "*relatively* irrelevant"; it is obvious that these words indicate at least traces of connotation not developed in detail. On the other hand, the words "no further development," "do not intrude," "unintelligible and meaningless" carry negative significance, and indicate that something of the original enotation is preserved innotatively within the enlarged field.

[43] *ibid.,* Vol. 2, pp. 325 *f.*

CHAPTER IX

Contradictories

"If language were as perfect and as copious as we could imagine it to be, we should have, for every name which has a positive signification. another which merely implies all other things: thus, as we have a name for a tree, we should have another to signify everything that is not a tree. In logic, it is desirable to consider names of inclusion with the corresponding names of exclusion: and this I intend to do to a much greater extent than is usual, inventing names of exclusion by the prefix not, as in tree and not-tree, man and not-man. Let these be called contrary or contradictory names."
—A. De Morgan, *Formal Logic, or the Calculus of Inference* . . . , 1847, p. 37.

A. The laws of contradiction and excluded middle, which belong with identity as the classical laws of logic, may now be considered in the light of what we have seen concerning negatives. We are here concerned more directly with contradictory terms, like *A* and *not-A*. Propositions affirming the same predicates of *A* and *not-A,* or affirming *A* and *not-A* as predicates of the same subject, may be called contradictions. Contradictory propositions involve the process of induction, with its universals†, like "All *A* is *B*," and particulars, like "Some *A* is *not-B*." The propositions of the square of opposition are considered at E (III), below, and further in Chapter XIII, C (f), and Chapter XVI, E (VI).

The law of contradiction, "*A* can not be *B* and *not-B* at the same time," is traditionally associated with the law of excluded middle, "*A* is either *B* or *not-B*." The two are variously interpreted and often confused.[1] We shall develop some distinctions between them in E (V) and after (IX), below.

B. (α) Sometimes there is skepticism as to whether any given contradictories really mirror or fit the world. Cohen says that in general the question whether the world falls within one or the other of opposite categories must be largely verbal.[2]

(β) Some modifications of earlier views concerning the formal status of contradiction have issued from certain studies of primitive and child psychology. We said in Chapter V that Lévy-Bruhl holds

[1] See R. M. Eaton, *Symbolism and Truth,* p. 205.
[2] M. R. Cohen, *Reason and Nature,* 1931, p. 321.

that primitive mentality, in its collective representations, operates in accordance with the "law of participation." Piaget says that until the age of seven or eight, children, while they do not believe what is self-contradictory, adopt, one after another, opinions which if they were compared would contradict one another. Piaget calls this "contradiction by amnesia." Besides this, there is "contradiction by condensation." The child, unable to choose between two contradictory explanations of one and the same phenomenon, agrees to both simultaneously, and even fuses them into each other. Piaget calls this insensibility to contradiction "transduction." But the habit of argument, when acquired, causes the need for inner unity and for systematization of opinions to make itself felt.[3] According to Lorimer, various terms gradually become established as habitually involved in contradictory statements, with appropriate inhibitions, reinforcements, etc. The fact that the rejection of one principle involves acceptance of its alternative in behavior is the foundation of decisive action and of accurate and consistent thinking. The law of contradiction might be more accurately described as a law of transition from the presymbolic to the symbolic structure of intellectual activity. The symbolic statement clarifies the issues, and makes possible the systematic exploitation of previously established knowledge in the control of new intellectual activity.[4]

According to a number of theories, contradiction is an ingrained characteristic or ultimate fact of knowledge, and any logical discourse must involve the principle of contradiction at the very start. Lewis, however, says that the excluded middle declares our purpose to make for every name a complete dichotomy of experience, instead —as we might choose—to classify on a basis of a tripartite division into opposites and a middle ground between them. Our rejection of such tripartite division represents only our penchant for simplicity and similar considerations.[5]

(γ) With reference to the relationships of mind and world, Eaton says that as a statement of general intentions in the use of symbols, the law of contradiction is *a priori* in the sense that it is a rule laid down by mind for its own guidance. It is a general rule of symbolism and therefore a general principle of thought, neces-

[3] J. Piaget, *The Language and Thought of the Child,* 1926, pp. 74 *f.*; *Judgment and Reasoning in the Child,* pp. 241 *ff.*

[4] F. Lorimer, *Growth of Reason,* pp. 131, 136 *f.,* 153-6.

[5] C. I. Lewis, *Mind and the World Order,* 1929, pp. 207-10, 247.

sary for the very existence of meaning.[6] Bradley, on grounds involving selection and neglect, and affirmative statements about the background, maintains that contradiction is appearance, as distinct from reality.[7] Sheldon holds that contradiction has a reflective character; it is not objectively observed but is a phenomenon of thinking, and thinking is just the region where errors come in.[8] Cohen holds that, strictly speaking, contradictions are always dialectical, *i.e.*, they hold only in a logical universe. Two statements, which taken abstractly are contradictory, may both be true of concrete existence, provided they can be assigned to separate domains or aspects.[9] The law of contradiction should be stated, "Nothing can be both *A* and *not-A* in the same relation."[10]

On the other hand, Eaton says that according to the existential interpretation the law of contradiction is a general truth about objects and expresses the minimal conditions for the being of objects.[11] For us, as was said in connection with negations, the basis on which the law of contradiction rests is the same whether in mind or world.

(δ) Contradictions have frequently been grounded in the supposed higher unities† of intuitional knowledge. Bergson says that from the subject as seized by intuition we may pass easily to two contrary concepts.[12]

Among the idealists, contradiction has served to point the way to the Higher Unity. For Bradley, the basic contradiction between appearance and reality can be reconciled only in and by the Absolute Mind.[13] To insist merely on the contradictoriness may be practically to miss the difference between insight and blindness.[14]

It has frequently been held that contradictions are unified in aesthetic experience. According to Baldwin, the aesthetic experience is a fusion and not a mixture, and issues in a meaning of its own sort and kind. Its essential characteristic is unity of compre-

[6] R. M. Eaton, *Symbolism and Truth*, pp. 205 *f.*, 209.
[7] F. H. Bradley, *Appearance and Reality*, pp. 571 *ff.*
[8] W. H. Sheldon, *Strife of Systems*, p. 454.
[9] M. R. Cohen, *Reason and Nature*, p. 166.
[10] *idem*, in *Jour. Phil.*, 24, 1927, p. 678.
[11] R. M. Eaton, *op. cit.*, p. 206.
[12] H. Bergson, *Introduction to Metaphysics*, transl. T. E. Hulme, 1912, p. 40.
[13] *cf.* F. H. Bradley, *Appearance and Reality*, pp. 571 *ff.*
[14] *idem*, *Principles of Logic*, 1922, Vol. 2, p. 672.

hension. It fulfils contrasts while comprehending and outliving them.[15]

With reference to ultimate problems, William James declared that that secret of a continuous life which the universe knows by heart and acts on every instant can not be a contradiction incarnate. If logic says it is one, so much the worse for logic.[16]

C. Problems involving contradictories are resolved into other horizon problems in numerous ways; to the examples mentioned here may be added some of those mentioned in Chapter VIII in connection with negatives.

(a, b) According to Eaton, the general condition of the use of symbols, that a negative is always distinct in meaning from its positive, is the principle of contradiction in its most abstract form.[17] The recognition of negatives of suspension, however, turns the edge of this distinction.

(e) De Morgan held that what he called a pair of contrary names, as "man" and "not-man," between them represent everything† imagin*able*† or real† in the universe.[18]

(e, f) Boole, in hitting on the analogy of 1 to "everything" and of 0 to "nothing," was able to express the basic logical principles of contradiction and excluded middle in terms of his algebra.[19] But as we shall see in Chapters XI and XII, 1 and 0 are limiting notions which carry enotative and innotative references.

(f) Contradictions are ordinarily held to have nothing in common or to reduce to zero. At least in logic and mathematics, says Cohen, where we deal with abstract determinations, two contradictories produce a result which is 0; the entity of which they are asserted is absolutely† impossible†.[20]

(g) Many treatments of contradiction make it involve truth-values, as when it is said that the law of contradiction asserts that it is impossible† for a proposition to be both true and false.

(h) Avey says that the law of contradiction in the form $A \cdot -A = 0$, serves as definition of the limit of the infinitesimal†.[21]

[15] J. M. Baldwin, *Thought and Things . . .*, Vol. 1, pp. x, xi. *cf.* J. Dewey, *Experience and Nature,* pp. 392 *f.*

[16] W. James, *Pluralistic Universe,* p. 207.

[17] R. M. Eaton, *op. cit.,* p. 205.

[18] C. I. Lewis, *Survey,* p. 37, n. 60.

[19] C. I. Lewis and C. H. Langford, *op. cit.,* p. 12.

[20] M. R. Cohen, *Reason and Nature,* p. 166.

[21] A. E. Avey, in *Jour. Phil.,* 26, 1929, p. 523.

(j) Sometimes solutions have been sought among infinites. In dealing with some particular problems, Dantzig maintains that the flat and straight and uniform can be adapted to its very opposite. the skew and curved and non-uniform, not by a finite number of steps, but "only by that miracle-maker, the infinite." But in the case of the "greatest transfinite" number, the problem of contradiction remains wide open.[22] Brouwer's treatment of the problem of contradiction, considered in Chapter XVI, involves an appeal to the problems of the infinite and of possibility†; the latter appeal is somewhat concealed in the term *Konstruirbarkeit*.

(n) Some authors link contradictories with impossibilities. Lewis and Langford make the law of excluded middle necessary; it is declared impossible that it could be false†.[23] But, as will be indicated later, their use of possible, impossible, and necessary is open to question.

(o, p, q, r) The work of Bradley may be cited again, as an outstanding modern attempt to look beyond contradictions, which he regarded as appearance, to a Reality which is one, a whole, and absolute.[24]

(s) Alexander derives the category of contradiction from the fact that the occupation of one piece of space-time is not the occupation of a different† one.[25]

(t) The physical universe is made the ground of contradiction when Northrop maintains that a principle of opposition and contradiction is inherent in nature; contradictions find their basis in the relations of the changeless macroscopic atom and the changing microscopic atoms.[26]

None of these treatments deals adequately with the basic fact, that contradictories involve horizons. The discussions merely translate the problem of contradictions into some of the other horizon problems.

D. (1, 2) Both affirmative and negative statements about contradictories were considered at the corresponding place in our study of negatives. When such statements are considered in relationship to contradictories, the emphasis, as we said, is upon the meaning or content of the terms rather than upon the more nearly

[22] T. Dantzig, *Number, The Language of Science*, pp. 137, 226.
[23] C. I. Lewis and C. H. Langford, *op. cit.*, p. 166.
[24] F. H. Bradley, *Appearance and Reality*, pp. 136, 140, 456.
[25] S. Alexander, *Space, Time, and Deity*, Vol. 1, p. 205.
[26] F. S. C. Northrop, *Science and First Principles*, 1931, pp. 143 *f*.

formal fact of exclusion. Such affirmations or denials, on one side or the other of controversial questions which are expressible in contradictions, characterize much of the dogmatism in science and philosophy.

E. (I) We saw, in connection with negatives, that any term or object *A* may be said to be individuated relatively to its milieu *non-A*, which is at first left enotative. As a result of denials and exclusions, *not-A* is then distinguished. But when *not-A* is distinguished within *non-A*, *not-A* is also left enotative, and, if described, should be described only in relationship to *A*. One difference between contradictory and contrary terms appears to be that the former is (00.111) a relationship between a term and its milieu, while the latter is (00.113) a relationship between two cognate terms.

(II) Porphyry's tree showed long ago that there are various orders of contradictory terms, and hence of contradictions. Attempts made to synthesize or dissolve contradictions in a "higher unity"†, with or without internal distinctions, leave the higher unity to front upon its own contradictory which is usually disregarded—*i.e.*, left enotative. Some of the contradictions and paradoxes which Whitehead and Russell attempt to solve by the theory of types are considered in Chapter X. A hierarchy of principles of excluded middle has sometimes been thought useful.[27]

(III) Although, as we said, they involve induction†, with universal and particular propositions, some problems of contradictory propositions, with excluded middle, may be considered here, from the point of view of selective interactions of subject-terms. The relationships between contradictory propositions indicated in the square of opposition are traditionally relationships of *classes* in extension. Thus if all *A*'s belong to the class *B*, some *A*'s can not belong to the class *not-B*. If no *A*'s belong to the class *B*, some *A*'s can not belong to the class *B*. The first modification to be suggested here is the one we have indicated. Except in simple cases, there *may* be a class *non-B*, enotative as regards *B*, innotative as regards *B* and *not-B* together; the question must be left open. The classes of the traditional square are exclusive but not exhaustive.

For us, relationships in intension—understood in the structural rather than the psychological sense[28]—are correlated with those of

[27] C. I. Lewis and C. H. Langford, *op. cit.*, pp. 466, 483 *ff*.
[28] See G. P. Conger, *Epitomizations*, p. 391.

extension; in other words, attribution is correlated with distribution. So, considered as processes of predication, *i.e.,* selective appropriation and rejection of attributes, contradictory propositions are exclusive, but not exhaustive. As regards predication, then, the relationships between contradictory propositions turn upon the fact that predicates which the members of a class have once rejected into the enotative are supposed not to be appropriated by members of the class. Thus, if all *A*'s are *B*'s, some *A*'s can not appropriate predicates from the class *not-B*'s, which the class *A* definitely excludes, but they may from the *non-B*'s which are not yet determined as *B* or *not-B*. Again, if no *A*'s are *B*'s, some *A*'s can not appropriate predicates from the *B*'s which all *A*'s reject, but they may from the *non-B*'s which are not yet determined as *B* or *not-B*. Nor is the situation essentially altered if the negative is in the copula rather than the predicate; it is the difference between attribution and distribution, or, more accurately, between rejection and exclusion. In either case, the enotative or innotative *non-B* must be reckoned with and left open. Let us repeat that *non-B* is to be left open; we must be quite non-committal about it, as to whether it is *B* or *not-B*.

(iv) The law of contradiction may be regarded as axiomatic. Russell says that it is accepted because it is self-evident.[29]

One of two contradictory terms may, by assumption or some more strongly authenticated procedure, preempt the field, so that the other term simply does not occur.[30] Examples abound in absolutist† systems, where descriptions are given in terms of one contradictory while the other is disregarded.

(v) We noted that the dichotomy of a negative of suspension, *A* and *non-A,* is at least somewhat sharpened in a negative of exclusion, which presents the contradictory terms *A* and *not-A*. Whitehead says that the term supplementary to any term *A* represents that region which includes all† the universe† with the exception of the region *A*. The two regions together make up the universe, but they do not overlap, so their region of intersection is nonexistent†.[31] There have been a number of attempts to make the classical law of contradiction less rigid, and, in one way or another,

[29] B. Russell, *Principles,* p. 455.
[30] *cf.* H. L. Hollingworth, *Psychology of Thought,* p. 275.
[31] A. N. Whitehead, *Universal Algebra,* p. 39.

to make possible the recognition of an intermediate or boundary zone between *A* and *not-A*.

One of the simplest of these attempts emphasizes intermediate differences of degree of contrast in our actual concrete experiences. Bogoslovsky says that, generally speaking, the beginnings† of the functioning of a pair of opposites may be traced to two experiences appreciably different along a certain line. When differentiated and contrasted the two experiences start to function as poles of the potential† continuity†. If any more extreme experience comes, it assumes the function of the pole, and the range between the poles in this way increases until the poles become only indicators of direction. The fact that inquiry along a certain direction is determined by its two poles is the fundamental and primary element in reasoning.[32]

Another way of easing the rigidity and recalcitrance of contradiction is by recognizing transitions from one judgment to another about a changing world. Cohen says that of things in transition there are times when opposite predicates are equally† true.[33] Lorimer says there is nothing sacred or absolute about logical contradictions. They have their foundations in previous habits of thought and are in need of constant revision and reorganization. The truth usually lies between crude logical contradictions.[34] Recognition of this may be combined with recognition of a process analogous to growth, as in VI, below.

Sometimes a distinction is made in terms of truth-value. Avey agrees with the view just mentioned, that the law of contradiction may be taken as representing the point of transition from one state to another. He goes on to say that as such, the logical product, common to *A* and *not-A,* is not a realm of complete unreality†, but rather the dividing line between *A* and *not-A* when these are interpreted as static divisions of the universe. Avey, however, thinks that this does not annul the law of contradiction in connection with the truth-value of the propositions. No proposition is true† and false† at the same time.[21] In the two-valued algebra, if x is not equal to 1†, x is equal to 0. As applied to propositions, this means that if x is not true, it is false, with no middle ground; but this hard and fast division does not apply to classes.[35] Eaton maintains that the

[32] B. B. Bogoslovsky, *Technique of Controversy,* pp. 125, 129 *f.*
[33] M. R. Cohen, in *Jour. Phil.,* 24, 1927, p. 677.
[34] F. Lorimer, *Growth of Reason,* p. 137.
[35] *cf.* C. I. Lewis, *Survey,* p. 66 and n. 5, above.

principle of excluded middle has no connection with the principle of contradiction, when truth and falsity are not considered in the definition of negation.[36]

Important in this connection has been the work of Brouwer, which is discussed in Chapter XVI.

For us, the correspondence between propositions and the classes of things for which the propositions are true or false makes it inconsistent to argue for an intermediate state in the latter and not for an intermediate condition in the former. According to the epitomization hypothesis, the logic of terms-in-propositions, with the process of predication, resembles that of classes so much that negatives, which are indeterminate anyway, will have analogous functions in both. Thus attribution and distribution will for our purposes be correlated; when a term has a given attribute, it belongs in the class of terms having that attribute, and when it does not have the attribute it does not belong to the class defined by that attribute. "*A* and *not-A*" differ from "*S* is either *A* or *not-A,*" but whatever is innotative between *A* and *not-A* in the one case is innotative in the other. The law of excluded middle differs from the law of contradiction in that excluded middle is affirmative in form, while contradiction is negative. Contradiction denies the antinomy or conjunction of *A* and *not-A*; excluded middle affirms an alternative and a disjunction of *B* and *not-B*. In excluded middle, the negative appears in the predicate, and since attribution and distribution are correlated, the result for our purposes is the same. The disjunction, like the conjunction, fails to take account of the enotative or innotative *non-B*. The proper form would be for contradiction, *A, non-A* and *not-A*; and for excluded middle, *A* is either *B,* or *non-B,* or *not-B*. But it must be confessed that *non-B,* always innotative, like a buffer state, tends to crumple and (perhaps by a kind of logical illusion akin to some optical illusions) tends to disappear. As in "perfect inductions," some cases are so simple that they are unimportant.

(vi) It is admitted on all sides that the content of any particular contradiction, as between *A* and *not-A,* may be open to revision. Bogoslovsky insists that the process of growth makes the law of excluded middle in static logic obsolete. Modern dynamic thinking can not avoid opposition to every limitation upon the possible† con-

[36] R. M. Eaton, *op. cit.*, p. 205.

stant growth of its concepts and is therefore rather hostile to definitions.[37]

The classical law of contradiction, after all, prescribes only that *A* can not be *B* and *not-B at the same time,* and consequently, as Cohen suggests, does not exclude the possibility† of a transition from one to the other. Robb holds that when the law of excluded middle refers to the occurrence of events, and when the events are past†, it is a disjunction of propositions, but when the events are not past it is a disjunctive proposition, except possibly† when they are at the instant† of assertion.[38]

There is a suggestion of the leapfrog procedure in the statement of Broad that the laws of contradiction and excluded middle apply to a fixed universe† of discourse, and we can† at any† moment† get a fixed universe of discourse by taking the sum total† of reality† up to that moment.[39]

The reliance of the mathematical formalists upon their so-called freedom from contradiction, resulting from procedure in accordance with their axioms†, is really a kind of recognition of enotation. As far as the procedure has gone at any given moment, no contradiction has been encountered; as to anything further, the principle of freedom from contradiction is just non-committal enough to carry an open reference. It appears that the only freedom from contradiction consists in continually excluding a contradictory into the region of freedom, namely the enotative.

Bogoslovsky suggests progress toward a limit when he says that dynamic logic contains static logic, since cases where *A* is 99.9999 . . . % *B* and 0.000 . . . 1% *non-B* "closely correspond" to static logic.[40]

(VII) Something suggesting the result of a process of reproduction between contradictories appears when Lorimer says that when explicit statement of the opposed concepts involved in any judgment-process has been achieved, refined judgment generally locates the most acceptable solution of the problem at some definite point of the continuum† scale between the two poles of the opposites.[41]

[37] B. B. Bogoslovsky, *op. cit.*, pp. 8, 28, 80, 140.

[38] A. A. Robb, "Time and Space," in collective work, *Evolution in the Light of Modern Knowledge,* 1925, p. 427.

[39] C. D. Broad, *Scientific Thought*, p. 83.

[40] B. B. Bogoslovsky, *op. cit.*, p. 18.

[41] F. Lorimer, *op. cit.*, p. 156.

(VIII) The synthesis of contradictory elements into a larger whole† has been a favorite doctrine of many philosophies, especially the Hegelian. One of the later expressions of such a view comes from Ouspensky, with a proposed *tertium organum.* He maintains that the recognition of the unreality of the divisions between *A* and *not-A* and the recognition of the unity† of all† opposites is necessary† for the comprehension of the higher logic. Such recognition when formulated into axioms† produces the impression of absurdities: *A* is both *A* and *not-A*, everything† is both *A* and *not-A*, everything is All†.[42] One difficulty, we should say, is that the All must also carry its enotative reference to the non-All.

(IX) The tree of Porphyry, again, is enough to show that subordinate contradictories are contained within broader contradicories, and that one or both members of the subordinate contradictories may be left innotative. When contrary terms, too, are distinguished within the range of contradictory terms, something is usually left innotative.

We may now sum up the position of this and related chapters. For us, the laws of contradiction and excluded middle are not concerned primarily with truth-value, but are attempted descriptions of even more fundamental logical structures and processes which subsist antecedently to the existent world and its problems of truth and falsehood.

In the descriptions of structures and processes involving contradiction and excluded middle, at least one member of each pair of contrasted terms must carry enotative or innotative reference—*i.e.*, must be indicated without being completely explored or described.

The enotative or innotative references in statements about the two laws will turn upon (i) the limitations of our thinking at the time the statement is made, and (ii) the nature of the negatives which the statements contain. With regard to the limitations of our thinking, at any particular moment thinking is selective-neglective. The content of the selection varies from moment to moment, but the formal condition of selection-neglect remains characteristic of the thinking process. It is the formal condition, rather than the content, which now concerns us; and, according to the realistic presuppositions, the same formal conditions apply equally to the

[42] P. D. Ouspensky, *Tertium Organum,* transl. N. Bessaraboff and C. Bragdon, 1924, pp. 262, 265.

monads of the mind and the world. So, for the present problem, the thinking process is as good as transparent, and we may look objectively at the world around us.

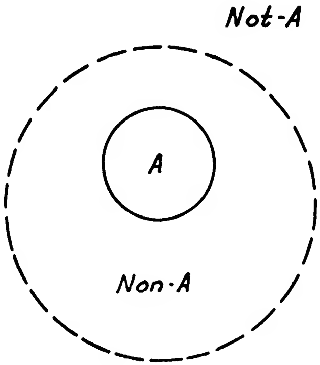
With regard to the negatives, the fundamental dichotomy, reflected in the process of selection-neglect, is a dichotomy of *A* and *non-A*. *Non-A* is merely a negative of suspension. The *A* is selected with reference to a neglected *non-A*, and the latter may or may not contain more *A* than that originally selected. It should be made clear again that the presence of further *A* in *non-A* is *not affirmed, nor is it denied. The question is left open.* The laws of contradiction and excluded middle do not necessarily apply to *non-A*. Any given thing may in a sense be *A* and *non-A* at the same time, since *non-A* may contain more *A*, although at the moment it may be not yet discovered, or even not yet objectively actual. Again, any given thing need not be either *A* or *non-A*, since it may be *not-A* instead.

A's contradictory, *not-A*, means definitely *that*‡ there is exclusion, although, being a negative, it is indeterminate as to *what* or *how much* has been excluded as *not-A*. The law of contradiction might be called the law of exclusion; we might then say that exclusion is exclusion, and *A* and the excluded *not-A* of course have nothing‡ in common. In this sense, the law of contradiction applies to *not-A*, but not necessarily to *non-A*. *Not-A*, although we do not know and can not describe its content, is excluded and contains no *A*. But *non-A* remains innotative or enotative; we do not know even whether *A* completely excludes it or not. *Non-A* may contain more *A*, or no *A*, or perhaps both *A* and *not-A*, or even something which is neither *A* nor *not-A*. In the case where *non-A* is a boundary region between *A* and *not-A*, the contradictories have *non-A* in common, and the innotative *non-A* may replace the innotative term "nothing" in the statement of the law. All these descriptions of *non-A* are possible‡ but not conclusive; any such description must carry enotative or innotative references.

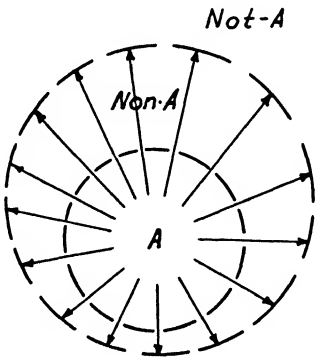
Attempts to render one or another of the descriptions conclusive occur in connection with universals. Thus "All *A*" and "No *A*" give at least the impression that *non-A* has been explored and that the entire range of *A*'s has been accounted for. The *not-A*'s have been accounted for after a fashion, but, as we shall see in Chapter X, only in relationships to the *A*'s.

We said that *non-A* may be a region intermediate between *A* and *not-A*, and may even serve as a "boundary" between them. Spatial

diagramming, however, is not a very satisfactory way to show this. The laws of figure, contour, and ground are against it. If diagrammed in circles, for instance, an intermediate region *non-A* will tend to be



(1) assimilated to the denotatively-connotatively known *A*, but may by an effort (2) be kept distinct, or assimilated to the enotative *not-A*. If *non-A* is regarded as a mere boundary line, it will, like a contour, tend to belong (3) to the figure, or the ground, or perhaps (4) to both, or (5) to neither. These interpretations correspond respectively to (1) affirmative statements, (2) negative statements, (3) disjunctive statements, (4) antinomial statements, or (5) nihilistic statements, such as are discussed in D of this and other chapters.



A more adequate diagram would use arrows to indicate the directions which the possible† progressively selective predications of *A* may take, and bars (to indicate what Brouwer, as we shall see in Chapter XVI, might call obstructions), admitting *A's* predicates to *non-A,* but excluding them from *not-A.*

CHAPTER X

Inductions, Generalizations, and Universals

"The completely unknown environment never enters into an inductive judgment."

—A. N. Whitehead, *Process and Reality*, 1929, p. 312.

A. In this chapter we are primarily concerned with the processes of induction and the structures of propositions arrived at by induction. Although it is impossible to draw the line with complete precision, these problems of induction are treated as far as possible independently of the problems of the material validity or truth of inductive propositions, which will concern us in Chapter XIII.

When a quality or property of a thing, or an attribute of a term, is considered in its own right, in quasi-independence of the thing or term, the quality or attribute thus "drawn off" is an abstraction. In this sense, "white" will constitute an abstraction, say from this paper; it is still more definitely indicated as an abstraction by the suffix "-ness" in "whiteness." Now "white paper" or even the "whiteness of the paper" is rather definitely localized; but "white" or "whiteness," considered as by itself, somehow eludes any such local bounds, and pertains to an indefinite range of objects, such as paper, houses, and countless others. The process of abstraction, as soon as it becomes indefinite in range, passes over into generalization, and the two are virtually inseparable. We may say that "white" is a generalized attribute, "whiteness" is a generalized term, and propositions about them are generalizations. In fact, any propositions, whether about qualities like whiteness or things like paper, if their range is wider than one particular instance, may be generalizations.

A generalization pertains to classes. The process of forming generalizations is called induction. A generalization may be definite or indefinite in its range; it may be applicable either to the instances examined and to no more, as in perfect induction, or it may be at least alleged to be applicable beyond the instances examined, as in imperfect induction.[1] Its range may be indicated denotatively, by

[1] W. E. Johnson, *Logic*, Part I, p. xxxvi.

demonstratives, like "these," or by words like "all" and "some," which signify universals or particulars. Its range may be indicated denotatively and connotatively by numbers or other more concrete descriptions. Often the range of a generalization can not be precisely determined, and there is a question as to whether it applies throughout a given class, examined or unexamined. In such cases we proceed by expressions of probability. The theory of probability raises so many questions concerning truth that it will be considered mainly in Chapter XIII.

Abstractions and generalizations are frequently confused with universals. Sellars says that connotation, which refers to the qualities belonging to an object, by itself presents us with abstract terms. For instance, the connotation of the class "animals" is animality. Logic, he says, calls such abstract terms universals, meaning by universal that which is separable from particular cases and repeatable. A connotation, for example, of a rose, is a complex of universals, like color, shape of leaves, etc.[2] This is akin to the view that each individual is determined by the complex of its universals,[3] and that as different individuals are considered, what is accidental is disregarded, leaving what is essential to them all as the universal.

According to Mrs. Swabey, in passing from the distributive to the collective significance of a group, thought exchanges the notion of a finite succession of empirical instances for that of a common essence or pervasive unity.[4] The doctrine of universals as pervasive finds a sort of counterpart in *Gestalt* psychology, for which the form is a determination which belongs to the whole figure, although it can not be localized at any place within this whole, not even in the contour, nor any part of the contour.[5] But just as the form, even though it belongs to the whole figure, is limited to the figure, so the pervasive unity of the group is limited to the group examined, and can not without risk go beyond the cases examined into those left enotative.

Sometimes a generalization is given the range of a universal. Keynes says that we mean by a generalization a statement that all of a certain defin*able*† class of propositions are true†.[6] For Ramsey,

[2] R. W. Sellars, *Essentials of Logic*, pp. 68 *f.*

[3] *cf.* R. M. Eaton, *Symbolism and Truth*, p. 72.

[4] M. C. Swabey, *Logic and Nature*, 1930, pp. 376 *f.*

[5] E. Rubin, *Visuell wahrgenommene Figuren*, Teil 1, p. 192.

[6] J. M. Keynes, *A Treatise on Probability*, 1921, p. 222.

general propositions must obviously be understood as applying to everything† for which we have names.[7]

For us, a universal marks a property not of individuals but of classes. A universal term is a term indicating that the corresponding class of terms or objects is marked by the property. A generalization is not a universal, unless marked by the affirmative "all," or the negative "no," or similar words. The range of a generalization is left open, enotative, but in a universal an attempt, at least, is made to reduce the range to definiteness. We shall try to indicate, at E (I and II), under what conditions such attempts proceed.

Where induction results in generalizations which fall short of the range of universals, we may have particular judgments, indicated by the term "some." The status of individual instances, as belonging or not belonging to such partial or particular ranges, may be indicated in expressions of probability.

B. (α) The problems of induction may be dismissed by skepticism, although most skepticism is directed against the more pretentious generalizations.

(β) Some psychological processes involved in abstraction and generalization are noted in Chapter III, section 16, and in A, above. For Lorimer, all ideas are general ideas in the sense that they are response idea patterns, which function in relation to varying situations and inwardly have varying degrees of synthetic intricacy. But the recognition of ideas as general is dependent upon conceptual analysis.[8]

(γ) Interpretations of universals in terms of realism and idealism have occupied philosophy for centuries. The Platonists regarded their eternal "ideas" as universals, and left it to the medieval philosophers to dispute about realism and nominalism. In our own day Whitehead has adopted the expression "eternal objects" in place of the ambiguous "universals."[9]

According to the traditional logic, a universal proposition can be independent of existence†. Lewis and Langford maintain that the countless universal propositions would all be true† if nothing† whatever existed.[10] The traditional logic, however, makes a particular proposition imply existence; thus the judgment "Some animals are bipeds" is held not to be true unless a class has members.

[7] F. P. Ramsey, *Foundations of Mathematics*, p. 34.
[8] F. Lorimer, *Growth of Reason*, p. 90.
[9] A. N. Whitehead, *Science and the Modern World*, 1925, p. 221.
[10] C. I. Lewis and C. H. Langford, *Symbolic Logic*, pp. 347 *f*.

The distinction between universals and particulars in this respect may be made to turn upon that between intension and extension. Thus intension is said to belong to the level of universals, while extension belongs to particulars.[11] But intensional meaning can easily be assigned to particulars. According to Lewis and Langford, in the logic of the intension of terms "All *A* is *B*" means that all possible† or conceiv*able* *A*'s must be *B*'s. "Some" in a proposition concerning intensions means "a definable species of," or "Some logically conceiv*able*."[12]

For us, both universals and particulars, as well as intensions and extensions, can be studied independently of truth-value, and of existence, and intension and extension can be studied independently of psychology; and in all these, whatever their relations, the horizon principles are found to be important.

(δ) For Johnson, the immediate apprehension of a universal in a particular instance affords an example of intuitive induction.[13] According to Sellars, intuitive induction is the basis for comprehension of mathematical and logical principles which are not directly existential in their import.[14] According to Russell, the notion of "every term" is indefinable and ultimate.[15] Doctrines of intuitionism here shade into those of the truth of intuitionism in general and will be discussed in Chapter XXI.

C. (a) Theories of induction inevitably turn upon similarities and differences. Every generalization must be based upon resemblance.[16] Further consideration of this point will be found in Chapter XIII.

(b) Lewis and Langford interpret universal propositions as purely negative, defining them as denials of particular, and, as they say, existential† propositions.[17] This view seems to us unwarranted, but it at least opens the way for universal propositions to be treated enotatively.

(e) The universe of discourse is taken into consideration when, according to Huntington, a class is said to be determined by any

[11] A. Uchenko, in *Monist,* 40, 1930, p. 413.
[12] C. I. Lewis and C. H. Langford, *op. cit.,* p. 66.
[13] W. E. Johnson, *Logic,* Part 2, p. 191 *f.*
[14] R. W. Sellars, *The Philosophy of Physical Realism,* 1932, p. 373.
[15] B. Russell, *Principles,* p. 40.
[16] L. S. Stebbing, *op. cit.,* p. 250.
[17] C. I. Lewis and C. H. Langford, *op. cit.,* pp. 269, 467.

test or condition which every entity in the universe considered must either satisfy or not satisfy. A null or empty class corresponds to a condition which is satisfied by no entity in the universe considered.[18]

(h) Induction and universals are by other writers understood with reference to limitations, although the limitations are often vague. According to Johnson, a summary or perfect induction, where the class number is known or the class is cap*able*† of being exhaustively enumerated, establishes conditions of limited universality by means of mere enumeration. It is the necessary preparatory stage in gathering together the relevant instances for establishing an unlimited generalization.[19] Reference to such classes is denotative or connotative.

(j) Further, for Johnson, a summary induction may apply to a finite number of cases which are enumer*able*†, or to an infinite number of cases which are non-enumer*able*.[20] We should find it necessary to leave an infinite number of cases enotative, and should understand summary induction to carry enotative reference.

For Uchenko, whenever the attempt is made to express a universal in terms of particulars, an endless or infinite way is open.[21] This takes for granted that the content of a universal is infinite and, in effect, enotative or innotative.

Poincaré made infinity here pertain to existent† objects. He pointed out that for the word "all" to have a precise meaning when the number of objects is infinite, it is necessary that there should exist† an actual† infinity. Otherwise all these objects can† not be conceived† as existing prior to their definition†; and then if the definition of a notion N depends on all objects A, it may be tainted with a vicious circle, if among the objects A there is one which can not be defined without bringing in the notion N itself.[22] We should say that precision and existence, whether prior to definition or not, involve each other only for certain constructions of the problems of epistemology, and that both infinites and existences must allow for enotation.

(l) Whitehead maintains that in every inductive judgment there is contained a presupposition of the maintenance of the gen-

[18] E. V. Huntington, *The Continuum and Other Types of Serial Order*, 1917, p. 4.

[19] W. E. Johnson, *Logic*, Part 1, p. xxxvi; Part 2, p. 198.

[20] *ibid.*, Part 2, p. 200.

[21] A. Uchenko, *op. cit.*, p. 413.

[22] H. Poincaré, *Science and Method*, transl. F. Maitland, 1914, pp. 194 *f*.

eral order of the immediate environment. Anticipations are devoid of meaning apart from the definite cosmic order which they presuppose.[23]

(p) Dubs says that the phenomenon of a limited† universal and its later inclusion in a more general statement seems to point to a monistic conception of the world. But he adds, quite properly, that while such may be the case, we should remember that such a theory is as yet a mere speculation.[24] According to our interpretation, the problem of monism, too, must be left open and enotative.

(q) Generalizations in terms of wholes seem sometimes to be implied in the reasoning of children. Piaget says that when a child listens to someone else talking, instead of analyzing what he hears in detail, he reasons about it as a whole. His lack of adaptation to other persons causes him to think in general schemata. A schema of understanding has been constructed, relatively correct, but resting on only a few points which have been spontaneously related.[25] This, after all, is not so different from Bosanquet's concrete universal and Whitehead's organic theory of induction, and is subject to the same persistent limitations.

(s) We shall see, at E (VIII), below, that Whitehead associates the problem of induction with considerations of space and time.

D. (1, 2) Affirmative and negative statements in which too much is claimed for induction and universals abound in many works we have discussed. But the extravagant claims are usually truth claims, and are best considered in Chapter XIII.

E. (1) In our studies of perception and thinking, we saw that we might consider the patterns either as (I) relatively individuated or as (III) selectively interacting, and that both characteristics are marked by enotation and innotation. It might be held that since induction is a process of thinking, the same procedure would be best here. But the process of induction is not so much an interaction as it is an integration,[26] and the relative individuation of such a new integrate, as covering or failing to cover the whole class with which it deals, is more important than the interactions which become apparent in the predicates which its subject-term takes.

[23] A. N. Whitehead, *Process and Reality*, p. 311.
[24] H. H. Dubs, *Rational Induction . . .*, 1930, p. 282.
[25] J. Piaget, *Language and Thought of the Child*, pp. 151 *f.*
[26] See G. P. Conger, *Epitomizations*, pp. 427 *ff.*

Thus we find, as in C (h) above, that an induction may be "perfect," or "imperfect." This, however, does not violate the more general principle of enotation, because the word "all," when used to indicate the result of a perfect induction, *i.e.*, a universal, indicates a horizon. It is the sign that here is a class which, unlike some other classes, is not to remain open or enotative. The horizon line is, so to speak, drawn tightly around it. We might say that the principle of enotation is recognized, but by the use of the word "all" declared, at least for the time‡ being, not to pertain to this class. The word "all," if its range is known, is denotative or connotative as regards its inclusions and enotative as regards its exclusions. If its range is not known, it is enotative as regards its proper locus, as well as its exclusions, and innotative as regards some of its inclusions.

The universal negative, "no," is in some respects quite similar. "No men" begins a proposition about men. "No men are immortal" does not mean that the subject-term "men" is not being considered; it means that it *is* being considered and found to exclude a certain property or attribute. It deals with a class considered, whether the class exists† or not. It does not mean that the class considered, and indicated by the term "men," is empty; the class is in fact, full, but full of members excluding the quality specified. Some other positive properties or attributes are doubtless included in the class, but they are left innotative. The term "no" is, like the term "all," the sign of a horizon, the mark of a class which is not to remain open. The predicate of a universal negative proposition represents a class which is said to be altogether excluded from the subject, but about which nothing† else is said. The word "some" in a particular proposition, affirmative or negative, is a sign that a portion of the members of a class include or exclude the given properties, but other members of the class are left enotative or innotative.

Generalization from a number of examined instances, which are not assumed to constitute all the instances of a given class, is now usually known by the name induction by simple enumeration, or imperfect induction.[27] The reference may be denotative with regard to the number of instances examined or included, or enotative as regards unexamined instances indicated by generalization. The question whether imperfect induction leads to universal propositions necessarily true should be considered in the study of validity, as in Chapter XIII, E (III).

[27] L. S. Stebbing, *op. cit.*, p. 245.

(11) There are well recognized degrees of generalization in propositions. Multiply-general propositions and functions involve a hierarchy of denoting.[28]

We saw that a generalization is not a universal unless marked by "all," "no," etc., and that the range of a generalization is left open, enotative, but in a universal the attempt at least is made to reduce the range to definiteness. Such an attempt proceeds subject to certain limitations, and some of these limitations mark sublevels of induction. Some of the limitations have been encountered, and some of the difficulties have been formulated in a number of paradoxes.

Three decades ago, Russell saw that difficulties arise whenever we try to deal with the greatest cardinal number, or the class of all entities absolutely, or with any equally numerous class.[29]

If, for instance, we take all the classes there are, then we have the class of all classes, which makes another class added to all the classes, and so on. Again, the number of classes in a given class is always greater than the number of members of a class.[30] Thus it has been maintained that there is a vicious circle involved in supposing that a collection of objects may contain objects which can only be defined by means of the collection as a whole†.[31]

One of the famous contradictions† is that of the greatest cardinal number. If zero† be taken as a number, then the class of cardinal numbers including zero is one more than the greatest cardinal. Russell at first thought that the contradiction could be solved only by abandoning some common-sense assumption†. The suggestion was that it was wrongly taken as axiomatic† that a class as one is to be found wherever there is a class as many, but that this axiom need not be universally admitted. The class as one does not always exist†, and the class as many is of a different type from the terms of the class.

But there were further difficulties, in which Russell thought that the natural suggestion would be to demur to the notion of all terms or all classes. It might be urged that no such sum total† is conceiv*able*†; and if "all" indicates a whole†, our escape from the contradiction requires us to admit this. But if this view were main-

[28] C. I. Lewis and C. H. Langford, *Symbolic Logic,* pp. 286, 301.

[29] B. Russell, *Principles,* p. 362.

[30] *idem, Introduction to Mathematical Philosophy,* 1930, pp. 135 *f.* This work will hereafter be cited as *Introduction.*

[31] A. N. Whitehead and B. Russell, *Principia Mathematica,* 1925, Vol. 1, p. 37.

tained against *any*† term, all formal truth† would be impossible†, and mathematics, the characteristic of which is statement of truths concerning any terms, would be abolished at one stroke. Thus the correct statement of formal truths requires the notion of any term or every term, even if not the collective notion of all terms.[32] This, by the way, might have been met by recognizing that "any" carries innotative reference, as "all" carries enotative reference.

Later came a solution involving a theory of knowledge†. Russell declared that classes are logical fictions. The supposition that a class is or is not a member of itself is meaningless in just this way. To construct symbolically any class whose members are not all of the same grade in the logical hierarchy is to use symbols in a way to make them no longer symbolize anything. Still, he does "not pretend to have explained the doctrine of types."[33]

Others, continuing and criticizing the work of Russell, have developed solutions based upon distinctions between things and names, or objects and words used to describe them. Ramsey holds that the contradictions† which Whitehead and Russell try to remove are really of two kinds, unified by being deduced in a rather sloppy way from the vicious-circle principle. The contradictions of one group are removed, as above, by pointing out that a propositional function can not significantly take itself as argument, and by dividing functions and classes into a hierarchy of types according to their possible arguments. But other contradictions, like Weyl's problem of the "heterological," contain some word like "means," and are due to an essential ambiguity of such words. They result from the obvious ambiguity of naming and defining. The name or definition is in each case a functional symbol, which is only a name or definition by meaning something. The sense in which it means must be made precise by fixing its order. The name or definition involving all such names or definitions will be of a higher order, and this removes the contradiction. For Ramsey, the second set of contradictions probably has a psychological† or epistemological† and not a purely mathematical solution. In particular, the proposed solution by the aid of the "axiom† of reducibility" is faulty.

Ramsey, with a clue from Wittgenstein,[34] admits differences of order (or type, or, as we should say, sublevel), but makes this difference apply to names or symbols as distinct from

[32] B. Russell, *Principles*, pp. 105, 368.
[33] *idem*, *Introduction*, p. 137.
[34] F. P. Ramsey, *Foundations of Mathematics*, pp. 24-9, 34, 37, 42, 48*f*.

objects.[35] We should say that obvious as the latter distinction often is, for a logical realism it is sometimes a needless complication. The sublevels need not be partly objective and partly subjective or symbolic. Universals are characteristic of the subsistent realm of pure logic, and the sublevels marked by them are as objective as their epitomizing parallels in the germ cells of organisms. In short, the way out of the difficulties with the doctrine of types does not seem to us to be through Ramsey's epistemology.

This applies to the somewhat similar view of Lewis and Langford, with their "conceptual complexes" involving possibilities† open to realization, as against purely logical propositions which have no significant contradictions†.[36]

The theory of Lewis and Langford is based upon several presuppositions which we should have to question. To mention three, they concern (1) the relationships of mind to its objects, as well as the theory of intension and extension; (2) the nature of possibilities†; (3) the nature of generalizations. With regard to (1), we should say that conceptual complexes, even when they do not accurately represent either the existent or the subsistent realms, reveal something of the relationships of the two. The conceptual complex of a centaur is to be referred to neither of the two, exclusively, but to their relations to one another. As noted elsewhere, we maintain that the relationship of intension and extension is not that of conceptual content and instances, but of attribution and distribution, whether in concepts, world, or both. As to (2), for us, purely abstract possibilities constitute the initial level of the logical realm. They are ontologically real, although not describable. As such they are enotative. Their persistence in later realms gives us what possibilities there are, for example, in the physical world and among our concepts; these possibilities, too, can be indicated but not described, because they are enotative and innotative. Then, (3) generalization always marks a difference of level or sublevel in propositions. We regard all adjoined contradictories† as significant, although most of them are significant only of enotative reference. The essential point about the doctrine of types is not that certain highly abstract generalizations evade it, but that the abstract generalizations are

[35] R. W. Sellars, in his *Philosophy of Physical Realism* (p. 349) calls the theory of types "the theory of what propositions are about," and says that it refers to the capacity of the human mind to make its primary propositions (about things and events) themselves the objects of secondary propositions.

[36] C. I. Lewis and C. H. Langford, *op. cit.*, pp. 463, 466, 471, n. 12, 472, 482 *ff*.

among those which show its essentially enotative reference. Any generalizations about generalizations, including those of the authors studied, or our own, merely exemplify this statement, and help to indicate a point of reference in such an enotative hierarchy.

Much of Ramsey's work, with its distinction between paradoxes which arise in mathematics and others which have nothing to do with mathematics, is valuable enough. He makes universals involve possibilities†, and sometimes infinites†, and describes what he thinks could be done with infinite resources for expressing atomic functions.[37] We should say that it is no safer to rely, in this difficulty, upon brief descriptions of infinite numbers of meanings, without allowing for enotative reference, than it is to rely upon a construction of propositions of infinite length or infinite numbers of propositions. Ramsey says that if indefinable classes are excluded, the meaning of statements about all classes will be fundamentally altered.[38] We should say, rather, that the meaning of "all" requires that some classes, especially those of high orders, be excluded and left indefinable, enotative.

Some restrictions are necessary, and some distinctions of order, type, or sublevel are indispensable in the broader problems of universals. Wittgenstein holds that the word "elementary" is not an adjective of the proposition type, but only of its instances.[39] Somewhat similarly, for us the word "universal" is misleading when applied to a term. Some terms are universals, just as some cells are germ-cells,[40] but the word "universal," properly speaking, characterizes a *class of terms,* or objects, or propositions. As Ramsey says, general propositions containing "all" and "some" are found to be truth-functions for which the arguments are not enumerated but given in another way.[41] For us, to universalize about any given term, object, or proposition *A,* is to construct classes, made definitive by assembling the full range of any selected *A* and adding to it any *A* at first concealed in *non-A, i.e.,* by establishing the mutual exclusion of *A* and *not-A.* The class may be described connotatively by its attributes or denotatively, with its extent merely indicated, but in any case it must involve an enotative reference. Universals are more nearly akin to finites than to infinites. Each universal is an expression of the solidarity which is characteristic of the class to which it applies. As the epitomization hypothesis has it, a universal may be compared

[37] F. P. Ramsey, *op. cit.,* pp. 33 *f.,* 39-42. [38] *ibid.,* p. 22.
[39] *ibid.,* p. 34. [40] See G. P. Conger, *Epitomizations,* p. 431.
[41] F. P. Ramsey, *op. cit.,* p. 8.

to a germ-cell containing a number of chromosomes characteristic of a given species. There are universals of various degrees of comprehensiveness of terms, just as there are species of various characteristic numbers of chromosomes.

Eaton has tried to avoid the difficulty of the universal class of all classes, by maintaining that there is a universal class of which everything except this class is a member.[42] But Eaton's "universal class" *taken together* with "the class of which everything except this class is a member," constitutes a new type or sublevel which is a new universal class—*i.e.*, all those classes mentioned in the foregoing part of this sentence. The point is that each new universal is a universal with some condition left enotative.

Eaton finds that though the universal class is not a member of itself, still it is identical† with itself.[42] In his effort to avoid the enotative reference proper to such a problem, he disregards the innotative reference involved in identity.

The process of universalizing runs through successive ranges, each at its own sublevel involving enotative reference. There is a hierarchy of universals, just as there is of atoms and organisms. The successive sublevels correspond to successive types, functions, etc. But at each added function, range, type, or sublevel, there is at least for the time being enotative reference. Any universal and any class indicated by "all," any totality, is but temporary and in the midst of an unexplored milieu, which the next higher function, type, etc., may penetrate still further but will in its turn leave open. This is the way in which the conception of a totality of things is, as Russell hinted, illegitimate;[43] it is illegitimate unless it carries an enotative reference. To make a class a member of itself is merely to disregard the difference of sublevel, and confuse the container with the contained, by reason of the fact that the same word ("class") is used indiscriminately for both. The contradiction of the "greatest cardinal" is the kind of contradiction that always arises when a mere enotative reference is treated denotatively or connotatively.

In short, the way to escape from the vicious circle principle is by the spiral route. The class-term is not in the circle which it sums up, but in the class just above, and so on. The "class of all classes" is open at the end and left enotative.

(III) The subject of an *A* proposition exhibits a selection in a class of predicates, a class which is undistributed. The undistrib-

[42] R. M. Eaton, *Symbolism and Truth,* p. 243, n. 2.
[43] B. Russell, *Principles,* p. 362.

uted predicate of the *A* proposition is properly indeterminate and enotative. If "All *A* is *B*," the class *B* is made up of the class *A* and an undetermined portion of the class *not-A*.[44]

(IV) The process of induction, with or without direct regard for its truth†, rests upon certain assumptions. Johnson says that in order to obtain the requisite premises for demonstrative induction, we must assume that by a preliminary induction, based upon general experience, a number of variable circumstances have been eliminated as irrelevant to the formula to be proved.[45] And irrelevant circumstances are left enotative. Creighton, seeking the ground of induction, says that we assume, and must, that the world† is a cosmos†, not a chaos. And this means that there are universal relations and connections of events which, if once discovered in their true† nature, may always† be depended upon. Our thinking assumes that identity† of law and identity of nature exist in and through the diversity† of things, and that it is in virtue of these universal principles of connection that the world is a coherent and intellig*ible*† system.[46] It is significant that in this statement, designed to be fundamental, these are so many terms requiring enotative reference. Keynes brings out the fundamental nature of certain assumptions made in the process of induction, when he says that we assume that the necessary† number of instances is finite, but we do not know what the number is. We know that the probability of a well established induction is great, but we can not name its degree. The probability of an induction is numerically definite only when we are able to make definite assumptions about the number of independent equiprobable influences at work. Otherwise it is non-numerical, though bearing relations of greater or less to numerical probabilities, according to approximate limits† within which our assumption as to the possible† number of these causes lies.[47]

(V) Boundary conditions in fields of induction do not at first sight seem likely, but in our study of negatives of suspension, we distinguished between the connotatively and denotatively known *A* and the enotatively and innotatively known *non-A*. Contradiction†, we said, sharpens this into *A* and the negative† of exclusion, *not-A*, but we may still consider an intermediate or boundary region

[44] C. L. Lewis, *Survey*, p. 61; *cf.* p. 186.
[45] W. E. Johnson, *op. cit.*, Part 2, pp. 218 *f.*
[46] J. E. Creighton, *An Introductory Logic*, 1932, pp. 242 *f.*
[47] J. M. Keynes, *op. cit.*, pp. 253, 259.

non-A, left innotative. Even though this be called "nothing†," it must still be recognized. Contradictions are established by processes of induction, which at least undertake to comb the *non-A*'s, assemble all the *A*'s, and distinguish them from the *not-A*'s. But even in the process of induction, we must maintain room for a possible intermediate, innotative *non-A,* except perhaps in some cases of induction so perfect that it is not important.

(VI) An induction may involve series formation and a process like that of growth, as in the so-called mathematical induction. This raises questions which, again, are best considered with other similar problems in Chapter XIII.

(VII) According to Johnson, demonstrative induction resembles deduction, in that it requires the conjunction of two types of premise.[48] But this is an explicit, rather than an implicit, duality.

According to the epitomization hypothesis, the process of exclusion, with formation of universal negatives in pure logic, is epitomized and paralleled by the process of extrusion in biology.[49]

(VIII) Whitehead criticizes the usual inductive procedure in terms of his more "organic" theory. Induction, he says, is more complex than Bacon thought. Either there is something about the immediate occasion which affords knowledge of the past† and the future†, or else we are reduced to utter skepticism† as to memory and induction. The key to the process of induction is to be found in the right of understanding of the immediate occasion of knowledge in its full concreteness.[50] But we should say that even the organic features of induction leave it under marked limitations. Whitehead says that he does not hold induction to be in its essence the derivation of general laws; it is the divination of some characteristics of a particular† future from the known characteristics of a particular past. The wider assumption† of general laws holding for all cognizable occasions appears to be a very unsafe addendum to attach to this limited† knowledge. All we can ask of the present occasion is that it shall determine a community of occasions which are in some respects mutually qualified by reason of inclusion within that same† community.[50]

(IX) Like all procedures involving or verging upon the statistical, induction sometimes leaves room for innotative reference.

[48] W. E. Johnson, *op. cit.,* Part 2, p. 241.
[49] See G. P. Conger, *Epitomizations,* pp. 166 *ff.,* 443 *ff.*
[50] A. N. Whitehead, *Science and the Modern World,* pp. 61-3.

According to Keynes, when our definite knowledge about the instances is limited†, we must begin to pay attention to their number rather than to the specific differences between them. When our previous knowledge is considerable and the analogy is good, the purely inductive part of the argument may take a very subsidiary place. But when our knowledge of the instances is slight, we may have to depend upon pure induction a good deal.[51]

The principle of innotation accounts for the difference between "any" and "all." The distributive "any" indicates that at least there is a selection in the midst of a class, where, for instance, one member is taken and the others are left enotative as regards the member selected, and innotative as regards the class. "Every" is the attempt to extend the distributive selection indicated by "any" to the class indicated by "all." It may or may not be justified, according as the induction is perfect or imperfect, as above. If there is no limitation† imposed by "every," then the selection indicated by "any" is made in the midst of conditions where still more is left enotative or innotative. Hasty generalization leaves too much.

Thus the process of induction may be seen to proceed within limitations. Sometimes its horizons are vague, as in generalizations. Sometimes the words used for precision are applied to results which are actually vague. But always there are horizons, and references to what may lie beyond those horizons must be guarded. The further discussion of these problems raises questions of the limits of induction and its truth-value; these questions will be considered in the next three chapters.

[51] J. M. Keynes, *op. cit.*, pp. 234, 241.

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CHAPTER XI

The Logical Universe

"Let us say that the whole idea under consideration is the *universe* (meaning merely the whole of which we are considering parts) and let names which have nothing in common but which between them contain the whole idea under consideration, be called contraries *in, or with respect to, that universe.*"
—A. De Morgan, *Formal Logic, or the Calculus of Inference . . .*, 1847, p. 38.

A. In this and the following chapter we discuss two terms of opposite significance in logic—"the universe," or "everything," and "nothing." This chapter is devoted to the logical universe; the physical universe, which is a special case within the logical universe, with its own characteristic problems, will be considered in Chapter XIX.

B. (α) Stebbing says that if by "world" we mean everything† that is the case, it may be doubtful whether the world is a system, since we can not empirically observe everything.[1]

(β) According to Keynes, a particular universe of reference may be defined by considerations which are partially psychological, but when once the universe is given, the theory of the relations in which other propositions stand towards it is entirely logical.[2] In mathematical logic, too, the symbol for the universe often carries a touch of psychology. 1 is interpreted intensionally as that whose negative† is inconceiv*able*†—a concept such that everything† conceivable falls under it.[3]

We shall argue that in all such cases the term "universe" should be left enotative. This is especially true if account is taken of the psychological processes which are involved and which are so plainly enotative.

(γ) The problems of idealism and realism are suggested when Lewis says that the most frequent universe of discourse is "phenomena."[4] We shall see, at E (v), that Wittgenstein's study of the logical world leads him to solipsism. But even with phenomenologi-

[1] L. S. Stebbing, *Modern Introduction to Logic*, pp. 197 *f.*
[2] J. M. Keynes, *Treatise on Probability*, p. 131.
[3] C. I. Lewis and C. H. Langford, *Symbolic Logic*, p. 67.
[4] C. I. Lewis, *Survey*, p. 329.

cal or solipsistic presuppositions, the thinking process and the objects which are thought involve horizons.

(δ) Wherever we have intuitional theories, we find that the objects of the alleged intuitions are regarded as so important that they are almost inevitably held to constitute a universe.

C. Writers dealing with the universe have sought to domicile their object within several other horizons of thought.

(a) In the Boole-Schröder algebra, any two elements which have the value 1 are identical, so far as their properties in the algebra are concerned.[5] The qualification affords ample covering for enotation as well as innotation. According to Whitehead, propositions which are implied in reasoning, without rising to explicit consciousness or needing explicit statement, might be equated to the universe.[6]

(b) One of the postulates† of the Boole-Schröder algebra of logic defines the universe by a negative reference to zero†; $1 = -0$.[7] This may be passed as merely a restatement of the relationship of exclusion, but strictly speaking, it is a negative statement about an enotative term.

(c) De Morgan, in the passage quoted at the head of this chapter, introduced the idea of the universe of discourse by taking a pair of what he called contrary or contradictory names, such as man and not-man. These have nothing† in common, but between them represent everything† imagin*able*† or real† in the universe.[8] But affixing the name "universe" to any two contradictory terms is not enough unless one recognizes that the universe, or whole idea under consideration, can not be exhaustively described, but itself carries enotative as well as innotative reference.

Even a subordinate system must be described as containing one contradictory, but not both.[9] For Lewis and Langford, a set of propositions may be said to be consistent, or the members of the set to be compossible†, if and only if the set does not imply a contradiction.[10] The contradictories not contained in a system are with respect to it left enotative. With respect to a universe they may be innotative. The fact that either a universe of discourse or a system

[5] C. I. Lewis and C. H. Langford, *op. cit.*, pp. 87 *f.*
[6] A. N. Whitehead, *Universal Algebra*, p. 109.
[7] C. I. Lewis, *Survey*, p. 119.
[8] *ibid.*, pp. 37, n. 60; 40, n. 66
[9] See L. S. Stebbing, *op. cit.*, p. 199.
[10] C. I. Lewis and C. H. Langford, *op. cit.*, p. 338.

within such a universe can be delimited by the help of contradictories is a case of the broadening and narrowing of horizons familiar in other connections. The range or scope within a horizon may vary, but each horizon, whether it is of the constituent system or the total† universe, carries its enotative reference.

(d) We find the universe defined in terms of a universal when Boole says that whatever may be the extent of the field within which all† the objects of our discourse are found, that field may properly be termed the universe of discourse.[11] The limitations here are those of the word "all," as discussed in Chapter X.

Failure to allow for horizons involved in the use of the terms "any" and "all" appears to be responsible for the view of McTaggart, that any two, or more, substances will form a compound substance, and that there must therefore be one compound substance which includes all substances. He holds that any content which is not† in any given substance A must be in some substance or substances outside of A; and by adding these to A we shall have a compound substance U which contains all content and all substances. Every compound substance other† than U will contain some but not all of the content of U, and will therefore be part† of U. This compound substance is what we call the universe.[12] In Lewis's treatment, the application of the algebra to classes involves truth-values, and will be considered at (g) below.

(f) For the Boole-Schröder algebra, the universe of discourse, "everything," is the negative† of the null class, "nothing." The horizon properties of this last term are discussed in Chapter XII.

(g) Lewis says that the term universe of discourse is pretty well understood, but may be defined as follows: If A_n be any† member of class† A and x represent the class concept† of the class x, then the universe of discourse is the class of all† the classes, x, such that "A_n is an x" is either true† or false†. For example, if "the fixed stars are blind" is neither true nor false, then "fixed stars" and the class "blind" do not belong to the same† universe of discourse. Again, in the application of the Boole-Schröder logic to propositions, the propositional sign A may be taken to represent the class of cases in which proposition A is true. 1 will be the class of all† cases or all "actual"† cases, or the universe of facts. "$A = 1$" represents "the cases in which A is true are all cases," or "A is

[11] G. Boole, *An Investigation of the Laws of Thought*, 1854, p. 42.
[12] J. E. McTaggart, *Nature of Existence*, Vol. 1, p. 147.

true in point of fact," or "*A* is true."[13] Besides involving truth-value, the first definition turns upon relationships between intension and extension, as these are usually interpreted; the second raises questions about actuality and possibility; both definitions involve the question of the significance of the word "all."

(h) Boole noted that in every discourse, there is an assumed or expressed limit within which the subjects of its operations are confined. The most unfettered discourse is that in which the words we use are understood in the widest possible† application; for then the limits of discourse are coextensive† with those of the universe itself.[14] But, as we have repeatedly suggested, the limits of any possible universe must carry enotative reference.

(j) Keynes suggests that the universe is infinite, but puts it negatively†, with ample room for enotative reference. He says that the assumption† that all† systems of fact are finite, at least in the sense in which he defines the term, can† not†, it seems plain, be regarded as having absolute†, universal† validity†. The most which can be maintained is that this assumption is true† of some† systems of fact and that there are some objects about which, as soon as we understand their nature, the mind is able to apprehend directly that the assumption† in question is true.[15] We find with the Boole-Schröder algebra, $1 = (a + -a) \cdot (b + -b) \cdot (c + -c) \ldots,$ but this is valid only if the number of elements involved be finite, since the proof depends on the principle of mathematical induction.[16] Here various conflicting views concerning the logical universe can be reconciled by recognizing that any connotative description of a universe should allow for enotative and innotative reference, and all the more if the universe is to be called infinite. Some of the advantages of symbolizing the universe by ∞ rather than 1 will be indicated in the note appended to this chapter.

(n) Views of the relationship of the universe of discourse to possibilities vary with different writers. Russell interprets Wittgenstein as unwilling to exclude the possible from the world. In logic we can not say there is this and this in the world, but not that, for to say so would apparently presuppose that we exclude certain possibilities, and this can not be the case, since it would require that logic should go beyond the boundaries† of the world.[17]

[13] C. I. Lewis, *Survey,* pp. 185, 213.
[14] G. Boole, *op. cit.,* pp. 42, 47.
[15] J. M. Keynes, *op. cit.,* p. 262.
[16] C. I. Lewis, *Survey,* pp. 131 *f.*
[17] B. Russell, in L. Wittgenstein, *Tractatus,* p. 18.

According to our view, possibilities are real, and belong in the logical universe of discourse as its initial level. They are not, however, fully describable. Some of them should be left enotative, and in this sense excluded from the remainder of the universe. Other possibilities may be accommodated among the actualities of the remainder of the universe, but these also are not fully describable, and should be left in some respect innotative. Logic can not go beyond the boundaries of the world, but it can indicate that the question of such boundaries is to be left enotative or innotative.

Adler makes the universe of discourse, as distinct from the realm of actuality, a realm of possibility.[18] He thus uses the term "universe of discourse" for what we would call only the initial level of the initial subsistent logical realm. In any case, however, both the universe of discourse and its possibilities should carry enotative and innotative reference.

(p) Mrs. Swabey holds that if thought is able to reveal the universal in the case of lesser kinds, we are led by a continuous† process to assume† that the universe itself, as the kind of all† kinds, can be apprehended in the same† manner, as a logical unity.[19] But the continuous process, passing from one unity to another, should make continual allowance for enotation and innotation, in the case of the universe as of every lesser kind.

(q) The universe of discourse is often made to involve the notion of a whole, or totality. For De Morgan, the universe is a range of ideas which is either expressed or understood as containing the whole matter under consideration. Stebbing says that in this sense the phrase "universe of discourse" is an unfortunate name for what would ordinarily be called the context within which a proposition is asserted.[20] The latter, we should say, is preferable, because a universe suggests a circumscribed or at least a unified region, but a context may easily suggest enotation.

Stebbing also sees the difficulty of trying to describe the whole world. Even if the actual† world were a completely† coherent system it would not follow that knowledge of a part of the system would yield knowledge of the whole system, nor even that it would involve knowledge of what kind of system it was.[21]

For us, the difference between part and whole must be left enotative as regards any part, and innotative as regards the whole.

[18] M. J. Adler, *Dialectic*, pp. 199, 204.
[19] M. C. Swabey, *Logic and Nature*, p. 377.
[20] L. S. Stebbing, *op. cit.*, pp. 55 *f.*
[21] *ibid.*, pp. 119 *f.*

Sometimes a region within the whole world is specified and left thus enotative and innotative. For Whitehead, any term x can be written $x + 0$. However, since x does not exhaust the whole region of discourse, say it is not 1, let the 0 be modified into ω. Then $x + \omega$ expresses the fact that x is not equivalent to 1. ω then may be looked upon as a limitative† symbol restraining from undue extension.[22]

Bosanquet has the organic view, but does not see that the total universe should carry enotative reference. He says that if we can intelligibly speak of the universe as a whole, we must take it as the totality of relations, and therefore as bearing no† relation to anything† outside itself. But this speculation is unprofitable, because what is out of relation is out of knowledge, or has at most only a negative† value. He prefers the doctrines which extend the relativity† which holds within the totality of relations to the ideal† totality of relations itself, and so discuss its origin† or the possibility† of its not having been†.[23] We should say that the former view is the more adequate, since a negative has clearer enotative reference than such an "ideal" totality or such an account of origins. Bosanquet shows the inadequacy of such accounts when he says that all explanation is within the universe, not of it.[23]

D. (1) Objections to what we should call the connotative description of the universe are stated by Mrs. Swabey. Thus it is said that to allude to the universe is to refer to something with no definite meaning. Even granting that the notion of a *summum genus* covers everything, it discovers nothing†. Though by definition all†-inclusive, as to the nature of what is included it affords no inkling. "Chaos" or "non-entity"† so named would enjoy as much unity† and definiteness of meaning as "universe" or "cosmos."[24] But Mrs. Swabey's answer, with a Cantor-like interpretation of the universe as infinite, involves the difficulties characteristic of such affirmative statements, discussed in Chapter XVI.

(2) According to Whitehead, the supplement of the universe is a non-existent† region.[25] Taken at its best, this may be said to reflect the characteristic of the universe which we have called inclusion and exclusion. But the definiteness of the symbol $\blacksquare A$ is mis-

[22] A. N. Whitehead, *Universal Algebra,* p. 84.
[23] B. Bosanquet, *Logic,* 1911, Vol. 1, pp. 136 *f.*
[24] M. C. Swabey, *op. cit.,* p. 366.
[25] A. N. Whitehead, *Universal Algebra,* pp. 36, 39.

leading. We can say that† the universe is exclusive without at all being able to deny existence to the excluded.

(4) Cohen says that of incompletely† determined existence, as in the case of the total† universe, contradictory† propositions do not annihilate each other, since they refer to a complex of existences; yet they can† not† always†, because of the indefiniteness† of the subject, be reconciled to each other. This gives rise to the antinomies of metaphysics.[26]

E. (I) The logical universe can not be exhaustively described, but must be left with somewhere an open reference. Whitehead, for example, takes De Morgan's universe of discourse to be limited†, but sees that the class† of propositions making up the universe is perhaps indefinite in number.[27] On the other hand, Ramsey says that it is only when we take, not a limited† universe of discourse, but the whole† world, that nothing† can† be said about the number of individuals in it.[28]

(II) Universes of various grades or sublevels of complexity may be distinguished, as when Schröder distinguishes between the universe of individuals, symbolized by 1^1, and the universe of binary relatives, symbolized by 1^2.[29] Moreover, as Mrs. Swabey suggests, the term "universe" can be included in assertions about the universe as the all†-inclusive object.[30] And for that matter, any number of limited universes of discourse may be grouped in a universe of universes, still exhibiting, however, the principle of enotation and innotation.

(III) The famous thesis of Bradley, that the universe is the real subject of all† propositions,[31] would ascribe all predicates to that subject. But the word "all" here can not be exhaustively described, nor the predicates exhaustively enumerated. Any which are described or enumerated are but selections from the "all-but-infinite"‡ number, some of which should be left enotative.

(IV) Adler postulates that† there is a comprehensive universe of discourse which has the characteristic of an infinite† class† of ordered† entities†.[32] The term "universe" is used by Whitehead in

[26] M. R. Cohen, *Reason and Nature*, p. 167.
[27] A. N. Whitehead, *op. cit.*, pp. 100, 109.
[28] F. P. Ramsey, *Foundations of Mathematics*, p. 61.
[29] C. I. Lewis, *Survey*, pp. 111 *f*. [30] M. C. Swabey, *op. cit.*, p. 370.
[31] F. H. Bradley, *Principles of Logic*, 1922, Vol. 1, pp. 41, 50 *f*.; Vol. 2, p. 662.
[32] M. J. Adler, *Dialectic*, pp. 203 *f*.

connection with the truth† of propositions. Such a universe may be fixed by convention. These propositions, or that class of perhaps an indefinite† number of propositions will be severally considered as equivalent† to the universe when their validity† has acquired some special absoluteness† of assent. Such a convention may have the force of an axiom. Whitehead says that the name "universe" is unfortunate; "truism" would be better.[33]

(v) Wittgenstein's view that the logical world can have no logical boundary leads to an identification of the boundary with the thinking subject and so to a curious solipsism. Logic, he says, fills the world; the boundaries of the world are also its boundaries. We can not give any definite description of the world, for this would involve an exclusion of which logic is not capable; it would, as we saw, require that logic should go beyond the boundaries of the world, as if it could contemplate these boundaries from the other side also. The boundaries of language indicate the boundaries of my world, and the metaphysical subject, which does not belong to the world, is the boundary of the world.[17] We should say that recourse to solipsism always raises many more problems than it solves. Here it is unnecessary, because enotation makes possible and in fact requires from within the world exclusion without description; moreover, any boundary between what is included and what is excluded is to be left innotative. We can not describe either the boundaries of the universe or the bounds of possibility; they may be identical†.

In Russell's preface, there is a slightly different view, without recourse to the transcendental subject. According to the different view, the world may† be bounded for some superior being surveying it from above, but for Russell, however finite it may be, it can† not† have a boundary since it has nothing† outside it.[17] In other words, the question is answered, but answered without recognizing that the answer carries enotative reference. Wittgenstein uses as analogy the field of vision. He says this field does not have a visual boundary, just because there is nothing outside it, and in like manner our logical world has no logical boundary, because our logic knows of nothing outside it.[17] This, for us, is an unfortunate comparison, as an investigation of perception suffices to show. "Nothing" is here enotative, or innotative, and quite non-committal; it

[33] A. N. Whitehead, *op. cit.*, pp. 109 *f.*, 113.

may as well as not mark a boundary, either for the field of perception or for the logical universe, or both.

(vi) Lewis says that in the applications of the Boole-Schröder algebra of logic to spatial entities, the elements of the algebra *A, B, C,* etc., will denote continuous† or discontinuous regions in a plane or in a circumscribed portion† of a plane. 1 represents the plane (or circumscribed region) itself.[34] The alternative here offered, *i.e.,* the plane or a circumscribed portion of it, indicates that the universe of discourse may, at different moments, be taken to be less or more inclusive. Lewis goes on to say, in connection with the Venn diagrams, that it is not really necessary to draw the enclosing square, 1, since the area given to the figure, or the whole page, may as well be taken to represent the universe. But when the square is omitted, it must be remembered that the unenclosed area outside all the lines of the figure is a subdivision of the universe.[34]

This is another instance of what we call the leapfrog procedure, which leaves the precise significance of the term "universe" ambiguous. When the universe is represented by the enclosing square, the area outside the square is the enotative non-universe. By a process reflecting the characteristic of growth, the universe may next be represented by the area given to the figure or to the whole page. Then the area not given to the figure, or the area off the page, is the enotative non-universe, and so on. The square may be, as Lewis says, omitted, and the unenclosed area may be a subdivision of the universe; but it is not the same universe, or not the universe at the same moment, as when it was represented by the enclosing square. The denotation, connotation, enotation, and also the innotation vary from moment to moment, without in the least disappearing as formal conditions. In fact, the uncertainties of such a diagram begin to be seen. According to Lewis and Langford, it is a principle of this style of diagram that an unmarked area can† not† be presumed† to represent something which exists†, and it can not be presumed that what it represents does not exist. If an area is neither struck out nor marked, then the data tell us nothing† about the subclass which it represents.[35]

A universe of discourse may contract as well as expand. For Whitehead, in spatial problems, the element called the universe must be identified with all† space; or, if discourse is limited† to an

[34] C. I. Lewis, *Survey,* pp. 175, 177.
[35] C. I. Lewis and C. H. Langford, *op. cit.,* p. 54.

assigned portion† of space which is to comprise all the regions mentioned, then the universe is to be that complete† region of space.[36]

(IX) Innotation, which appears within De Morgan's universe, may be illustrated by the help of the Venn diagrams. Such a diagram initially leaves room for any possible† relation of the classes represented by regions within it, and the actual† or given relation can then be specified by indicating that some particular region is null† or is not-null. Initially the diagram represents simply the "universe of discourse," or 1. When the universe of discourse is represented by a square, and an element A is represented by a circle included within the square, then the remainder of the square represents $-A$, and so on for subclasses. The negative of an entity is always the plane exclusive of that entity.[37] But where the plane is used, $-A$, lying within it, is not the only negative involved. There is also the "non-plane" outside it. With respect to the plane, $-A$ may be regarded as innotative, while the non-plane is enotative; and so on for each increment added to the universe. Innotation may also be indicated, especially by the help of 0, without the spatial interpretation.

Studying equivalent† forms of the equation $A = B$, Lewis says that for any equation there is a set of equivalent equations in terms of elements which appear in the given equation. Every such set has its member $= 1$ and its member $= 0$. These are the functions in terms of which an arbitrarily chosen t is determined. Any t is the t which contains the function $\{= 0\}$ and is contained in the function $\{= 1\}$. The validity of this law of the theory of equations depends simply upon the fact that for any t, $t = 1 \cdot t + 0 \cdot -t$. It is rather surprising, says Lewis, that a principle so simple can yield a law so powerful.[38] According to our view, the law is powerful merely insofar as it embodies the successive inclusions characteristic of the universe of discourse. So far as its content goes, its power is that of the indefinite resources of enotative or innotative reference. It is noteworthy also that in solving such equations, we are to get the completest† possible† expression equivalent† to 0 or the least† inclusive possible expression equivalent to 1.[39]

Positive and negative terms within or with reference to the universe are involved in the "law of consequences." The "law of

[36] A. N. Whitehead, *op. cit.*, p. 39.
[37] C. I. Lewis, *Survey*, pp. 176 *f.*
[38] *ibid.*, p. 146.
[39] *ibid.*, pp. 162 *f.*, 165.

consequences" expresses any inference from $A = B$ by taking advantage of the fact that if $A \cdot -B + -A \cdot B = 0$, then $(A \cdot -B + -A \cdot B)\, v = 0$, and if $A \cdot B + -A \cdot -B = 1$, then $A \cdot B + -A \cdot -B + u = 1$.[39] We should interpret these expressions by saying that an indeterminate element 0 has in common with an arbitrary element v only an indeterminate element; and that in a given universe of discourse 1, there is always room for the inclusion of an arbitrary element u hitherto left innotative.

Note on Some Symbols of Mathematical Logic

We find that allowance for enotation and innotation at least helps to explain the striking fact, first made prominent by Boole, that certain laws of logic can be formulated in symbols of mathematics and that there results from this an algebra, which, with some exceptions and limitations, yields a calculus of classes, propositions, and relations.

In the first place, there is a reason for the application of the same formulae to propositions and to classes. It is that, according to the hypothesis of epitomization, terms-in-propositions and classes (bodies of propositions, with generalizations) are monads of two different logical levels (13.00 and 14.00, respectively) and like other monads, exhibit typical monadic characteristics, with analogous structures and processes. The fact that in the interpretation for propositions, x indicates the class of propositions for which the relationship φ is true really needs the above for its justification; otherwise the class relationship is taken for granted, and not regarded as a case of integration and difference of level, such as is characteristic of the cosmos.

It is familiar that in the two-valued algebra the values are restricted to two numbers 1 and 0, the properties of which are peculiar. It is said that the distinctive law of Boole's system, $xx = x$, holds of 0 and 1, and of no other numbers.[40]

From this index law, Boole derives the law of contradiction. Since $x^2 = x$, $x - x^2 = 0$; then, by factoring, $x\,(1 - x) = 0$. But Lewis says that this transformation hardly represents any process of logical deduction.[41]

To us, the result of the transformation appears to be due to the fact that the symbols have a number of properties which

[40] C. I. Lewis and C. H. Langford, *op. cit.*, p. 13.

[41] C. I. Lewis, *Survey*, p. 55.

here coincide. In the first place, by convention x^2, representing repetition or reiteration, is set equal to x, with differences left enotative or innotative. Then by taking one from the other, the result is said to be zero—another term with enotative or innotative reference. But where reiteration is represented by the symbols of higher powers, it is hard to see for what the symbols or processes of division or factoring might stand, unless it is for some process of reduction whose only importance is that it retracts the error of the exponentiation. Lewis says that some of the algebraic processes of the system and some resultant expressions will not be interpretable in terms of logic.[41] Conversely, some of the logical interpretations will not follow the numerical algebra. In Boole's demonstration, logically, $x\ (1 - x) = 0$, but only because here in the non-numerical algebra the term 1 has no numerical value, and $1 - x = -x$. If 1 is to have numerical value, in order to secure this result we must say $1 = 0$.

Some of the other relationships between Boole's algebra and numerical algebra are quite surprising. We consider the four equations, $1 \cdot x = x$; $0 \cdot x = 0$; $x + 0 = x$; and $x + 1 = 1$. Involved in these is the general principle that if in any theorem, each element be replaced by its negative, and $\times$ and $+$ be interchanged, the result is a valid theorem. Thus for every valid theorem in the system, there is another got by interchanging the negatives 0 and 1 and the symbols $\times$ and $+$. This principle is called the Law of Duality.[42]

In the first of the four expressions, when 1 and x represent classes, 1 must represent such a class that all the individuals which are found in any proposed class x are also all the individuals $1 \cdot x$ that are common to that class x and the class represented by 1. The class 1 must be the universe, since this is the only class in which are found all the individuals that exist in *any* class.[43] Or the fact that 1 and x have members in common, with its symbolization in logical multiplication, may be interpreted in terms of selection; $1 \cdot x$ means that selecting the x's from the universe gives x. The point here is that whatever belongs to 1 but not to x is utterly neglected and enotative, and does not appear in the result, which is therefore x. We may say that, since everything not x is neglected, $x = 1$, and $1 \cdot x = x$; or, we may say that the x is taken only "1 time," and it is in this way that $1 \cdot x = x$ happens to be true for arithmetical multiplication, too.

As to the second, that which x and 0 have in common is postulated as 0,[44] regardless of enotation or innotation. Elsewhere,

[42] *ibid.*, pp. 125 *f.* [43] G. Boole, *op. cit.*, p. 48. [44] C. I. Lewis, *Survey*, p. 119.

Lewis notes that 0 as a factor can be otherwise expressed.[45] We should say that $0 \cdot x = 0$, because an enotative or innotative term can have nothing describable in common with any x which excludes it; if there is anything in common, it remains enotative or innotative.

In the third, the result of adjoining x and 0 is x; since, by definition $x + b = -(-x \cdot -b)$;[44] substituting 0 for b, although 0 should carry innotative or enotative reference, we have $x + 0 = -(-x \cdot -0) = -(-x \cdot 1) = -(-x) = x$. Elsewhere, Lewis notes that 0 in a sum disappears; $x + 0 = x$. 0 disappears,[45] *i.e.*, is disregarded here, because it is merely a sign of innotation or enotation. That which is either something or nothing, must be something; otherwise the something would be excluded and become nothing—at least nothing which could be described.

The last of the four, $x + 1 = 1$, is not true of numerical algebra, unless as a special case we have $x = 0$. The proof of the logical equation proceeds by much manipulation of negatives. There is, first, the postulate $x \cdot 0 = 0$; then substituting $-x$, $-x \cdot 0 = 0$. Then $-(-x \cdot 0) = -0$. Then by the law of duality [where $-(-x \cdot y) = x + -y$], $-(-x \cdot 0) = x + -0$; hence $x + -0 = -0$. Substituting 1 for -0, we have $x + 1 = 1$. The first 1 represents the universe as known connotatively; and since there can be for this algebra nothing beyond that universe, the only symbol left to indicate the result of the adjunction is the 1 at the right.

Thus the four equations, and many other relationships, are established by disregarding the enotative or innotative references of negatives, 0, and 1. What the Boole-Schröder algebra does is to take from the cardinal numbers an indivisible finite number 1 and assign it the value of the universe. The number 1, since it is indivisible, serves as a symbol for the universe without provision for innotative reference. Since it is finite, it should be understood to carry enotative reference. Innotative reference is in part supplied to the system by the interpretation of $1 \cdot x$ in terms of selection from 1, and also by 0 and the convention that 0 shall be contained in every class. But enotative reference is overlooked.

Our interpretation also explains the peculiar "law of duality," when it is remembered that 0 and 1 are limiting concepts, opposite in sense and (in spite of the apparent exactness of the number 1) not definite as to content; furthermore, logical addition, in the sense of adjunction, and logical multiplication, in the sense of selection, may be opposite processes; in

[45] *ibid.*, p. 286.

$x + y$, x and y may be taken separately but in xy they are combined and their common element picked out from the combination. And any term, especially when indefinite, may be operated on by operating in the opposite sense upon its opposite, as we may go northward by going the opposite of southward.

We may consider the results if, by some changes in the symbolism, both innotative and enotative references are accommodated. The Boolean algebra, after all, did not use all the possible values of x for which xx is numerically equal to x. If we take infinity as a number, and take $x = \infty$, we have $\infty \cdot \infty = \infty$. To make the Boole-Schröder algebra more adequate, it should be at least three-valued; 0 to represent "nothing," with innotative reference, 1 to represent everything denotatively or connotatively known, and ∞ to represent everything with enotative reference.

Lewis says that Boole's failure to include ∞^2 in his index law and let ∞ equal the universe, and 1 equal a particular, crops out when his successors, considering his system as a logic of propositions, say that if x is not equal to 1, x is equal to zero. As applied to propositions, this means if x is not true, it is false, with no middle ground; but this hard and fast division does not apply to classes.[46] We should say that the failure to make ∞ the symbol of the universe crops out, not so much in differences between classes and the truth-values of propositions, as in difficulties due to the disregard of enotative and innotative reference.

In such a system, we should have $\infty \cdot x = x$, $0 \cdot x = 0$, $x + 0 = x$, and $x + \infty = \infty$, with the possibility that the selected $x = 1$, the connotatively known universe. Of the four equations one is not true numerically. There seems to be no improvement in the correspondence between the logical and the numerical algebras; the logical algebra is if anything rendered more flexible and thus less amenable to precise symbolization and manipulation. It is more like the universe, but less like the traditions.

[46] *cf.* C. I. Lewis, *Survey*, p. 66.

CHAPTER XII

SOMETHING WITH REFERENCE TO "NOTHING"

"Great difficulties are associated with the null class and generally with the idea of nothing. It is plain that there is such a concept as nothing and that in some sense nothing is something."

—Bertrand Russell, *The Principles of Mathematics,* Vol. 1, 1903, p. 73.

A. Some distinctions should be made between the term of reference "nothing," the number 0, the zero of magnitude, 0 as the symbol for a false proposition, and the null class,[1] but for our present purposes their logical properties are similar. The connection between a false proposition and the null class is shown by Lewis. If the propositional sign a is taken to represent the class of cases for which a is true, 0 is the class of no cases, and $a = 0$ will mean a is true in no case—*i.e., a* is false.[2] Russell gives six definitions of the null class—(1) the class having no terms; (2) the class of terms which belong to every† class; (3) the class which does not exist†, in the sense in which a class is said to exist when it has at least one term; (4) the class which is contained in every class; (5) the class K which is such that the propositional function "x is a K" is false for all† values of x; (6) the class of x's satisfying any† propositional function φx which is false for all the values of x. All these definitions, says Russell, are easily shown to be equivalent†.[3]

B. (α) The statement that we can know nothing may stand as an expression of skepticism. If thus taken, it sets out both the essence of skepticism and its own status, in an unusual but accurate light. In the statement, the term "nothing," is the object of the verb, and is at least a term of reference. It is an object referred to, but not described. When the skeptic says that we can know nothing, he means not that we have no knowledge, but that our knowledge is enotative or innotative, not denotative nor connotative.

(β) Zero is interpreted in psychological terms when Whitehead says that $x = 0$ comes to mean the rejection of x from any process of reason or from any act of assertion. If a term represents the null

[1] *cf.* B. Russell, *Principles,* p. 184.

[2] C. I. Lewis, *Survey,* p. 213.

[3] B. Russell, *Principles,* p. 23.

element, it symbolizes the fact that the mind, after apprehending the component regions (if there be such) symbolized by the term, further apprehends that the region placed by the term before the mind for apprehension does not exist†.[4]

Most prominent among those who have offered psychological interpretations of the term "nothing" has been Bergson. He maintains that the idea arises when, in seeking something, we find not an absence of everything, but merely something else.[5]

(γ) A good deal can be said on the apparently hopeless question of the reality of nothing. Others besides Bergson give "nothing" at least the status of an object of reference, as when Russell, in the passage quoted at the head of this chapter, says that in some sense nothing is something, and Alexander says that the nothing we can think of is not nothing at all, but is a department of being.[6] For us, the essential point is that nothing is an enotative term, doubtless objective but not described in detail.

Sometimes the problem is stated and answered in terms of the difference between intension and extension. For Lewis and Langford, o, interpreted intensionally, is the logically inconceivable or self-contradictory. Interpreted extensionally, o is the extension of any† false proposition.[7] Lewis and Langford insist that the logic of the null class is one of those points concerning which it is important to remember that we are here dealing with the *extension* of terms. The term "unicorn" and the term "centaur" have very different connotations; they are not equivalent† in intension. But, the authors continue, the class of things denoted by "unicorn" is the same† as the class denoted by "centaur"; namely, the class of nothing† existent. Thus the extension of the two terms is identical†; there is only one class "nothing." The consequence is that if no *A*'s exist, then all the *A*'s there *are* will be anything you please.[8] This extravagant result will be considered later, especially in Chapter XIII, C (f) and (n); it may be avoided, as we shall see, by other views of the relationships of intension and extension, and by treating zeros or null classes as enotative or innotative.

For Russell, if we are to take in serial order the class of all† magnitudes of a kind, the null class is the first† term, and, if we

[4] A. N. Whitehead, *Universal Algebra,* pp. 38, 109.
[5] H. Bergson, *Creative Evolution,* transl. A. Mitchell, pp. 273 *ff*.
[6] S. Alexander, *Space, Time, and Deity,* Vol. 1, p. 199.
[7] C. I. Lewis and C. H. Langford, *Symbolic Logic,* pp. 67, 87.
[8] *ibid.,* pp. 28, 31.

take the classes as quantities, the null class is zero quantity. In this way, it would seem that "no pleasure" has the same relation to pleasure that the various magnitudes of pleasure have, though of course it has also the special relation of negation†.[9] Russell attempts to evade the difficulties of a zero magnitude by making the problem epistemological. The zero magnitude of a series of magnitudes is declared to be in a peculiar way conceptual.[9]

We have repeatedly pointed out that whatever be the relationships between mind and the world, the horizon problems are only postponed when they are transferred from the one to the other. Whether a zero magnitude is a concept or an existence, or both, it involves enotation and innotation, and its difficulties and paradoxes can be solved in those terms rather than in terms of subjectivism or its opposite.

(δ) The term "nothing" is even used to indicate the object of intuitionism or mysticism. According to R. Otto, mysticism contrasts the numinous with all† that is of nature† or this world, and with Being† itself, and ends by calling it that which is nothing. By this "nothing" is meant not only that of which nothing can be predicated, but that which is absolutely† and intrinsically other† than and opposite of everything† that is and can† be thought.[10]

C. (a) Attempts are made to elucidate the problem of nothing or zero by the aid of expressions of identity, equivalence, sameness, difference, etc. Russell criticizes Meinong's definition of zero because it gives a zero which is the same for all† kinds of magnitudes. He goes on, however, to say that where magnitudes are differences or distances, zero difference or distance has at first sight an obvious meaning, namely identity—the identity, that is, of one zero with another. Still, zero distance is no distance, and Russell says that this is only correlated with identity.[11] It is often held that two or more zeros or null classes are identical or equivalent. In the Boole-Schröder algebra, any† two elements having the value 0 are identical, so far as their properties in the calculus are concerned.[12] Whitehead says that insofar as rejected propositions are rejected, they are all equivalent. Elsewhere, Whitehead says that it may be assumed† that $A - A = B - B$. The fundamental property of the null ele-

[9] See B. Russell, *Principles*, pp. 186 *f.*, 195.
[10] R. Otto, *The Idea of the Holy*, transl. J. W. Harvey, 1924, pp. 29 *f.*
[11] B. Russell, *Principles*, pp. 185 *ff.*
[12] C. I. Lewis and C. H. Langford, *op. cit.*, pp. 87 *f.*

ment is that the addition of this element and any other element A of the manifold yields the same element A. Still, it would be wrong to think of 0 as necessarily† symbolizing a mere non-entity. There can† be no difference in non-entities, so $A - A$ would be absolutely† identical with $B - B$, whereas they are palpably different.[13] According to our view, the difficulties about identity, equality, sameness, difference, etc., of zeros, can be met only by enotative or innotative reference.

(b) Zero is defined in terms of negation by Russell, who says that we shall have to hold that any concept which defines a kind of magnitude defines also, by its negation, a particular magnitude of the kind, which is called the zero of that kind and is less than all† other members of the kind.[14] We should say that any magnitude thus "defined" by negation is indefinite, and would have to be left enotative or innotative, as indeed is indicated by Russell's concluding words.

Bradley says that with "nothing," the stress falls on the aspect of exclusion, and we have not a mere defect, but a denial of positive qualification. We have gone beyond a mere emptiness and absence. We have now the abstraction of an object which, rejecting all qualification, is forced, by consequence, to exclude itself.[15] But to add the last touch is, very decidedly, a qualification. "Nothing" is our word for the result of an exclusion which utterly eludes description.

(c) The exclusion used to interpret "nothing" may signify contradiction. According to Jevons, 0 indicates that which is contradictory or excluded from thought.[16] Somewhat more elaborate, and involving several horizon concepts, is the statement of Whitehead, who professes clearly to define the class, necessarily of an indefinite† number of propositions, which are to be equated† to the null element. If the universe† be reduced to the laws of thought, then all propositions equated to null are self-contradictory. With a more extended universe, all propositions equated to null are those which contradict the fundamental assumptions† of our reasoning.[17] We should admit that "nothing" and a contradictory have in common the fact of exclusion; the important point here, however, is not the

[13] A. N. Whitehead, *Universal Algebra,* pp. 24, 109.
[14] B. Russell, *Principles,* p. 187.
[15] F. H. Bradley, *Principles of Logic,* 1922, Vol. 2, pp. 670 *f.*
[16] C. I. Lewis, *Survey,* p. 73.
[17] A. N. Whitehead, *Universal Algebra,* p. 111.

exclusion, but the enotation. Since both are enotative, they may not be equivalent.

(d) Zero is defined in terms of a universal by Eaton. "All the numbers which when added to a number give that number" designate a totality† of one member—the member zero.[18] In order for this statement to hold, it must be understood that it does not apply to transfinite numbers, which should be left enotative. There is also some question as to the propriety of the universal "all," when countless numbers have not been even tentatively added to a given number to see what result they produce. The universal carries enotative reference, and zero may conceal countless innotations.

(g) For Whitehead, the null element of the algebra (in the calculus of propositions) corresponds to the rejection of the validity of the proposition. We saw that $x = 0$ comes to mean the rejection of x from any process of reason or from any act of assertion.[19]

(h) For the Boole-Schröder algebra, 0 and 1† are the two limits of class extension,[20] and the limiting values of variables generally.[21] This, for our discussion, amounts to saying that they must be left with enotative and innotative references.

(i) Continuity may be invoked to help interpret zero. Russell says that zero seems to be a radically distinct conception, according as the magnitudes concerned are discrete or continuous. It is hard to define without contradiction†. In any† class† of magnitudes which is continuous, in the sense of having a term between any two, and which also has no† limiting† magnitude, we can introduce 0 in the manner in which real numbers are obtained from rationals. Any collection of magnitudes defines a class of magnitudes which is less than all† of them. This class of magnitudes can† be made as small as we please, and can actually† be made to be† the null class.[22]

We shall see in Chapter XV that rational and real numbers also involve horizon problems. And when Russell says that such a class can be made to be the null class, here again the last phrase is the one which claims too much. If the class can be made as small as we please, that is "all" there is to it—that is to say, the series is left enotative.

[18] R. M. Eaton, *Symbolism and Truth,* p. 143, n. 1.
[19] A. N. Whitehead, *Universal Algebra,* p. 109.
[20] G. Boole, *Investigation of the Laws of Thought,* p. 47.
[21] C. I. Lewis, *Survey,* p. 147.
[22] B. Russell, *Principles,* pp. 184, 186.

(j) According to Russell, zero and infinity under certain circumstances may be introduced as limiting† cases of segments of rational numbers,[23] but we should say that this procedure, too, needs to face its horizon conditions.

(k) Zero may be interpreted as the beginning of a series. Whitehead says that since $A = 0 + A$, we may consider A or $+A$ as degenerate forms of $0 + A$. From this point of view, every element of the manifold is defined by reference to its relation to the null element. But 0 at the beginning of some expressions may be disregarded. In combination with any other element, the null element disappears. The symbolism may be rendered more convenient by writing $-A$ for $0 - A$.[24] This amounts to saying that 0 at the beginning of an expression is left enotative. The same characteristic appears in Russell's statements that when a collection consists of all† magnitudes of a kind, and classes are so related as to form a series, the null class is definitely the first term. The zero magnitude is less than all† other magnitudes of a kind; but zero has to be excluded before the axiom† that there is no least magnitude becomes true.[25] The zero thus excluded, like a limit†, is gratuitously referred to as the least magnitude. We shall see in our study of limits that such a reference at most is only denotative and incapable of precise determination without allowing for enotative and innotative reference.

(m) Bradley says that "nothing" is perhaps best described as the idea of a "that" which excludes and is excluded by any† and every† "what."[15] This is, for us, another way of stating the principle of enotation.

(n) Again, "nothing" is more abstract than what is impossible, and in a sense underlies it.[15] The statement of Bradley wrongly suggests that we can describe the content of the impossible. What he should say is that the term "nothing" denotes extreme or limiting cases of enotation.

(o) Once more, Bradley says that "nothing" differs† from the idea of mere being, in which emphasis falls on the positive side. With "nothing" we have not a mere defect, but a denial† of positive qualification.[15] But "nothing" is more abstract than what is *un*real, and in a sense underlies it.[15] For our view, all there is left is to say that "nothing" marks a denial and exclusion into the enotative,

[23] *ibid.*, p. 273. [24] A. N. Whitehead, *Universal Algebra*, p. 25.
[25] B. Russell, *Principles*, pp. 186 *f.*, 190 and n.

so complete that it can not be described by the term "being," nor even by the term "unreality."

(q) Bergson maintains that annihilation applies to parts of the world but not to the whole, and that the extension of such an operation to the totality of things becomes self-contradictory† and absurd.[26] In other words, one can not regard the whole world enotatively, without denotation or connotation.

(s) Time is invoked to help describe zero when, according to Poincaré's interpretation of Coutourat, zero is the number of objects that satisfy a condition never fulfilled.[27] But in a world of relativity and contingency, no one can say what those conditions are, nor make any other than enotative or innotative statements about them.

D. (1) There is a very common form of statement which is expressed in affirmative form, but is usually negative in meaning, where the term "nothing" is used as subject or substantive. Such expressions as "Nothing is further from the truth," "We have nothing to fear," etc., are supposed to be extreme in their exclusions, but in them the term "nothing" is curiously open to interpretations which give to it a content left enotative or innotative. Properly speaking, the term signifies not absence of everything, but absence of everything which we are at the moment selecting and considering. It signifies the open possibility† concerning something which we are neglecting.

A sentence containing the term "nothing" may exhibit a shift of sense comparable to the shift in an optical illusion. Thus the statement "Nothing is both *A* and -*A*,"[28] ordinarily understood to mean that contradictories are exhaustive or that the law of excluded middle holds, may be taken as a description of the paradoxical character of a region denoted by "nothing," but otherwise left enotative or innotative. In fact, an affirmative statement about "nothing" fails to carry weight just because its subject is enotative and offers "nothing" to which weight can be attached.

In mathematical logic, the transformation of an equation† not in the form in which one member is 0 into an equation in which one member is 0 may be interpreted in terms of our general thesis. It is

[26] H. Bergson, *op. cit.*, p. 280.
[27] H. Poincaré, *Science and Method*, transl. F. Maitland, p. 158.
[28] C. I. Lewis and C. H. Langford, *op. cit.*, p. 11.

effected by multiplying each side into the negative of the other and adding the products. Thus

$$A = B \equiv A \cdot \blacksquare B + \blacksquare A \cdot B = 0.$$ [29]

This amounts to taking as elements what each member has in common with that which it excludes, and assembling these elements as a description of 0. Multiplying each side into the negative of the other amounts to exclusion into the enotative, and after that enotatives may be added with impunity. But 0, as an enotative or innotative term, may, for instance, represent an innotative boundary between A and $\blacksquare B$. Moreover, equality is also enotative or innotative, and perhaps it is only as this boundary or some other innotative feature of A is disregarded that can A be equal to B.

The peculiar propositions, discussed in Chapter XIII, yield fantastic descriptions of the null class. Eaton maintains that what is true of any class is true of the null class, but that the converse does not hold.[30] We shall attempt later to show that such "descriptions" are unwarranted. A diagrammatic analogue of affirmative statements about zero appears when Lewis interprets certain fundamental propositions by intersecting closed circles and tries to indicate portions of the diagram which are null.[31]

(2) A curious statement, in negative form, is indicated by Russell, when he says that the proposition "Nothing is not nothing" is undoubtedly capable of an interpretation which makes it true†. It is necessary to consider whether the contradictions† which naturally arise can be avoided.[32] We should say that they can be avoided, provided interpretations be supplied for the subject and predicate with enough enotative or innotative reference so that they possess different contents. "Nothing," as ordinarily understood, is not "nothing," in the sense of no object of thought.

(3) Alternatives are provided in the Boole-Schröder algebra, where any relation can be completely specified by taking advantage of the fact that its expression can always be reduced either to the form of an expression equivalent† to zero or not† equivalent to zero.[33]

(4) Zero admits of antinomial interpretation. Lewis says that when $A = 0$, "All A is B" and "No A is B" are both true†.[34] It is more accurate to say that when zero is left with proper enotative or

[29] C. I. Lewis, *Survey*, p. 144.

[30] R. M. Eaton, *op. cit.*, p. 245.

[31] C. I. Lewis, *Survey*, pp. 181-3.

[32] B. Russell, *Principles*, p. 73.

[33] C. I. Lewis, *Survey*, p. 181.

[34] *ibid.*, p. 189.

innotative references, one description of its content fits the rest of the world as well as another, even when the descriptions are contrary propositions.

E. (I) When we come to study the permissible uses of the puzzling term "nothing," we find that it is best understood in contrast to the term "universe."† The universe is selected as distinct from a non-universe, which is neglected and left enotative and innotative. Everything is selected as against "non-everything." It is most convenient to regard the term "nothing" as a contraction for the latter. It is not to be interpreted as a denial of the existence of anything outside the universe, but is to be regarded non-committally. We can not say whether or not it has any content. It has open reference to objects, states, or results which in some way lie beyond the range of definite description.

In what might be a simple example of enotation, Russell invites us to suppose that the total† number of individuals in the universe were, say, 10. Then, proceeding to a negative description of the enotative region, which occurs to him because of the artificiality of the example chosen, he says that there would be no class of 11 individuals, and that the number 11 would be the null class. So would the number 12. He then proceeds to call the two equivalent†; we should, he says, have $11 = 12$, and the successor of 10 would be the same† as the successor of 11.[35] Somewhat similarly in the theory of aggregates, Brouwer says that the possibility† of ending† the process of counting can† be supplanted by the possibility that after a certain fixed count every† further counting, instead of generating even a row of signs, will over and over again generate nothing.[36] It is hard to see from these arbitrary and restricted examples, but what we ought to strive for in our knowledge is a denotative-connotative world "*with nothing‡ left out.*"

(II) If any one wishes, he may write a sentence, or a book, which says nothing about nothing. Reflection upon the first of the two terms will make clear its nature. Or, he may contemplate the equation $0^0 = 0$, to see what is its degree of indeterminateness.

(III) In this and the preceding chapters, we have seen that several attempted logical or arithmetical operations with 0, in one way or another, "amount to nothing." These operations can easily be translated into propositions with "nothing" as subject. When

[35] B. Russell, *Introduction,* p. 24.
[36] L. E. J. Brouwer, in *Math. Annalen,* 93, 1925, p. 244, n. 2.

this is done, there seems to be, as indeed we should expect, little or no reason for supposing that this subject would select one predicate rather than another. If there were evidence, this would amount to descriptions of the subject. Such evidence as there is amounts to description only in that it states the principles of enotation, etc. "Nothing" is evidently *the* subject-term with the least selectivity. If any one cares to engage in the mistaken attempt, practically any descriptive attribute can be palmed off on this subject, and the null class can be made to appear to be, as in current symbolic logic, contained in every class.

(IV) The use of zero and the null class requires a number of assumptions and postulates. Lewis says that the idea of zero in any branch of mathematics seems a little more of an arbitrary convention than the other numbers, but once zero is defined, its queer properties are inevit*able*†.[37] According to our view, zero should not be defined, but designated approximately and then left enotative or innotative, where any untoward result, otherwise inevitable, can‡ be evaded.

The Boole-Schröder algebra postulates that 0 is unique. Again, in the derivation of the logic of classes from the calculus of propositional functions, the postulates of the existence of the class 0 must be supposed to be satisfied by the fact that we have exhibited in definitions the logical functions which determine 0.[38]

In the theory of aggregates, the null aggregate contains no element. It is no aggregate at all, but on grounds of special conventions, amounting to a broadening of the definition, it functions as an aggregate, and as a finite aggregate.[39] All of these cases, we should say, allow the opposite assumptions concerning 0, when once the artificially fixed conditions are removed.

(V) It is familiar that 0 serves as a boundary between the series of positive and the series of negative cardinal numbers, even when both series are left enotative at the further ends. In connection with contradictories† and excluded middle, it appeared that boundary zones were possible†. When in spatial applications of the Boole-Schröder algebra, A and B do not overlap, it is said that $A \cdot B$ is the null region;[40] here $A \cdot B$, equal to zero, may be regarded as a boundary region or even a boundary line.

[37] C. I. Lewis, *Survey*, p. 327. [38] *ibid.*, pp. 119, 267.
[39] A. Fraenkel, *Einleitung in die Mengenlehre*, 1928, p. 21.
[40] C. I. Lewis, *Survey*, p. 175.

(VI) The notion of progression in a series is often employed to clarify the term "zero" or "null," as when 0 is defined as in a regular sequence a term less than any assigned number.[41] Russell says that where a kind of magnitude is continuous† and has no† limiting† magnitude, although we have apparently a gradual and unlimited† approach to zero, yet magnitudes of this kind are such as essentially have no minimum. Hence we can not without express contradiction† take zero as their minimum.[42] (In other words, to take zero as their minimum would terminate this essentially enotative series of magnitudes.) Russell offers a way of avoiding the contradiction by saying that there is always† a magnitude less than any† other, but not zero, unless that other be zero. This, he says, avoids the contradiction but gives rather a mark of zero than its true meaning. What makes zero obviously incapable of any further diminution?[42] His reasoning here amounts to saying that if, contrary to hypothesis, in the enotative series, zero is the least of the magnitudes, then zero must be less than zero. But, apart from the contradiction, this is again introducing a description of a region once left enotative. Even the assigned "mark" of zero is enough to burden it with a description. And zero is not obviously incapable of any further diminution, as long as its content is left enotative or innotative. We may regard zero as the limit of such a series, but if so we should remember that both "zero" and "the limit of such a series" are not to be connotatively described. This point with regard to limits will be discussed in Chapter XIV.

(IX) Zero lends itself readily to interpretation as an innotative term. The Boole-Schröder algebra of logic postulates† a unique element, 0, in a class K such that $A \cdot 0 = 0$ for every element A.[43] Eaton expresses it by saying that any† class† covers as great a logical area as a class of no† members. If it is not† a class of no members, it covers a wider area.[44]

In the interpretations of the Boole-Schröder algebra for regions in a plane, zero will represent the null region contained in every region.[45] Recognition of the principle of innotation here would clarify Lewis's difficulty with spatial representation of class relationships, referred to in D (I), above. He says that in a given diagram where all† possible† classes or regions in the universe† are

[41] H. B. Fine, *The Number System of Algebra*, 1902, p. 29.

[42] B. Russell, *Principles*, p. 185.

[43] C. I. Lewis, *Survey*, p. 119.

[44] R. M. Eaton, *op. cit.*, p. 247.

[45] C. I. Lewis, *Survey*, p. 120.

initially represented we can† not† presume† that a given subdivision is null or not null. All we can say when we have struck out the null regions is that, so far as the premises represented are concerned, any† region not struck out may† be not-null.[46] In other words, the question of more null regions is left innotative.

Innotative also is Zermelo's axiom†, that an aggregate contains the partial aggregate o, then, distinct from this, the partial aggregate $\{ 0 \}$, then $\{ \{ 0 \} \}$, whose sole element is the aggregate $\{ 0 \}$, and so on.[47]

Again, the values of o may be regarded as fixed within certain ranges or limits†, and otherwise left innotative. Bolzano held that in the two equations $A - A = 0$ and $A + 0 = A$, we set up an idea, which on the one hand is as broad as usage and scientific interest demand and on the other hand is restricted enough to prevent misuse.[48] Here zero is used denotatively with the suggestion of approximation, the degree of which is left innotative.

Summing up, we may say that "nothing" is the limiting case of exclusion, where description utterly fails, and where even mere indication is at a minimum. It is the denotative term of minimum connotation, used for reference to what is properly enotative and innotative.

[46] C. I. Lewis, *Survey*, p. 183. [47] A. Fraenkel, *op. cit.*, p. 308.
[48] B. Bolzano, *Paradoxien des Unendlichen*, ed. F. Prihonsky, 1920, p. 56.

CHAPTER XIII

Validity, Truth, and Probability

"Logic is certainly formal, or at any rate non-material, in the sense that it can not guarantee the objective or material truth of any particular conclusions."
—J. N. Keynes, *Studies and Exercises in Formal Logic,* 1906, p. 2.

"The first investigator [Francis Bacon] could not have anticipated that induction, in spite of its apparent certainty, would prove so elusive to analysis."
—J. M. Keynes, *A Treatise on Probability,* 1921, p. 266.

A. No problems are more subtle than those of logical validity, truth, and probability. To discuss them is to put forward, as claiming validity, truth, etc., for themselves, propositions about the validity, truth, etc., of other propositions. To challenge the propositions thus put forward is to put forward propositions with fresh and mounting complications. Whatever else the process does, it opens all the problems of the nature of propositions and the nature of mind and world. This is one reason for the present disquieted interest in logic and the appearance of so many books seeking to dispose of the many-sided questions which are inevitably raised.

Validity, truth, and probability are involved in all the problems we have been considering, and may be treated by any or all of the methods we have indicated. Thus (A) to define the problems of validity, truth or probability, is to discuss (B) some psychological processes, and sooner or later to bring in (C) other problems involving horizons, with inevitable (D) affirmative or negative statements. Against these, we maintain that the proper procedure is to leave questions of validity, truth, and probability enotative and innotative. It is as inadequate critically to condemn our reasoning as blindly to follow it. *Ultimately we do not know whether our reasoning is valid, or true, or even probable, or not. We have to proceed within horizons.* This statement does not mean that we have no ground on which to stand. The universe attended to that before we appeared. The statement means that the ground on which we stand, or proceed, is subject to continual re-survey. Nor does this fact rob our knowledge of its basic assurances, for even though the content of our knowledge is continually changing, it does not by

any means all change at the same rate, and many important and many humble features of it are relatively permanent.

It is important to remember that, although the distinction can not be made with complete precision, we are here not primarily concerned with the validity, truth, or probability of our horizon principles; the discussion of those is the purpose of the whole book. In most of this chapter we are concerned with the application of the horizon principles to problems of validity, truth, and probability as they come up in other contemporary investigations. Some questions about the horizon principles themselves are discussed at the end of the chapter.

To make a long story short, we introduce with apparent arbitrariness an important distinction. In the epitomization hypothesis we have maintained[1] that the data of logic constitute an objectively subsistent realm, and that the data of the physical sciences, for example, constitute an objectively existent realm. One result of this is that there are two aspects of validity, one having its roots in the logical realm and the other in the relationships of that realm and the existent cosmos. We call the two aspects formal and material validity, respectively. In practice they are intertwined. In pure logic, propositions, whether universal or particular, have nothing to do with existence, but only with subsistence. The *structure* of propositions and even their processes, especially in inference, should be studied quite irrespective of whether the propositions are true or not, or whether they are even entertained or not.

The question of formal validity concerns the structure of propositions, and for the epitomization hypothesis becomes a question of propositions as monads of the logical realm, exhibiting the monadic characteristics. Much of Chapter X was concerned with the formal validity of generalizations, universals, etc.

Material validity is a question not so much of the form of our propositions as of their content and applicability in our adjustments to the existent universe. The question is whether, under actual concrete conditions, their content is socially dependable or not. Some of the best known errors arise when structures and processes which belong within some particular realm, existent or subsistent, are treated as if they were actually fully adjusted to the structures and processes of other realms. Material validity may be

[1] G. P. Conger, *Epitomizations*, pp. 585 *f*. On the distinction between validity and truth, *cf*. J. N. Keynes, *Studies and Exercises in Formal Logic*, 1887, p. 2.

called truth, and the problem involved may be called that of truth-value.[2] Various approximations to truth give us what we call probabilities.

B. (α) Skepticism in various forms and degrees ensues when the probability† of true knowledge is thought to be low. But on the other hand, induction seems to presuppose and also help to develop a certain confidence in our experience. From this point of view, skepticism appears to be "an exercise, not a life."[3]

(β) Skepticism and various forms of relativism are criticized, if not answered, by some of the psychological theories of truth and probability. Keynes says that the judgments of probability, upon which we depend for almost all our beliefs in matters of experience, undoubtedly depend on a strong psychological propensity in us to consider objects in a particular light.[4]

Other writers attempt in more detailed fashion to ground certainty in psychological considerations. For example, according to Washburn, correct reasoning involves the full development of movement systems.[5]

Mention should be made here of the pragmatic view, that truth virtually consists in the operation of some of the non-cognitive functions of mind. But pragmatism, like other theories of truth which are based upon psychology, neglects more fundamental facts about the universe, and makes its suggested solutions superficial.

(γ) Problems of truth and falsity of propositions are traditionally involved with those of idealism and realism, in the theories of coherence and correspondence.[6]

Problems of correspondence are suggested when Keynes says that we can not deny that what appears true is sometimes false, but unless we can assume† some substantial relation of probability between the appearance and the reality of truth, the possibility† of even probable knowledge is at an end.[7]

Coherence and correspondence theories are combined when Ogden and Richards maintain that true reference is to a set of referents as they hang together.[8]

[2] Material validity, or truth, may be abstract, as in the case of $2 + 2 = 4$, or concrete, as when 2 pages + 2 pages = 4 pages.

[3] G. Santayana, *Scepticism and Animal Faith*, 1923, p. 69.

[4] J. M. Keynes, *Treatise on Probability*, p. 52.

[5] M. F. Washburn, *Movement and Mental Imagery*, p. 182.

[6] See G. P. Conger, *Course in Philosophy*, pp. 278, 374, 383.

[7] J. M. Keynes, *op. cit.*, p. 239.

[8] C. K. Ogden and I. A. Richards, *Meaning of Meaning*, p. 179.

Montague, studying methods of knowing, says that there is a large class of judgments which lend themselves with almost equal ease to evaluation by the criteria of authoritarianism, mysticism, rationalism, empiricism, and pragmatism.[9] This recalls his suggestion of a "federation of methods," which is surely a desirable working arrangement, even though it falls short of the ideal of an organic union of all the methods.

In accordance with the epitomization hypothesis, it appears that any complete act of thought or any sustained discussion combines all of the methods in an organic union. The union may be adumbrated and the methods combined in the statement that any idea which is socially dependable for our adjustments to the universe must in the long run belong within the corpus of knowledge, must be substantiated by nature around us, and must be fruitful in producing results which, as thought out, worked out, and immediately felt, are progressively more and more socially satisfying. The process is by no means chaotic and arbitrary; it goes on and takes place in the structured cosmos, with its monads of various realms and levels. And the main lines of these structures and processes, as exhibited and explored in metaphysics, assuredly have a bearing upon the problem. The epitomization hypothesis itself, whatever is thought of its employment of analogy, must ultimately be judged by such criteria as we have indicated.

In discussions of the problem of truth-value we often find appeals to the alleged differences between the intensional logic, or logic of concepts, and the extensional logic, or logic of classes. This is not necessarily a difference between the subjective and the objective, but, as we saw in Chapter VI, it can be regarded as such. According to the current view, when proceeding intensionally, one deals with a predicate or concept of a class; when proceeding extensionally, one deals with all those things denoted by a class concept.[10] We shall see, at C (f), C (n), and D (1) below, how much importance is ascribed to this distinction in some problems of truth-value, and how the usual treatment turns out to be inadequate.

For us, material validity is to be sought and found not by a separation of concepts and objects with separate rules for each, but

[9] W. P. Montague, in Columbia University, Department of Philosophy, *Studies in the History of Ideas,* Vol. 1, 1918, p. 223.

[10] C. I. Lewis, *Survey,* pp. 13, 185.

rather in some kind of active and reciprocal relationship between concept and object. This relationship is unique and complex. The theories of coherence, correspondence, and consequences all express parts or aspects of it. The real distinction between intension and extension, according to our view, does not involve material validity, but merely formal validity, *i.e.*, membership in certain subsistent structures in pure logic. The distinction is essentially the difference between the interactions of terms-in-propositions, on the one hand, and, on the other hand, their aggregations or integrations. In the working out of this theory in accordance with the epitomization hypothesis,[11] attribution in intensional logic can be correlated with distribution in extensional logic, without confusion due to admixtures of psychology and epistemology. Again, the difference between propositions with their structures and processes and classes with their structures and processes turns out to be a difference of one monadic level,[12] and the similarities of essential structures and processes at the two levels yields just such repetitions, although for other reasons not precisely the same theorems, as were hit upon when it was found that the Boole-Schröder algebra applies not merely to classes but also to propositions. Lewis sees that the relations of intension and extension are generally analogous.[13] The epitomization hypothesis supplies fresh details for such analogies.

The theory of probability usually involves somewhere questions of the relationships of mind and world. Probability may be understood, first, as the degree of rational belief accorded a proposition[14] —that is, the degree of belief that the proposition meets, whatever tests of truth are regarded as adequate in the particular adjustment between mind and world that may be involved. This is the psychological aspect of probability, difficult to describe or to measure. Stebbing, following Keynes, says that probability is a relation holding between propositions, and that assertions of probability are logical principles which are independent of matters of fact.[15] But, while a relationship between propositions need not involve belief nor any other psychological process, the whole point of a *judgment* of probability is its application to the facts of the real world. The judgment is *not* independent of matters of fact.

[11] See G. P. Conger, *Epitomizations,* pp. 391 *ff.*

[12] *ibid.*, pp. 425-33.

[13] C. I. Lewis, *Survey,* p. 327.

[14] J. M. Keynes, *op. cit.*, p. 6.

[15] L. S. Stebbing, *Modern Introduction to Logic,* p. 409.

More objectively, probability means the statistical frequency of an event or events which give rise to or which substantiate a belief. Here may be included beliefs and predictions concerning events such as the flipping of a coin. Naturally a high probability in this sense promotes a high probability in the other. Measurement here tends to be more accurate, and, while sometimes only indicated by greater or less, is often reducible to numerical expressions which are at least approximately precise.[16] According to Keynes, statistical induction either asserts the probability of an instance selected at random from a series of propositions, or else it assigns the probability of the assertion that the truth-frequency of a series of propositions is in the neighborhood of a given value.[17] Establishment of such measurements may be called the calculus of probabilities. We shall see later that the statements of probability offer abundant opportunity for enotative or innotative references.

The measurement of probability as the statistical frequency of events may have reference merely to our knowledge and mean the statistical frequency of events known to us. But it may go along with the doctrine that probability is a feature of the objective world, without anything essential to do with our knowing it or not knowing it.[18]

By way of combination of the various senses, it may be said that propositions, rather than events, should be taken as the subject matter of probability, and that the probability of a proposition always depends upon referring it to some class whose truth-frequency is known,[19] but known by reason of the statistical frequency of certain objective events, as compared with certain other events. Keynes says that probability is a logical relation between propositions, but that it is an objective relation. Elsewhere he says that we may believe that our judgments can penetrate into the real world, even though their credentials are subjective.[20]

(δ) Perennial in the history of thought is the tendency to seek and hold as valid a knowledge which comes by the direct light of intuition.[21] Discussion of the value of intuition is best deferred until

[16] *cf.* J. M. Keynes, *op. cit.*, pp. 34, 95.

[17] *ibid.*, p. 406.

[18] *cf. ibid.*, pp. 32, 289.

[19] *ibid.*, p. 101.

[20] *ibid.*, pp. 4 *f.*, 52.

[21] See, *e.g.*, W. E. Johnson, *Logic*, Part 2, p. 192; A. N. Whitehead, *Aims of Education*, p. 230; J. M. Keynes, *op. cit.*, pp. 52 *f.*, 70, 263 *f.*; W. E. Hocking, *Types of Philosophy*, 1929, p. 208; C. I. Lewis and C. H. Langford, *Symbolic Logic*, p. 252.

Chapter XXI. The intuitionism of Brouwer and his followers is treated in Chapter XVI, E (vi).

C. (a) Eaton distinguishes conceptual validity from truth by saying that a system is conceptually valid, whether or not it is true, if it does not violate the principles of identity and contradiction†.[22]

Truth-value is interpreted by the aid of equivalence, when any two true propositions are declared to be logically equivalent, and when similar statements are made concerning propositions false, impossible†, or necessary†.[23] Even here there is innotation, for there may be genuinely distinct propositions which are logically equivalent, and there are pairs of equivalent propositions one of which contains a concept not contained in the other.[24]

(b) In the Boole-Schröder algebra of logic, the truth-value of propositions is symbolized in terms of negatives; $-p$ signifies "p is false." There is good reason for this in the structures and processes of pure logic. When a negative proposition denies a predicate, the subject-term rejects the predicate into the enotative *not-A*. That a given proposition is false means that if the subject continues to take the given predicate or attribute, that attribute will be excluded from the particular structure or universe of propositions which are regarded as true, and will be left in the enotative region. Since the enotative region is indeterminate, one of these facts about rejection into it will not be distinguished from the other, except by the difference in the connotative aspects of the process. This difference of aspect is that which we have just indicated between the negative of denial and the false proposition. They both involve a region referred to negatively and treated enotatively. Broad says that "not-true" is a wider term than "false."[25] But this puts a negative of exclusion where we should have a negative of suspension, "non-true." "Non-true" is indefinite, whereas "false" involves a definite rejection and the development of some "non-true" into "not-true" propositions.

In Chapter VIII the theorem A implies B $=$ $-B$ implies $-A$, was interpreted spatially or for classes, as an affirmative statement about negatives. In this connection it is worthy of note that in the system of strict implication a postulate of a form corresponding to that of this theorem has to be emended so that it expresses a relationship of implication rather than equivalence.[26] In fact, whatever

[22] R. M. Eaton, *op. cit.*, p. 218.
[23] *cf.* C. I. Lewis and C. H. Langford, *op. cit.*, pp. 103, 176.
[24] *ibid.*, p. 285, n. 2.
[25] C. D. Broad, *Scientific Thought*, p. 82.
[26] C. I. Lewis, *Survey*, pp. 124, 295, and *Jour. Phil.*, 17, 1920, pp. 300 *f.*

is said to be the meaning of implication, and whether it is interpreted in terms of being contained in, or of conjunctive truth-value, or of deducibility, it is safer to assert such relationships with the enotative than to assert the relationship of equivalence.

(c) For Lewis and Langford, the logically inconceivable is the self-contradictory, and there is no decisive test of invalidity, except in the special case where the conclusion drawn contradicts something which genuinely follows from the premises.[27] Some possible limitations of the law of contradiction are discussed at E (v) in Chapter X, and E (vi) in Chapter XVI.

(d) A peculiar problem arises in propositions which concern the validity of all propositions. In the face of some of the difficulties, Russell in 1903 suggested denying that there are any true propositions concerning all objects or all propositions.[28] Here we should first distinguish between formal and material validity, or between validity and truth. To say that all‡ propositions are valid is to state a fact about their structures and processes, their formal validity. It is little more than a truism or a tautology, as if we said "All͂ propositions are propositions." If *p* is a proposition, of course it is valid. Something is left enotative here, but it is the possible (or non-possible) propositions which are non-valid, and which, at least in our logical universe‡, would be outlandish. If this statement seems utterly fantastic, that is just the way it ought to be, in order to be true.

All propositions are formally valid, but they do not necessarily belong to the much narrower range of propositions socially dependable, or true. Taking the universal as applying to the range of all propositions, with the non-propositions left enotative, we may say that all propositions are true or non-true. Russell says "true or false,"[28] but this again makes the negative† one of denial and exclusion rather than mere suspension.

Sometimes, as in the Epimenides problem, the falsity of propositions is dealt with in universal propositions, which result in paradoxes. According to Russell, no single proposition is self-contradictory; the nearest approach would be "No proposition is true," since this implies its own truth.[29] Propositions like these are best discussed by calling attention to the psychological and social situation involved

[27] C. I. Lewis and C. H. Langford, *op. cit.*, pp. 58, 67.
[28] B. Russell, *Principles*, p. 368. [29] *ibid.*, p. 454.

in any discussion. Not as a presupposition, but as a condition of having any presuppositions, we may say that the conditions under which any discussion takes place impose certain limitations upon the content of the discussion, especially when the content is about the truth of propositions. In any discussion, at least some propositions are explicitly or implicitly proposed by some participants for acceptance by others. This limits the too sweeping universal negatives which are at first so puzzling.[30] Another way of limiting their range is by the recognition of sublevels or types, discussed in E (11). Thus in the problem raised by the proposition "All rules have exceptions," there is, for Russell, a difference between rules about things and rules about rules about things.[31]

(e) To the real group which contains all the propositions which are known to be true, Keynes assigns the old logical term "universe of reference."[32] The theories of coherence tend to ground truth and validity in the properties of a self-consistent universe of mutually related parts.[33]

Russell treats the problem of generalization in terms of the universe. He declares that if there were not† universe, all† general† propositions would be true; for the contradictory† of a general proposition is a proposition asserting existence†, and would therefore be false if no universe existed.[34] This, for us, reduces to the fact that anything may be asserted of a non-universe, which must be left with enotative reference. The contradictory of a general proposition, however, does not necessarily involve existence, nor does any given universe necessarily involve it.

(f) In his discussions of the validity of certain propositions, Lewis makes conspicuous use of the notion of zero, or an empty class. He says that since it is a law of the algebra under consideration that for every x, 0 implies x, we must accept in this connection the convention† that the null class is contained in every class. All the immortal men are members of any class, since there are no such. Lewis says that this, however, is hard to understand in other instances; we should hardly say, "All sea serpents have red wings, because there are no sea serpents." Yet, he says, we understand

[30] See D. F. Swenson, in *Jour. Phil.*, 29, 1932, p. 525, n. 5.

[31] C. I. Lewis and C. H. Langford, *op. cit.*, p. 450.

[32] J. M. Keynes, *op. cit.*, p. 129.

[33] See B. Bosanquet, *The Principle of Individuality and Value*, 1912, pp. xviii, xxv, and Chap. VI.

[34] B. Russell, *Introduction*, p. 204.

"Whoever enters here must pass over my dead body." His explanation is that we readily accept and understand the inclusion of an empty class in some other, when the *concept* of the one involves the concept of the other—when the relation is one of intension. But in this sense an empty *class* is not contained in any and every class, but in some only. In order to understand this law, we must understand that the algebra treats of relations in extension only; it is a necessary law of the relation of classes in extension, that for every y, y implies 1. One can not accept this last without accepting by implication the principle that the null class is contained in every class.[85]

We should say, first, that here the appeal to intension resolves itself simply into psychology, involving a confusion of the conceived with the conceiv*able,* and, second, that the inclusion of zero in every class, which, as we shall see, is not necessary for other theories of implication, is, even when thought to be inevitable, at most an innotative procedure, where any explicit description is equivocal. It is the partial and fragmentary description of such an innotative region which gives rise to the difficulties. Thus, if all the immortal men are members of any class, they are, *when one comes or cares to think of it,* members of the class of animals with red wings, and so they are like the sea serpents, and if one's imagination is strong enough, all of them are members of the class of those who enter here, or who pass over my dead body, and so on.

This relieves a difficulty of the Boole-Schröder algebra. Lewis says that if "Some A is B" be so interpreted that it is false when class A has no members, then it will not follow from "All A is B" which is true when $A = 0$. But if "Some A is B" be true when $A = 0$, it does not accord with ordinary usage, and it does not contradict "No A is B," which is true when $A = 0$. The solution of the difficulty, for Lewis, lies in observing that "Some A is B," as a relation of extension, requires that there be some A—that at least one member of the class A exist.[86] But this solution is encumbered with epistemology. If we interpret 0 with enotative reference, neither "All A is B" nor "No A is B" is necessarily true when $A = 0$. To set A equivalent† to zero is to banish the problem; but a banished problem can† hardly be "solved," except, for instance, by convention†.

Lewis maintains that for the Boole-Schröder algebra the traditional relations of the square of opposition hold whenever the sub-

[85] C. I. Lewis, *Survey,* pp. 185-7.

[86] *ibid.,* p. 188.

ject of the four propositions denotes a class which has members. When the subject denotes a null-class, the two universal propositions are both true, and the two particular propositions are both false. Immediate inference runs against the difficulty of the class without members. For example, this sequence would be accepted by some logicians.

No *A* is *B* gives No *B* is *A*.
No *B* is *A* gives All *B* is *not-A*.
All *B* is *not-A* gives Some *B* is *not-A*.
Some *B* is *not-A* gives Some *not-A* is *B*.
Hence No *A* is *B* gives Some *not-A* is *B*.

Lewis holds that this is invalid, because *B* is an empty class and because it was presumed that "All *B* is *not-A*" gives "Some *B* is *not-A*." The inference of any particular proposition from the corresponding universal requires the assumption that either the class denoted by the subject of the particular or its predicate ("*not-B*," regarded as the predicate of "Some *A* is not *B*") is a class which has members. Lewis says that those who consider the algebraic treatment of null-classes to be arbitrary will do well to consider this logical situation with some care.[37] For us, the algebraic treatment of null-classes, like any enotative terms, is arbitrary, and arbitrary even in Lewis's own procedure at this point. In the above propositions, the negatives should be interpreted merely as stating, or upon conversion, obversion, etc., merely restating, the relationship of exclusion.

When we say "No *A* is *B*," we mean that *A* excludes *B*. Conversion to "No *B* is *A*" signifies that the exclusion is mutual and that the duality of *A* and *B* can be rendered explicit within the denotative and even the connotative field. The difficulties which especially concern us begin with the next step, the obversion of "No *B* is *A*" to yield "All *B* is *not-A*," with the negative in the predicate. If we said "All *B* is not *A*," with the negative in the copula, we should be merely restating the fact that *B* rejects and excludes *A*; but the traditional obverse, "All *B* is *not-A*," means that *B* gets affirmative, positive characteristics from that which is excluded from *A*, and which, because excluded, should be left enotative. When we go on to "Some *B* is *not-A*," we make a statement with the same enotative reference. So the following conversion, to "Some *not-A* is *B*," is

[37] C. I. Lewis, *Survey*, p. 190; *cf.* C. I. Lewis and C. H. Langford, *op. cit.*, pp. 275 *ff*.

subject to qualifications. The qualifications are connected with questions of enotation, more fundamentally than with questions of existence. The formal difficulty is not that any class has no members, but that connotation can not be safely derived from enotation.

The difference is not between intension and extension, in Lewis's sense, nor even in the sense in which we use these terms. Lewis interprets exclusions in such a way as to make them suggest positive content, and then reinforces the suggested positive content with assumptions of existence†. In general, the affirmative, and still more the particular form of proposition strongly suggest existence, but any subject or any predicate in a valid proposition may be subsistent, or may even be a kind of logical mirage due to refraction between realms, some of which are subsistent and some existent. Whatever we may have found out empirically about classes having members or no members will help us with problems of truth, but it will not throw much light upon more fundamental problems of validity. Lewis once says that the question whether, of an x which does not exist, φx is always true or always false or sometimes true and sometimes false or never either true or false, is in part one of convention†.[38]

(i) According to Keynes, if a probability $\varphi(x)$ is not necessarily† numerical, it is not a generally justifiable assumption† to take its continuity for granted. In a game where we have a number of compartments arranged in a circle and colored alternately black and white, the assumption of continuity amounts to the assumption that for every† white alternative there is a black alternative whose probability is very nearly equal† to that of the white.[39] But this, even if assumed, might not be continuity, nor, if it were, would it be free from innotative reference.

(j) Ramsey says that no writer previous to Wittgenstein has considered truth-functions as capable of more than a finite number of arguments. Of course, if the arguments are infinite, they can† not† all† be enumerated and written down separately, but there is no need to enumerate them, if they can be determined in any other way, as by using propositional functions. Just because we can not consider a thing individually does not mean that we can have no concern with it at all. Thus, although an infinite indefinable class, not defined by propositional functions, can not be mentioned by

[38] C. I. Lewis, *Survey*, p. 330.

[39] J. M. Keynes, *op. cit.*, p. 49.

itself, it is, nevertheless, involved in any statement beginning "all classes," or "there is a class such that."[40]

When we come to consider infinite classes, in Chapter XVI, we shall find difficulty in the use of the terms "all" and "any" in connection with them. Such references are at most denotative, but not connotative.

The work of Keynes on induction is noteworthy in this connection, as an attempt to avoid the pitfalls involved in infinites. He calls the "inductive hypothesis" the absolute† assertion of the finiteness of the system under consideration. His attempt to justify the initial probability which analogy seems to supply depends first upon the assumption† that the objects in the field over which our generalizations extend do not have an infinite number of independent qualities. There is thus some limitation† of independent variety. If the aggregate variety of the system is finite, the possible† plurality of causes must also be finite. And the limitation of independent variety seems necessarily to carry with it some degree of atomic uniformity.[41] The two principles amount to the assumption that nature is fundamentally finite.[42]

We shall see in Chapter XVI that in dealing with infinite classes the intuitionists of the Brouwer school reject the principle of perfect induction, and demand pure construction, which by means of finite processes can lead only to judgments of finite character.[43]

(1) According to Keynes, the so-called magnitudes or degrees of knowledge or probability, in virtue of which one is greater and another less, really arise out of an order in which it is possible† to place them. Certainty, impossibility†, and a probability which has an intermediate value constitute an ordered series.[44]

(n) Questions of possibility, impossibility, etc., are involved when Lewis's strict implication sets up, along with p (proposition p is true) and $-p$ (p is false), another truth-value, $\sim p$ (p is impossible).[45] In order to discuss this, we must bring together at this point a number of considerations which might properly be distributed elsewhere. They involve particularly a discussion of the contrast between material implication and strict implication.

[40] F. P. Ramsey, *Foundations of Mathematics*, pp. 7f., 22.
[41] J. M. Keynes, *op. cit.*, pp. 238, 256, 258-61.
[42] L. S. Stebbing, *op. cit.*, p. 413.
[43] A. Fraenkel, *Einleitung in die Mengenlehre*, p. 243.
[44] J. M. Keynes, *op. cit.*, p. 35.
[45] C. I. Lewis, *Survey*, p. 292.

Material implication usually has been said to mean that "it is not the case that *p* is true and *q* false."[46] It is taken in extension, as a relationship of classes of cases in which propositions *p* and *q* are true or false. But Lewis and Langford, in effect, point out that if this description of material implication be made into a definition of it, then implication, the relationship when it is *not* the case that *p* is true and *q* false, occurs when *p* is true and *q* is true, when *p* is false and *q* is true, and when *p* is false and *q* is false.[47] These relationships, considered with respect to *p*, show that when *p* is false it implies both *q* is true and *q* is false, and when *q* is true it is implied both by *p* is true and *p* is false. Hence we have the peculiar propositions of material implication—a false proposition (*p*) implies any proposition (*q*, true or false), and a true proposition (*q*) is implied by any proposition (*p*, true or false). These peculiar propositions are discussed at D (1).

Moreover, material implication defined as above and holding, for instance, when *p* is false and *q* is true, must hold sometimes when *q* is not deducible from *p*—as when *p* = "Roses are green" and q = "Sugar is sweet." Hence in material implication there can be no propositions which are both consistent and independent.[48]

Lewis and Langford attempt to set more precise safeguards for the truth of inference. According to their restatement, there are certain facts of our meanings in the use of language which are very fundamental because they exhibit the order† of our chosen ways of ordering. Thus there are logical facts, and there are relations between them, whether the relations are observed or not. The structure of these facts and relations can be examined without regard to their meaning, truth, or usefulness, but ordinarily we select some of the relations, in accordance with our purposes, and find them to be logical principles, explicated in analytic propositions.[49]

The chief point is that our deductive inferences should not be on a basis of truth-value, which we may happen to know empirically or materially, but should be on a basis of the logical relations discern*ible* in analytic propositions, which are tautologies, or necessary† truths, *i.e.*, principles which exhaust all† the possibilities. Truth-value itself must be exhibited as tautology, or necessity—in other words, as strict implication, taken intensionally and defined as

[46] C. I. Lewis, *Survey*, p. 243.
[47] C. I. Lewis and C. H. Langford, *op. cit.*, pp. 137, 199.
[48] *ibid.*, pp. 134, 154, 178, 247. [49] *ibid.*, pp. 33, 71, 122, 127, 211 *f.*, 251, 256 *f.*

equivalent to the statement, "if *p* implies *q*, it is impossible that *p* be true and *q* false."[50] Thus the attempt is made by exhausting all the possibilities, to formulate a theory of implication, in which the conjunction or disjunction of possible truth-values will have the force of a statement of deducibility.

For Lewis, formal implication is simply a class or aggregate of material implications. It makes, however, a distinct difference whether the cases comprehended are all possible cases, all conceiv*able* individuals, or only all actual cases, all individuals which exist† in the universe† of discourse.[51]

From our point of view, there are several defects in the system of strict implication. They involve disregard of enotative or innotative references, especially in the attempted descriptions of possibility, impossibility, and necessity.

Lewis and Langford mention the sense in which "possibility" means "consistent with the data or consistent with everything† known,"[52] *i.e.*, known up to date, with allowance for the incompleteness† of our knowledge. But the meaning which they assign is logical conceiv*ability* or absence of self-contradiction†. This is said to be absolute† (in the sense of logically conceivable or self-consistent) and to concern only the relation which the fact or proposition has to its negative†.[53]

For us, each of the words marked with the dagger involves enotative reference. When possibility is described in terms of the conceivable, we must distinguish between what is (actually) conceived and what is (possibly) conceiv*able*. Anything denoted at all is obviously conceived, although often it is not connotatively represented or described, nor in any significant way describ*able*. We can not specify just what† is conceiv*able*—"to say nothing‡ of" what is *in*conceiv*able*.

To be sure, something is to be said for the inconceivable and even about it. The authors say that if any one asserts that meaningful statements can not be made about the inconceivable, he seems to be making one himself.[54] But such paradoxes become familiar in the study of logical arguments, as in E (11), and even when they are admitted the fact remains that the inconceivable can not be connotatively described.

[50] *ibid.*, pp. 238, 241-4, 262, 479.
[51] C. I. Lewis, *Survey*, pp. 243, 328.
[52] C. I. Lewis and C. H. Langford, *op. cit.*, p. 161.
[53] *ibid.*, pp. 161, 215, n. 4.
[54] *ibid.*, p. 68.

The difficulties due to disregard of this fact begin to appear when Lewis and Langford raise the questions as to what is true of what they call the logically inconceivable; for example, are the propositions "All round squares are round," and "All round squares are square," true or false, or what? The answer of Lewis and Langford is that these propositions are both true; in fact, since round square = 0, anything one may please may be truly predicated of it in intension. They go on to say that with respect to concepts which have possible though no actual denotation, the case is different. In intension, "All unicorns have a single horn" is true; it follows from the concept of this animal that it must have a single horn. But "All unicorns have two horns" is false. That is, there is a definite intensional truth or falsity about non-existent but conceivable things like unicorns. And it is not the case that concerning the merely non-existent, anything one may please is true.[55]

For us, intension has reference not to the psychology of concepts but to the logic of attributes, which should be studied in terms of process and structure, primarily without regard for considerations of truth and falsehood.[56] The fact that the process is progressive and admits of various steps brings out the difference between what is actually conceived and what is possibly conceiv*able*.

> Let us consider a mortal man, a one-horned unicorn, and a round square. These are all conceived. They are all alike in logical structure and all valid as examples of such structure; there is in each case a term and an attribute. In the first example, the term happens to represent an existent biological species; the attribute is familiar, and we might even say, essential, although the term can be denoted without making the attribute explicit; and the term with the attribute has actual denotation. In the second example, the term does not represent an existent, but an imaginary species; the attribute is familiar, and we might say, essential, although the term can be denoted (*e.g.*, as the animal pictured opposite the lion in a certain coat of arms) without making the attribute explicit; and the term with the attribute has actual (not, as Lewis and Langford say, merely possible) denotation, although not actual biological denotation. In the third example, the term represents an existent or subsistent species of geometrical figures; the attribute, although familiar, is anything but essential; the term, in its usual relationships, is denoted without making the inappropriate

[55] C. I. Lewis and C. H. Langford, *op. cit.*, p. 67.
[56] See G. P. Conger, *Epitomizations*, pp. 390 *ff.*

attribute explicit or implicit; and the term with the attribute has actual denotation (*e.g.*, as the entity used for illustration in certain discussions), although it has no existent, or picturesque denotation. The difference between the third example and the other two is not in their structures, nor in the fact that they are denoted, but is in the relationships that we find between such terms and their respective attributes when we look for them in the existent objective world. The first is found without difficulty, the second in pictures, and the third in logical discussions.

To get at the point here, let us consider the fact that predication, or attribution, whether employed in true propositions or not, is progressively selective and correspondingly neglective. When we say "Man is mortal," so far as the process of pure logic is concerned, a term selectively appropriates a relation among other relations, etc., which are neglected, left enotative, and indicated, if at all, as "non-mortal." By further selection and neglect we can go on to say "Mortal man is mortal"—with allowances for differences left innotative in so-called identity—or "Mortal man is prone to error," and so on. By rejection, rather than appropriation, we can say that "Man is not eternal," "Mortal man is not eternal," etc. These propositions are not merely valid, but true; that is, they encounter no difficulty when we approach the existent objective world with them. In them, conception is correlated with perception without deception. But if we say, as we can readily enough, "Man is eternal," or "Eternal man is mortal," or "Mortal man is equidistant," we do have difficulty. Any term exemplified in the existent objective world, with the selected attribute or property "mortal," as thus exemplified, automatically rejects and excludes "eternal"; when "mortal" is appropriated, "eternal" really belongs in the enotative. Again "man" and "mortal" ordinarily have nothing‡ to do with "equidistant." When these attributes are joined, correspondence and contact with the existent objective world is lost, because something which ordinarily in this connection is enotative is treated as if it belonged in the connotative field. Again, when we say, "A unicorn is one-horned," there is no essential difference in the logical form; the relations, etc., indicated by "non-one-horned" are left enotative. We may go on to say "A one-horned unicorn is one-horned," or "A one-horned unicorn is an animal pictured in the coat of arms," and so on. By rejection, we can say "A one-horned unicorn is not two-horned." These propositions, too, offer no difficulty in heraldry, although they would if we approached the zoological garden.

But we should expect difficulty in heraldry, if we insisted that "A unicorn is two-horned," or "A two-horned unicorn is one-horned," or "A one-horned unicorn is trivalent." In the existent, and even the conventionally pictured or otherwise represented imaginary world, the term with the selected attribute "one-horned" automatically rejects and excludes "two-horned." This is the point in Lewis and Langford's statement that what is true by definition can not conceivably be false, since it merely explicates or follows from a meaning which has been assigned.[57] With different views of conceivability and falsehood, we should say that when "one-horned" is appropriated, "two-horned" really belongs in the enotative. Again, "unicorn" and "one-horned" ordinarily have nothing to do with "trivalent." When these attributes are joined, correspondence and contact with the imagined objective world is lost, because, again, something which ordinarily in this connection is enotative is treated as if it belonged in the connotative field.

Now if we say "A square is rectangular," etc., there is no difficulty. But if we say "A square is round," or "A round square is round," or "A round square is virtuous," etc., we do have difficulties. Under all ordinary conditions of existence or imagination, "rectangular" automatically rejects and excludes "round;" when "rectangular" is appropriated, "round" really belongs in the enotative. And "square" and "rectangular," and "round" have ordinarily nothing to do with "virtuous." Some combinations of these attributes mean that correspondence and contact with the ordinary objective world is lost, because something which ordinarily in this connection is enotative has been treated as if it belonged within the connotative field. So the round square and its troubles are not so unique, after all. The propositions about it, like their analogues above, involve some rejections into the enotative and may thus be called self-contradictory. They may be indescribable but they are not inconceivable. Once the square has been said or conceived at least denotatively, to be round, any other self-contradictory attribute may be added, but such complications do not give us anything which we can call true. So far as the question of truth goes, there is, quite literally, nothing‡ to say. If any one maintains that there is something to say, that is because for him the range of truth has been fixed by some alleged impossibility or necessity.

Concerning the supposed necessities or restrictions upon possible truth-values here, the same criticisms which we have made concerning possibilities seem to be in place.

[57] C. I. Lewis and C. H. Langford, *op. cit.*, pp. 211 *f.*

First, it appears from what we have just seen that, for even stronger reasons than those applying to possibility, impossibility can not, in any important sense, be described. It is, then, not strange that in Lewis's strict implication, $\blacksquare p$ can be substituted for $\sim p$; this gives the system of material implication. It comes out satisfactorily because it narrows its range from the more remote to the less remote enotative region. The opposite transformation encounters various difficulties,[58] as we should expect from its attempt to pass from a restricted to a broader region, as well as to describe a negative†, properly left enotative, by attributing to it the characteristics of the impossible (whatever they are).

Several important considerations appear with regard to necessity and related topics here. We find that (*1*) necessity is defined in terms doubly negative; "*p* is necessarily true" is equivalent† to "it is impossible that *p* be false."[59] This suggests dealings with an enotative region, in which alleged equivalences are equivocal and proposed definitions are in some sense indefinite.

Again, (*2*) necessity, like possibility, can be analyzed only in terms of the conceived, not the conceiv*able*. When we conceive necessity, unless the range is narrow, it is easy to conceive possibilities outside it. For instance, the number of combinations of four elements, like the numbers 1, 2, 3, 4, taken two at a time, without regard to order, is six. It is impossible to make more than this number of combinations from the numbers thus indicated. The range is necessarily restricted to those numbers. But we deal here with only four numbers, whereas in logic we deal with a universe†, existent and subsistent, and a universe which, however analyzed, exhibits no ultimate restrictions or boundaries.

Within such a universe, any supposed necessary system is only a selection, and most of the logical necessities are highly abstract and sometimes relatively unimportant selections. According to Lewis and Langford, there are an unlimited number of truth-value systems, and truth-functions are not exhaustive of logical truth.[60] So if there is to be anything ultimate or fundamental about necessity, some other basis must be found for it.

Once Lewis and Langford say that (*3*) the source of necessary truth is in definitions† arbitrarily assigned.[61]

[58] C. I. Lewis, *Survey*, pp. 313 *f.*, 317; *cf.* p. 295, and *Jour. Phil.*, 17, 1920, p. 301.

[59] C. I. Lewis and C. H. Langford, *op. cit.*, p. 160.

[60] *ibid.*, pp. 199, 212, 229.

[61] *ibid.*, p. 211.

Again, (4) the system of strict implication is grounded upon assumptions†. For instance, Lewis and Langford set up not merely the usual postulates, but an "existence† postulate," in order categorically to distinguish between material implication and strict implication. This postulate is to the effect that there is some pair of propositions so related that p implies nothing† about the truth or falsity of q.[62] In other words, p and q are postulated as independent, with neither deduc*ible* from the other. But this for us is only another example of the familiar procedure of dealing with an innotative reference by some arbitrary assumption.

Strict implication, we should say, makes too much of its so-called deduc*ibility*; this is another of those terms which conceal in a suffix an appeal to possibility. The fact is that when two propositions appear to be independent of one another, so that q is seemingly not deducible from p, it is a fair question whether there is not a linkage or series of linkages between them left innotative—as in the case of the famous pair concerning the number of old maids and the state of the clover crop. Insofar as we have to deal with a universe† of discourse, it may be questioned whether any propositions are mutually independent. Independence, and consistency, too, require innotative reference. Lewis and Langford say that it is very likely true that there are an infinite† number of propositions which are consistent and independent, but that it would be a very awkward assumption†.[63]

The appeal to assumption brings up the important point that (5) no matter what may be said or not said, the logical principles are presupposed in any discussion of them. Lewis finds as examples of necessary propositions implied by any proposition, the necessary principles of logic and mathematics, and says that these are implied in the "honest-to-goodness" sense of presuppositions.[64] For Lewis and Langford, the fact is inevitable that when deductions are to be made, logical principles themselves are already implicit. The propositions of the system have a double use; they are used not only as premises from which conclusions are drawn, but all the time as principles of inference in accordance with which conclusions are drawn.[65]

Once Lewis and Langford say that in some sense there must be a truth about deducibility which is not relative to the variety and

[62] C. I. Lewis and C H. Langford, *op. cit.*, p. 179.
[63] *ibid.*, p. 181, n. 8.
[64] C. I. Lewis, *Survey*, p. 338.
[65] *cf.* C. I. Lewis and C. H. Langford, *op. cit.*, pp. 118, 252 *f.*

difference of systems and their relations. But they conclude that logistic deducibility has no general logical properties, but only such as are relative to the antecedently given modes of inference. The logistic rule which controls or allows this operation of inference reveals its externality to the facts within the system itself by being incapable of expression in the symbols of the system.[66]

Then, in accordance with this conclusion, (*6*) there is an attempt to base the assumed† necessities in an epistemology and metaphysics of pragmatism. Lewis at least appears to think that these forms of inference are dictated antecedently, in *a priori* fashion, by mind for its convenience.[67] For Lewis and Langford the problem of circularity in logic is epistemological.[68]

But according to a realistic interpretation, which makes possibilities subsistent, the involution of logical forms presents a problem somewhat like that of the theory of types. As the discussion becomes abstract, it soon ceases to concern the truth of the truth of propositions, and comes to deal with the validity of the validity of propositions. And here, as always, we are caught in the net of discourse, of language, of meanings, of subject-predicate structures or their thin disguises in symbolism. Every attempt to deny this, as well as every attempt to affirm it, only illustrates it again.[69] The pragmatic assumptions are as much subject to it as are any assumptions. This is what Lewis and Langford might call necessity; it appears to be impossible for this very abstract and general statement about logical structures to be invalid. If it were invalid, it would apparently belong in a realm outside discourse. But even the statement just made about necessity can be discussed as regards *its* logical structure, and so on—with always the open possibility that structure somewhere becomes so abstract that it is reducible to some other than recognizable logical form. But this open possibility can not be described, not merely for lack of data, but for lack of syntax. It must be left open and enotative, with all the facts this side of it indicating that as far as any one wishes to carry the process, the logical forms persist, until the conviction grows that they are not our arrangements but are ingrained in the structure of the cosmos, and are even basic features of that structure. The primary data discerned in the

[66] *ibid.*, pp. 236, 255, 258.
[67] C. I. Lewis, *Mind and the World Order*, 1929, pp. 237-49.
[68] C. I. Lewis and C. H. Langford, *op. cit.*, p. 118.
[69] G. P. Conger, *Epitomizations*, pp. 346 *ff*.

structure of the cosmos seem to be pure abstract possibilities. To talk about possibilities of possibilities, etc., is to reduce the world to a series of near-tautologies, which do not exhaust the possibilities, but which extrapolate them to the point where they have nothing left in them which is significant, or which has not already been discerned, and therefore to the point where they exhaust the interest and value of any discussion.

Again, (*7*) there is an appeal to a more rationalistic basis, when once Lewis and Langford speak of certainty as a status of logical believ*ability*†, and say that one may†, as a matter of fact, disbelieve a proposition whose probability value is 1, but one may not "rationally" do so.[70] But this is psychological rather than logical; and eventually Lewis and Langford admit that (*8*) their ultimate basis is in logical insight or intuition. The tautological character of tautologies is something which ordinary logical procedures assume. In that sense, all tautologies are presumed as immediately given.[71]

But we would contend that rationalism and intuition, if taken to be the basis of a doctrine of necessity, or any other doctrine, work as well in an objectively real world of possibilities left enotative as they do in any other system.

(o) The terms "being" and "non-being" are not synonymous with existence and non-existence, because "being" may include the subsistent. This makes "being" of broader, more general service for problems of validity than for those of truth, since truth, as we have defined it, requires adjustment to the existent world. Empty classes, of which no members exist, are treated at (f) in connection with their symbol, 0.

(q) Several doctrines of truth are grounded in totalities or wholes. For example, Keynes calls the collection of propositions which are logically involved in the premises in the sense that they follow from them, the group specified by the premises. Such a group contains all† the propositions logically involved in the premises or in any† conjunction of them and excludes all the propositions of which the contradictories† are logically involved in any of the premises or in any conjunction of them.[72] The difficulty with such truth-enclosures is, of course, the possibilities of future refutation from propositions now excluded and left enotative. Many idealists have held that, in Hocking's words, anything short of the whole

[70] C. I. Lewis and C. H. Langford, *op. cit.*, p. 226.

[71] *ibid.*, pp. 252, 355.

[72] J. M. Keynes, *op. cit.*, p. 124.

truth is untrue.[73] The difficulty here is that even the whole as denotatively or connotatively known, must carry enotative reference.

(r) Whitehead links validity with the universe in an absolute way. The validity of any proposition equivalent† to the universe† being taken† as absolute, the validity of the disjunctive complex formed of this proposition and some other proposition x can† not† be anything else but the absolute validity of the universe. Hence $x + 1 = 1$.[74]

The attempts to secure absolute certainty have constituted an historic quest, which the pragmatists have been eager to show was, and must be, futile. We are accustomed to exhortations to be content with a "practical absolute."

On the matter of revision of some ideas by others, Hocking says that for every† idea which expects correction there is another idea determining how that correction must come. Hocking, however, fails to leave the succession of validating ideas with an enotative reference. He says that since there are some ideas subject to correction, there must be some ideas or idea not subject to correction at all.[75] But such ideas, according to our view, must at any given stage in the argument be held with enotative reference.

(s) Keynes holds that in induction we judge directly that the resemblance between instances, which consists in their being past, is in itself irrelevant and does not supply a valid ground for impugning a generalization.[76] We should say that with space-time left enotative, this affirmation of irrelevance becomes gratuitous; the question of futures should be left open, in a way that is noncommittal.

Whitehead appeals more positively to space-time to support the validity of judgments. He says that space-time relations must† possess a systematic uniformity in order to know of nature as extending beyond isolated cases subjected to the direct examination of individual perception.[77] Broad virtually recognizes enotation when he says that no judgment about the future is absolutely certain; with the possible† exception of the judgment that† there will be events of some kind or other.[78]

[73] W. E. Hocking, *The Meaning of God in Human Experience,* 1912, p. 577.
[74] A. N. Whitehead, *Universal Algebra,* p. 110.
[75] W. E. Hocking, *op. cit.,* p. 568.
[76] J. M. Keynes, *op. cit.,* pp. 255 *f.*
[77] A. N. Whitehead, *The Principle of Relativity,* 1922, p. 64.
[78] C. D. Broad, *Scientific Thought,* p. 70.

(t) For Northrop, the physical universe, composed of the macroscopic atom and the microscopic atoms, is the ultimate reality; truth and error arise from the fact that a form of the possible†, which we use to assert the existence† of a similar form in the actual†, may or may not have the same† formal properties as the form of the actual.[79] But, according to our view, the macroscopic atom itself must involve enotative reference, and, even if all possibilities are actually existent within it, they must be left innotative. Hence truth and error, if this is their source, are subject to these limitations.

D. (1) Affirmative statements, in which unwarranted truth-claims are made, abound in the literature of philosophy. Examples may be found in many of the views criticized in this book. The question whether our criticisms themselves are open to the same charge will be discussed at the end of this chapter.

In current mathematical logic, false propositions, symbolized by negatives, are assigned what for us is too much positive content. Thus, in *Principia*, it is maintained that if p is an elementary proposition, $\sim p$ is an elementary proposition, and any proposition which is assumed or proved for p may also be asserted to hold for $\sim p$. In fact, $\sim p$ may even be substituted for p or q or r in any proposition of the system.[80] Such substitution, however, opens the way for widespread disregard of the enotative-innotative features essential to negatives.

Difficulties connected with negatives appear in the application of the Boole-Schröder algebra to propositions, where, in order to avoid ambiguity, it is required that $\sim p$, the negative of p, shall be its contradictory.[81]

The peculiar propositions of material implication can be approached from a study of over-statements concerning 0 in the Boole-Schröder algebra. Taken extensionally, 0 is the null class, and, by postulate, $A \cdot 0 = 0$; "that which is both A and nothing is nothing." But $(A \cdot B = A)$ and, by definition, $(A \cdot B = A) = A \supset B$; "'that which is both A and B is A' is equivalent† to 'A is included in B.'" Substituting 0 for A, we get $(0 \cdot B = 0) = 0 \supset B$. Hence the null class is contained in B, or the null class is contained in every class. Lewis says this seems queer and arbitrary because it is a relation in extension, with no analogue in intension. If we substitute 1 for B, we obtain the more plausibly

[79] F S. C. Northrop, *Science and First Principles*, p. 244.
[80] C. I. Lewis, *Survey*, p. 284. [81] *ibid.*, pp. 124, 213, and theorem 3.3.

appearing $A \supset 1$. When the Boole-Schröder algebra is applied to propositions, the relations $0 \supset A$ and $A \supset 1$ are used to establish the peculiar theorems
$(p = 0) \supset (p \supset q)$; "if p is false, then p materially implies q," and
$(q = 1) \supset (p \supset q)$; "if q is true, it is implied by any proposition p."[82]

But, granting that postulates and definitions such as the above, although they are arbitrary, may be adopted, and that there are proper analogies between a logic of classes and of propositions, the substitutions of 0 and 1, upon which the whole reasoning rests, amount to descriptive statements concerning terms which ought to be left with enotative reference. An empty class is not to be described as substitutable for A, nor in any other way; there is "nothing"‡ to say about it. Similarly, once a proposition is shown to be false, and is relegated to the limbo of the enotative, if it is referred to at all, it must be used with great caution. It may be used to establish the falsity of other propositions, or with certain reservations, the truth of their contradictories†, but it is not to be used indiscriminately in statements about subjects altogether different.

Again, the universe does not lend itself to complete description, and when it is substituted in an expression carries its enotative reference with it. This does not mean that the expression taken as equivalent† to it is true, but that the expression, while taken as true within a given system, is open to the possibility of subsequent revision. And the term representing the universe should not be used in affirmative statements about any of its subordinate classes or propositions without such reservations.

Even under the guarded condition of their strict implication, Lewis and Langford also find themselves encumbered with peculiar propositions; some of the reasons were indicated at C (n), above. They are obliged to say that an impossible proposition (one which implies its own negation) implies anything and everything, and a necessary proposition (which is implied by its own denial) is implied by anything and everything.[83] We find, however, that the same kind of faulty substitutions arise in strict logic. For instance, we have $p \cdot -p$ strictly implies p, but it rests upon the substitution of the negative $-p$ for q in pq strictly implies p.[84] Later in the development of the system, without regard for the difficulties of identity†

[82] *ibid.*, pp. 119, 229, 327.

[83] C. I. Lewis and C. H. Langford, *op. cit.*, p. 248; *cf.* C. I. Lewis, *Survey*, p. 338.

[84] *ibid.*, p. 337; *cf.* Lewis and Langford, p. 180.

or excluded middle†, it is assumed† that the impossible† $p \cdot \sim p$ strictly implies p and $p \cdot \sim p$ strictly implies $\sim p$. So the result, in effect, can be whatever the exigencies of the argument demand. Then by appropriate disjunction of p and q and substitutions of $\sim p$ for p, q can be "shown to be true."[85]

> Lewis discusses the peculiar propositions in the light of the difference between extension and intension.[86] For us, the difference between extension and intension is, in the nature of things, irreducible, but at the same time the two can be correlated. The peculiar propositions are not due to the fact that extension is confused with intension, but to the fact that implication as conjunctive truth-value is confused with implication as deducibility.[87]
>
> According to the hypothesis of epitomization, the problem here is precisely paralleled in certain relationships of heredity in the germ cells of multicellular organisms. Conjunctive truth-value is paralleled by the (54.38132) linkage of chromosomes in cells which survive meiosis and develop as the result of fertilization.[88] But the former is only one feature of implication, as the latter is only one feature of heredity. The great essential is transference from one set of entities to another.[89] As Swenson says, genuine or noetic implication is a matter of content.[90] Just as linkage of chromosomes may occur in an organism without reproduction at all, so conjunction is not the essential mark of implication.

(2) Sometimes statements made as regards truth-value are too sweepingly negative; we have seen repeatedly that in the case of an enotative reference denial is as uncalled for as affirmation.

We saw that in Lewis's strict implication, necessity is defined by the help of two negatives. The same is true of consistency; "$p \circ q$" means "It is false that it is impossible that p and q are both true." Later on there are more complicated expressions, for example in the theorems $\blacksquare\ (p \circ q)$ strictly implies $\blacksquare\ (pq)$, and p strictly implies $q = \blacksquare\ (p \circ \blacksquare q)$.[91] These may be interpreted critically, as above, or

[85] C. I. Lewis and C. H. Langford, *op. cit.*, p. 250; *cf.* Lewis, *Survey*, p. 337.
[86] C. I. Lewis, *Survey*, pp. 328, 335 *f.*
[87] See D. F. Swenson, in *Jour. Phil.*, 29, 1932, p. 520, n. 3.
[88] *cf.* G. P. Conger, *Epitomizations*, p. 457, end of paragraph.
[89] *ibid.*, 14.3803, p. 452.
[90] D. F. Swenson, in *Jour. Phil.*, 29, 1932, p. 521, n. 4.
[91] C. I. Lewis, *Survey*, pp. 293, 307 *f.*; Lewis and Langford, *op. cit.*, p. 153, make the consistency of p and q equivalent to "It is false that p implies the falsity of q."

may be allowed merely as restatements of the fundamental relationship of exclusion.

(3) Of considerable importance in the theory of probability is the "principle of indifference." If there is no known† reason for predicating of our subject one rather than another of several alternatives, then, relatively† to such knowledge, the assertions of each of these alternatives have an equal† probability. Equal probabilities must be assigned to each of several arguments if there is an absence of positive ground for assigning unequal ones. The criterion for the principle of indifference is, as Keynes points out, negative†. If no ground for unequal probabilities seems to follow, the probabilities must be equal; but if they need be neither equal nor unequal, this method of reasoning fails.[92] We should say that such absence of ground for unequal probabilities should be taken not as requiring a positive statement of indifference or equality, but as affording an open reference which must be left indeterminate and enotative.

(4) We need not linger here with antinomies, since a good portion of the problem of truth-values consists in attempts to resolve them.

E. (I) So far as we are concerned at this point, considerations of validity or truth reduce to consideration of valid or true propositions. The relative individuation of propositions as against non-propositions, etc., treated at C (d), above, is not as important as the processes of selective interaction, which are discussed in (III) below. Relative individuation is characteristic of any subject-term, whether that term is the universe or some part of it.

(II) In dealing with propositions concerning validity, the question presents itself as to whether various sublevels, orders, or types are of any importance. Four cases suggest themselves: (1) It is true that *p* is true; (2) It is false that *p* is true; (3) It is true that *p* is false; and (4) It is false that *p* is false. The first and the third are usually dismissed as of only reiterative effect, and as such better left enotative. The second is merely a reversal, marking the exclusion of what was included. With regard to the fourth, the falsity of a false proposition may mean merely a preservation or maintenance of its original or primary falsity. But this meaning would not differ essentially from that of the truth that a false proposition is false. This amounts to a refusal to recognize any distinctions of sublevel, order, or type, in the falsity of a false proposition. But taken in

[92] J. M. Keynes, *op. cit.*, p. 42; *cf.* pp. 60-5.

another sense, the falsity of a false proposition means a reversal of its truth-value, so that it becomes a true proposition. This is objectionable, because it derives a positive value from a value essentially negative and enotative. The fact that it is hard to decide between these two meanings of the falsity of a false proposition may be taken as another example of the indefiniteness of anything essentially enotative.

As an illustration of an infinite† regress in the problem of validity, we may take the circular pair

$$p_1: \text{"}p_2 \text{ is false"}$$
$$p_2: \text{"}p_1 \text{ is false"}$$

If in either of these statements we replace the second p_2 or the second p_1 by its meaning as defined in the other, we have, according to Lewis and Langford, an infinite sequence of more and more complicated expressions, each of which requires explication before its meaning can become definite. No one of the expressions can be definite unless the sequence terminates, which it does not do.[93] In other words, the series must be left enotative.

The probability of a probability offers another problem. Dubs thinks that unless certainty is somewhere attainable, all probabilities reduce to probabilities of probabilities, and everything becomes equally† improbable. No difference can be made between the probabilities of the most absurd and the most reasonable statements. Contradictories† become equally probable.[94] In cases where it is very probable, etc., that a view is very probable, Dubs would express the original probability at 99%, and the probability of the probability of the probability, etc., would be expressed by the infinite† product 0.99^{∞}. But the limit† of 0.99^n, $n = \infty$, is 0†. Whence all† infinite series of probabilities of probabilities have the same† limit, 0.[95] Here, again, there seems to be a difference of sublevel or type; the probability of an event is of a different order of reality from the probability of that probability. The "probability of a probability," etc., need not be interpreted as a product of decimals diminishing in the direction of zero; the expression may be understood differently, as if the first word "probability" referred not to the condition recognized in the second, but *to the margin of uncertainty still remaining.* Thus one might say, if the original probability is

[93] C. I. Lewis and C. H. Langford, *op. cit.*, p. 440.
[94] H. H. Dubs, *Rational Induction*, pp. 355 *f.*
[95] *ibid.*, pp. 107 *f.* See A. N. Whitehead, *Process and Reality*, pp. 306, 313.

0.99, the next is more like 0.9999, and so on. When viewed in this way the margin between probability and certainty is left innotative.

(III) Propositions are valid according as their subject-terms appropriate or reject relations, attributes, etc. Propositions are true according as their subject-terms appropriate (affirm) or reject (deny) relations, attributes, etc., which are adjusted to the universe. Thus the proposition "trees are unhappy" is valid but not true, and the propositional judgment "trees are green" is both valid and true. Thus selective appropriations and rejections come to be of importance. And since any selection carries with it a correlative neglect, it follows that no predicate of any proposition contains the whole† of truth. If there are any propositions of which the universe† is predicate, the term "universe" itself carries enotative reference.

That imperfect induction does not lead to universal propositions necessarily true, is a view widely accepted.[96] As Stebbing says, induction by simple enumeration is of the form "All observed *A*'s have been *B*, therefore all *A*'s are *B*." Since the observed *A*'s are supposed *not* to be all the *A*'s there are, this inference proceeds from a premise with regard to some *A*'s to a conclusion with regard to all *A*'s. It is consequently formally invalid.[97]

On the other hand, if an inductive conclusion be stated in terms of probability no logical fallacy is involved, since it is not invalid to pass from the premise "All observed *A*'s are *B*" to the conclusion "That all *A*'s are *B* is (more or less) probable." The argument then requires as a premise some proposition concerning probability. Nor is numerical expression of probability always precise. Thus if all observed *A*'s are *B*'s, it is probable that the next *A* to be observed will be *B*, and it is usual to express probability by $\frac{m+1}{m+2}$, but ordinarily we can not evaluate the fraction because there are an infinite† number of ways for *A* to be *not-B*.[97] All these statements, for us, amount to saying that induction is involved in a set of conditions which are at any given moment left open, unexplored, enotative. The essential point will be covered, according to our view, if where a numerical expression of probability approaches 1, or certainty, as a limit†, we leave open the possibility‡ that probability does not†

[96] *cf.* J. M. Keynes, *op. cit.*, pp. 107, 245, 266-8.

[97] L. S. Stebbing, *op. cit.*, pp. 408 *f.*

equal† certainty, and then make that possibility vanishingly small, as small as we please, or as small as possible.

(iv) The validity of propositions is often made to rest initially or primarily upon the acceptance of axioms or postulates. According to Whitehead and Russell, the reason for accepting an axiom, as for any other proposition, is always largely inductive, namely, that many propositions which are nearly indubit*able*† can be deduced from it. No equally plausible way is known by which these propositions could be true if the axiom were false, and nothing which is probably false can be deduced from it. Infallibility is never attainable, and therefore some element of doubt should always attach to every axiom and to all its consequences. In formal logic, the element is less than in most sciences, but not absolute.[98] Whitehead says that statistical probability must ultimately rest on the ground presupposed in all our reasonings, although he adds that this is limited† to the relevant portion† of the cosmic epoch.[99]

(v) The question as to possible regions intermediate between true and false propositions does not occur in the classical systems. In the Boole-Schröder algebra $(p = 1) \cdot (p = 0) = 0$. No proposition is both true and false. Also,

$$(p = 1) + (p = 0) = 1$$
$$(p \neq 1) + (p \neq 0) = 1$$

Every proposition is either true or false.[100]

There appears to be at least some variation in the content of contradictories, depending on the universe of discourse. In the calculus of propositions, Whitehead defines the supplementary proposition $-p$ of proposition p by the properties $p \cdot -p = 0$ and $p + -p = 1$†. But from further considerations, it appears that both 0 and 1 here are indeterminate. Whitehead goes on to say that whatever the propositions of the universe† may be, even if they are reduced to the minimum† of the laws of thought, the logical contradictory† of p satisfies these conditions and is therefore a form of the supplementary proposition. But by the aid of the propositions of the universe, there are other more special forms into which the contradictory can be "converted." Any† such form, equivalent† to the contradictory, is with equal† right called the supplement of p.[101]

[98] A. N. Whitehead and B. Russell, *Principia Mathematica,* 1925, Vol. 1, p. 59.

[99] A. N. Whitehead, *Process and Reality,* pp. 306, 313.

[100] C. I. Lewis, *Survey,* p. 228.

[101] A. N. Whitehead, *Universal Algebra,* pp. 110 *f.*

The problem of contradiction and excluded middle, mentioned in Chapter X, E (v), will be considered in Chapter XVI, E (vi).

(vi) For Broad, the universe† of actual† fact is continually increasing through the becoming of fresh events. But he adds that changes in the truth which are mere increases in the number of truths through this cause are logically unobjectionable.[102] It is perhaps difficult to see how the universe of fact could be increased in any other way, but this only illustrates the difficulty of describing the enotative.

Many writers have felt that with the aid of a tendency or doctrine of growth or progress, the enotative can be approached with a certain confidence. For example, Sheldon thinks that a sort of ultimate attribute of reality is its tendency to spread, to cover all† cases not† ruled out by the circumstances.[103] Here also belong the "progressively verified" doctrines of pragmatism.

Important in mathematical reasoning is the process of mathematical induction. Dantzig says that it is better called reasoning by recurrence, and consists in showing that if a proposition is true for any member of a sequence, its truth for the successor of that member would follow as a necessity†. In view of its hereditary nature, the proposition, being true of the first term, must be true of the second, and being true of the second, must be true of the third, etc. We continue in this way until we have exhausted the whole† sequence, *i.e.*, reached its last† member . . . and so indefinitely†.[104] But mention of the "whole sequence" and the "last member" is not compatible with the words "and so on indefinitely," which reveal the enotative reference of the process.

Some have sought to escape these difficulties by appeal to a principle or law within the series.[105] But there is no point in hypostasizing such a law. There is, to be sure, a relationship between the terms of a series, but this relationship, defining property, or law of formation of the series, if not an indefensible abstraction, itself carries—as one might say, packed away within it—an enotative reference.

Somewhat less presumptive is the value attached to the continued growth of a series when it does not encounter anything which

[102] C. D. Broad, *Scientific Thought,* p. 83.
[103] W. H. Sheldon, *Strife of Systems . . . ,* p. 486.
[104] T. Dantzig, *Number, the Language of Science,* pp. 68 *f.*, 71.
[105] For example, S. Alexander, *Space, Time, and Deity,* Vol. 1, p. 222.

essentially modifies it. How, says J. W. Young, can we determine of any given class† whether or not it involves a contradiction?† It seems that with reference to the classes ordinarily considered in mathematics all we can say is that, in spite of the enormous amount of work that has been done on them, no contradiction has ever appeared. Our belief in the validity of our reasoning appears to rest on nothing more substantial than this.[106] We say certainty carries an enotative reference, but enotative reference does not give us certainty. To say that contradictions will not appear is, again, unwarrantedly to describe an enotative region, albeit in negative† terms. This will appear in connection with the work of Hilbert, considered in Chapter XVI.

Against over-confidence with regard to the enotative Dewey enters a *caveat,* maintaining that sophistication is often as irrational as the superstition it replaces. Our magical safeguard against the uncertain character of the world is to deny the existence† of chance, to mumble universal† and necessary† law, the ubiquity of cause and effect, the uniformity† of nature, universal progress, and the inherent rationality of the universe. But when all is said and done, the fundamentally hazardous character of the world is not seriously modified, much less eliminated.[107] Keynes thinks that even by means of induction itself, we may discover something about the nature of the universe, the knowledge of which will have the effect of destroying the further utility of induction.[108]

Whitehead virtually recognizes the enotation involved when he says there is a limitation† upon generality† which applies equally† to all† general statements. No† statement except one can† be made respecting any† remote occasion which enters into no immediate relation with the immediate occasion so as to form a constitutive element of the essence of that immediate occasion. The one excepted statement is, if anything out of relationship, then complete† ignorance† as to it. Here by ignorance he says he means *ignorance*; accordingly no advice can be given as to how to expect it or to treat it in "practice" or in any other way. Either we know something of the remote occasion by the cognition which is itself an element of the immediate occasion, or we know nothing†.[109] This is hardly

[106] J. W. Young, *Lectures on Fundamental Concepts of Algebra and Geometry,* 1911, p. 220.

[107] J. Dewey, *Experience and Nature,* p. 44.

[108] J. M. Keynes, *op. cit.,* p. 245.

[109] A. N. Whitehead, *Science and the Modern World,* 1925, pp. 36 *f.*

consistent with another statement of the same author, that we have to seek for a discipline of the speculative reason, and that it is of the essence of such speculation that it transcends immediate fact. Its business is to make thought creative of the future†. It effects this by means of systems of ideas, including observation but generalized† beyond it. The speculative reason turns east and west to the source† and to the end†, alike hidden below the rim of the world.[110] More in line with interpretations in terms of growth and limits, Whitehead, in still another passage, maintains that exactness is an ideal of thought, and is only realized in experience by the selection of a route of approximation.[111]

(VII) Not much is said in this book concerning the validity or truth of syllogistic and similar deductive reasoning, because the duality involved in the two premises is explicit, and because the truth of the conclusion requires the truth of the premises, which has to be established inductively and is subject to the limitations of induction. Processes of reasoning, like other psychological pattern reactions, are continued and repeated until they become habits, and we tend to rely upon them automatically, and to regard them as justifying any initial assumptions†, and, in short, as true. By idealists this regularity is usually credited to the mind and by realists to the world; but there seems no reason why it may not be regarded as a feature of both mind and world in their interactions, all of which proceed within horizons.

(VIII) Since we are dealing here with neuropsychological processes, the problem of integration or more inclusive synthesis suggests again the question of intuitional theories of knowledge, which we defer until Chapter XXI.

(IX) Innotation may be recognized in several ways in connection with problems of validity. For example, the particular subject-terms subsumed under a general or universal term may be regarded as innotative within it. The same is true for conclusions implied by any given premises, or propositions implied by any given propositions. We noted in Chapter X that the term "any" provides for innotation, while "every" seeks to avoid it. In many an accepted truth lurks a possible or probable error.

So we come back to the statement at the beginning of this chapter, that the proper way to deal with the problems of validity, truth,

[110] A. N. Whitehead, *The Function of Reason*, 1929, pp. 51, 65.
[111] *idem, Concept of Nature*, p. 59.

and probability is to leave them enotative and innotative. It is as inadequate critically to condemn our reasoning as blindly to follow it; *ultimately we do not know whether our reasoning is valid, or true, or even probable, or not.* We have to proceed within horizons. The question then may fairly arise, What about the validity and truth of our whole discussion of the horizons of thought? If studied carefully, our discussions at various points will be found to employ or to imply, most, and probably all of the terms which it criticizes. For example, the identical‡ principle of horizons, or its equivalent‡, is discussed in different‡ aspects. It is not‡ left open to contradiction‡. All‡ statements about it apply to a logical universe‡, and seek to omit nothing‡ from the scope of its validity‡, truth‡, or probability‡. The principle is held to apply without any de-finite‡ limit‡, and with no break in the continuity‡ of its applications. Any discussion of it must‡ be ordered‡, actual‡, affirmative‡, and offered as absolute‡, with reference to the whole‡ of being‡.

The ultimate question then is, What is the value of such a discussion, which is obliged to use without limitation the very terms and categories in which it seeks to recognize limitations? Some would say that the whole discussion is paradoxical and shows, if anything, that our thinking collapses into a mass of wreckage. Others would see in the thinking process something so subtle and elusive that it defeats its own purposes—a series of dissolving views, which can not be said to reach even the nothing they set out from. But the epitomization hypothesis, with its views about realistic data within relativistic horizons, and recognition of similar and recurring structures in mind and world, and in the world existent and subsistent, makes possible another answer. It is that statements about validity, truth, etc., can be underwritten by other statements about validity, truth, etc., and the process can go on in a non-finite‡ regress, with nothing‡ to stop it. But precisely as it goes on enotatively and innotatively, and becomes divested of material content, it exhibits more clearly than ever the pattern of that cosmic monadic structure which can not be gainsaid. Any attempt to escape from it but involves us in fresh instances of it. Our so-called truth doubtless reduces to probability; but truth and probability together reduce to a formal validity in which we live and think and have our being, and the existent world, as it arises within the horizons of the subsistent and presents its problems of truth, seems to show the same principles in its structures and processes.

PART THREE

Some Problems of Mathematics

"Experience has taught most mathematicians that much that looks solid and satisfactory to one mathematical generation stands a fair chance of dissolving into cobwebs under the steadier scrutiny of the next."

—E. T. Bell, in *Scientific Monthly,* 32, 1931, p. 204.

CHAPTER XIV

Limits

"The doctrine of limits, though beautiful and entirely satisfactory, can not profitably be presented to the beginner. It is better to reserve it until familiarity with inferior methods of investigation has made him feel the need of something better . . . . The results obtained by the lower type of reasoning are quite sufficient for all practical purposes."

—T. P. Nunn, *The Teaching of Algebra . . .*, 1914, p. 247.

A. Passing to some problems of mathematics, in this chapter we consider limits. Several definitions will be examined as we proceed. Some light is thrown upon the problem of limits by the nature of limitations, and accordingly some features of these will be included in our treatment.

B. (α) Current agnosticism about the traditional notion of limits is illustrated when Broad, introducing Whitehead's method of extensive abstraction, says that we do not know that a series of volumes has a limit which can serve as the definition of a point,[1] or that some series of rational numbers have limits.

(β) Perhaps the tendency to picture or to seek the limit of a series may be traceable to the tendency to complete an incomplete figure, as studied in the *Gestalt* psychology.

(γ) Locke's distinctions between mind and world were followed and amplified by his successors, until Kant spread upon that framework his doctrine that our understanding prescribes limits to itself by admitting that it can not know the noumena by means of the categories, but can only think of them under the name of something unknown.[2] Fichte was less modest, and it was his disregard of limitations which paved the way for the objective and absolute† idealism of Schelling and Hegel.

(δ) Mathematical limits are sometimes treated by quasi-intuitive methods. Rignano speaks of the difficulties which belong to every passage to the limit, implying as it does the "vision" of the result

[1] C. D. Broad, *Scientific Thought*, 1923, p. 42.

[2] I. Kant, *Critique of Pure Reason*, transl. F. M. Müller, 1896, p. 209.

towards which we indefinitely approach by a given series of merely imagined operations of algebraic calculations.[3]

C. (a) Limits and problems which involve them are sometimes interpreted in terms of sameness, identity, or equality. According to Russell, the rationals less than a variable term of the series .9, .99, .999, etc., are the same as the rationals less than 1.[4]

Nunn says that in calculating a rate function (or ordinate PQ under a curve), the ordinates on each side may be made to approach PQ and differ from it by as little as we please. If in accordance with Wallis's law, they differ from a given expression $3ax^2$ by as little as we please, it follows, says Nunn, that PQ must be exactly $3ax^2$. Nunn holds that other "proofs" of Wallis's law are open to the criticism that they give approximations.[5]

Some writers have committed themselves to inequalities for an answer. Russell says that the Achilles paradox proves that two variables in a continuous† series which approach equality from the same side can not even have a common limit.[6]

(d) The problem of limits is treated in terms of classes, when, according to Russell, instead of defining the limit of a function, we may define a class of limits. A number z belongs to the class of limits y for $x = a$ if within any interval containing a, however small, y will approach nearer to z than a given difference. This method, according to Russell, has the advantage that a class of limits always exists. The limit can be the only member of the class of limits if the class has only one member.[7]

But we should say that where there is doubt about the existence of a limit beyond an interval which is properly left innotative, it is at best only a makeshift to refer to a class of members of the series, or a class of limits, which are alleged to fill the interval. Such a class can be referred to only denotatively, and not fully described. The formation or construction of such a class encounters the difficulties of induction and the doubtful use of the word "all." Such a class can not be connotatively known nor described, except in part; something must be left enotative and innotative. A similar difficulty appears when it is said that, although Cantor's ω is preceded by (the class of) all† finite† numbers, it is not preceded

[3] E. Rignano, *Psychology of Reasoning,* transl. W. A. Holl, 1923, p. 175. *cf.* C. K. Ogden and I. A. Richards, *Meaning of Meaning,* p. 170.

[4] B. Russell, *Principles,* p. 272.

[5] T. P. Nunn, *Exercises in Algebra . . . ,* Part 2, 1914, pp. 327, 349.

[6] B. Russell, *Principles,* p. 359.

[7] *ibid.,* p. 328.

immediately by any† of them[8]—*i.e.*, they do not avail to fill the gap which must be left innotative. This is also the difficulty when a limit is defined generally as a term which immediately follows (or precedes) some class of terms belonging to an infinite† series, without immediately following (or preceding, as the case may be) any† one term of the series.[9] Plainly the notion of class is here used with enotative and innotative reference. This should be understood when, for instance, it is said that the limit of 1, 2, 3 . . . ν, is Cantor's ω, the next number greater than all† of them;[10] or, in the work of Broad, it is said that the limit of an endless series might be described as the first term that comes after all† the terms of an endless series.[11] Russell says that when it is forgotten that Cantor's ω has no immediate predecessor, all sorts of contradictions† emerge.[12]

In a later attempt to fill the gap which should be left innotative, Russell uses two universals. He says that the upper limit of a set of terms A with respect to a series P is that term x (if it exists) which comes after all the A's, but is such that every† earlier term of the series P comes before some of the A's.[13] But it does not essentially change the situation to have x coming after all the A's and every term earlier than x coming before some of the A's. First, as before, something is left innotative between the last A (of all the A's) and x. But then it turns out that since every term earlier than x comes before some of the A's, every term earlier than x comes before the last A. As the terms earlier than x are multiplied, the only effect significant for the problem is to crowd the last A nearer to x. But we can not construct the whole† class which contains every term earlier than x, nor do we know to deal with the infinite series as a whole.

Still more precisely, with another generalization preceding the two universals, he says that where "has the relation P to" = "precedes," a term x is an upper limit of class A with respect to a relation P, if (1) A has no maximum in P; (2) every member of A which belongs to the field of P precedes x; (3) every member of the field of P which precedes x precedes some member of A.[14] But to add to the former definition the provision that A shall have no

[8] *ibid.*, p. 361.
[9] *ibid.*, p. 277.
[10] *ibid.*, p. 313.
[11] C. D. Broad, *Scientific Thought*, p. 42.
[12] B. Russell, *Principles*, p. 361.
[13] *idem*, *Introduction*, p. 99.
[14] *ibid.*, p. 70.

maximum is merely to open the way for indefinite growth, not for definite attainment of the limit x. The need for enotation and innotation still remains.

Russell once appears to think that by appealing to the notion of classes of terms in a series he can find a condition which does not apply to the terms of the series. He says that in a compact series it is always possible to find two *classes* of terms which have no term between them. If P be the generating relation and x any term of the series, then the class of terms having to x the relation P is one between which and x there is no term. The class of terms so defined is one of the segments determined by x.[15] In other words, the generating relation requires proximity and immediacy of transition, but the definition of the compact series, on the contrary, requires intermediate or interpolated terms. To say that every terminated segment in a compact series has its defining term as a limit[15] disregards regions of the series properly left innotative. And to refer to the class, or "all" the members of a compact series makes the term "class" or "all" mask an innotative reference.

J. W. Young presents a more general definition, which includes the limit of a sequence as a special case. Let C be any† linearly ordered class, and let the variable x represent any element of this class. A segment of such a class may be defined as the elements of the class which lie between any two elements of the class. Given an element A (which need not be an element of the class C, but ordered† with reference to C), a neighborhood or vicinity of A is defined as any segment of the class C such that A lies between two elements of this segment. The element A is then said to be a limit of the class C, provided every† neighborhood of A contains elements of C.[16] But a neighborhood, as above defined, includes innotative reference, and the statement that every such innotative region contains elements of C disregards this.

(e, f) The notions of the universe and zero as the limits of class extension, etc., are familiar. If a variable in approaching a limit actually attains the limit, then the difference between the variable and the limit is zero. Zero fulfils the general definition, since it is less than any number or distance which may be chosen, however small. But as we have seen, zero itself should involve enotative reference.

[15] B. Russell, *Principles*, p. 276.

[16] J. W. Young, *Lectures on Fundamental Concepts of Algebra and Geometry*, p. 208.

(i) Russell says that when a limit is not known, it can not be proved to exist† at all unless we introduce some axiom† of continuity.[17]

(j) Infinites are frequently invoked to help with the difficulties of limits. No class, says Russell, can have a limit unless it contains an infinite number of terms. He defines the limiting point of a series as a term such that in any† interval containing the term there are an infinite number of terms of the series.[18]

Whitehead uses with reservations, the notion of the "sum to infinity" of a series,[19] but as Nunn declares, the use of the term involves a convenient fiction.[20]

We may consider here the question of the definiteness of a limit. According to Nunn, when properly stated the arguments of the calculus do as a matter of fact give results which are as unequivocally exact as multiplication. But he says that the appearance in an argument of the phrase "infinitely small" or "infinitely great" should always put the reader on his guard against an illegitimate deduction.

For practical purposes, Nunn proposes to follow the general custom of mathematicians who, when they say that a limit is "infinity" do not mean by the term any definite infinite number, but mean simply that the values of the variable increase forever†, so that no finite number can be named which they will not exceed, if x be taken small enough. He says that it is unfortunate that a term which has so clear a meaning should ever be used loosely, but that the student will recognize that in this case the advantage of using a uniform notation for all problems of the same kind is very great, and may justify a certain amount of logical irregularity.[21]

Whitehead says that the idea of a limit has a precise meaning in the theory of series and in the theory of the values of functions.[22] But we would maintain that the adjectives "definite" and "precise," used here of a limit, refer to it at most denotatively rather than connotatively, and that even denotative reference is across a gap left innotative.

(k) Sometimes the notion of beginning and ending is employed in the work on limits. Granville points out that in integration be-

[17] B. Russell, *Principles,* pp. 293 *f.* [18] *ibid.,* pp. 277, 290.

[19] A. N. Whitehead, *Introduction to Mathematics,* pp. 201 *f.*

[20] T. P. Nunn, *Teaching of Algebra,* p. 226.

[21] *ibid.,* pp. 246 *f.; Exercises in Algebra,* Part 2, pp. 336 *f.*

[22] A. N. Whitehead, *Aims of Education,* p. 207.

tween limits, the word means merely the value of the variable at one end of its range (end value), and should not be confused with the meaning of the word in the theory of limits.[23] According to Nunn, a limit is always the limit of a sequence S' which is thought of as *part† of a wider sequence* S. The sequence S' must† have either no† first or no† last term. If S' has no first term, let there be a term L of S which is not a term of S', but is the last term of S before all† possible terms of S'; or if S' has no last term, let there be a term U of S which is not a term of S' but is the first term of S after all possible terms of S'. Then L and U are respectively the lower and upper limits of S' in the sequence S.[24] Nunn emphasizes that the limits L and U are not themselves members of the sequence S', but are terms of S which lie outside S'.[24] The uncertainty as regards members of S' is reflected in his statement that the question whether or not S' actually has a limit and what that limit is depends upon the sequence S of which it is regarded as forming a part†.[24] It should be noted that members left enotative as regards S' are left innotative as regards S, and may be equally hard to describe in terms of the latter.

(n) In the work cited under (k) above, Nunn refers to a limit as the first term beyond the possible values of the variable.[25]

(q) Whitehead places within a totality his doctrine of a factor as a limitation. Any factor as a limitation within totality necessarily refers to factors of totality other† than itself,[26] but the reference must somewhere be innotative.

(r) The absolute rather than approximate character of limits used to be insisted upon in mathematics.[27] The more modern view is indicated by Nunn who says that in his exercises integration and differentiation are taught as a "calculus of approximation." The investigations, though giving results which may be regarded as true to any degree of approximation, do not give and must not be represented as giving absolute results.[28]

D. (1) As an example of an affirmative statement, advanced and then qualified, we find that when the limits of a definite integral are graphed in the curve called "the witch," with the end of the

[23] W. A. Granville, *Elements of the Differential and Integral Calculus,* 1911, p. 316 n.
[24] T. P. Nunn, *Teaching of Algebra,* p. 542.
[25] *idem, Exercises in Algebra,* Part 2, p. 336.
[26] A. N. Whitehead, *Principle of Relativity,* p. 17.
[27] W. Parkhurst and W. S. Kingsland, Jr., in *Monist,* 36, 1926, p. 524.
[28] T. P. Nunn, *Teaching of Algebra,* p. 246.

curve not quite touching the abscissa, Granville says that "we call the result the area which is bounded by the curve, the ordinate, and the abscissa, although strictly speaking this area is not completely† bounded."[29]

(2) A similar diagram may be interpreted negatively, as when it is said by Vaihinger that in strict logic, we could never subsume the curve under the straight line. All the laws of rectilinear figures hold only for such, and rectilinear figures remain rectilinear, even if we increase the number of their angles to infinity†. Parkhurst and Kingsland see that if the two figures fall short of agreement, albeit by as little as we choose to have them, the basis of the calculus is approximation, and our variable can† never† attain its limit.[30]

(4) According to Parkhurst and Kingsland, in the past some writers asserted that such a limit could "ultimately" be attained; some, never; whilst still others of equal mathematical eminence, unable to rest in either conclusion, essayed the precarious task of straddling two horses that galloped in opposite directions.[30]

E. (I) That limitations and limits involve enotative reference is well illustrated even in the number series, when Dantzig says that it is not a logical necessity† that any† number has a successor. A "bounded" arithmetic is just as ten*able*†. If the limit were the physiological or psychological limits of counting, say 500,000, then 500,001 would be meaningless and the meaningless cases would far exceed those which have a meaning.[31] But we should say the meaningless cases would not be considered at all; they would be left enotative. As far as the series was continued, of course, the very act of writing down the meaningless would have given it a meaning.[32]

(II) Multiple integrals, etc., bring with them various orders of limits.[33]

(IV) Doctrines of limits may be strengthened by assumptions or insured by definitions. Russell says the inference that there is a limit is precarious. It may be supported either by positive knowledge or by some axiom.[34]

(V) A limit may obviously serve as the boundary of its series; but the description of the boundary must be left somewhere innotative or enotative.

[29] W. A. Granville, *op. cit.*, p. 322.
[30] W. Parkhurst and W. J. Kingsland, *op. cit.*, pp. 528 *ff.*
[31] T. Dantzig, *Number, The Language of Science*, pp. 74 *f.*
[32] *ibid.*, p. 88.
[33] *cf.* W. A. Granville, *op. cit.*, p. 394.
[34] B. Russell, *Principles*, p. 293; *cf.* p. 283.

(VI) Thinking of a process rather than its results, and with a view which suggests Bergson's notion of "canalization," Whitehead holds that a factor is a limitation of fact in the sense that a factor refers to a fact canalized into a system of relata to itself.[35]

Russell says that the notions of limit and continuity† in definitions of perfect series, condensed series, etc., must not be confused with the notions of the limit of a function for approaches to a given argument, nor the continuity of a function in the neighborhood of a given argument.[36] One difference here lies in the peculiarities of numbers, which if specified, for instance in writing, must be discrete, and, even if referred to, must be grouped in classes. But the process of growth, progression, etc., is to be distinguished from the discrete units or stages reached, just as a function is to be distinguished from the coordinates used in its graph. If the process is described, the stages reached must for the time being be left enotative or innotative; if the stages reached are described, the process similarly must be left enotative or innotative. It is like the Heisenberg principle, according to which the position and the velocity of an electron can not be specified at the same time.[37]

We considered, at C (a) above, some cases where variables are said to be equal† to their limits. Some writers are content if the notion of limit expresses an approximation to equality. For Whitehead, l is the limit of the terms of the series $S_1, S_2, S_3, \ldots S_n$, if, corresponding to each real number k taken as a standard of approximation, a term S_n of the series can be found so that all† succeeding terms $S_{n+1}, S_{n+2}, \ldots$, approximate to l within that standard of approximation. After a while you come to terms all of which are nearer to l than any† number you may like to assign.[38] Similarly, it is to be noted that of Keyser's four definitions of the term "limit," three contain the words "however small a neighborhood be chosen," and the fourth expresses a similar idea in terms of any predecessor, any successor, and terms between terms.[39]

(IX) Innotative reference is plain in several definitions of limits, besides those cited above. Thus, a member x is a limit of a sequence

[35] A. N. Whitehead, *Principle of Relativity,* p. 16.

[36] B. Russell, *Introduction,* p. 104.

[37] See G. P. Thomson, *The Wave Mechanics of Free Electrons,* 1930, pp. 124 *f.*

[38] A. N. Whitehead, *Introduction to Mathematics,* p. 201; *cf.* p. 229.

[39] C. J. Keyser, *Mathematical Philosophy,* 1922, pp. 256, 259, 272 *f.*, 281; *cf.* T. P. Nunn, *Teaching of Algebra,* p. 205.

of members of the series, if there is no† member of the sequence beyond x (above x in the case of an upper limit and below x in the case of a lower limit, above and below being stated in terms of the relation which orders† the series), and if between any member of the sequence and x there are other† members of the sequence.[40]

Nunn's definition of the limit of a sequence of numerical values of a function is: Let L be a number such that as the variable assumes values which approach a, the values of the function eventually come and remain within the range $L \pm h$ and do so, however small h is taken to be; then L is the limit of the function as the variable approaches a.[41]

As regards limits, when we put together the results brought out in our examination of the foregoing views, it appears that our horizon concepts help to meet the difficulties. A limit of a series is a term between which and certain connotatively known terms of the series something is left enotative with respect to the connotatively known terms of the series and innotative with respect to those terms and the limit, taken together. In cases like 1 as the limit of decimal fractions, and perhaps where there are "definite" and "precise" ideas of limits noted above, the limit may be said to be known denotatively, but not connotatively.

[40] C. W. Morris, in *Jour. Phil.*, 26, 1929, p. 451.
[41] T. P. Nunn, *Exercises in Algebra*, Part 2, pp. 333 *f*.

CHAPTER XV

Continuity

"Wenn es überhaupt gelingen sollte, das Kontinuumproblem einer endgültigen Lösung zuzuführen, so werden dazu wohl neuartige Beweishilfsmittel oder irgendein Umdenken grundsatzlicher Art erforderlich sein, das *z. B.* zu einer ganz neuen Form der Fragestellung nötigen könnte."

—A. Fraenkel, *Einleitung in die Mengenlehre,* 1928, p. 301.

A. The problems of continuity are, as we shall see, closely allied with those of limits. The distinction between structure and process appears again in the distinction between the continuity of a series and the continuity of a function in the neighborhood of a given argument.[1]

Of the various examples of continuity and continua, the series of real numbers and the points on the segment of a line are among the most familiar. Most of the discussion concerning points on a line is for convenience grouped in Chapter XIX, C (i).

B. (α) The intuitionists are skeptical, and think that the difficulty with the continuum is in any fruitful sense impossible† to overcome.[2]

(β) Sometimes the difficulties of continuity are traced to differences between perception and thinking. Rubin points out that the perception of a continuous stroke, like a line, can not distinguish segments lying next one another. This shows a kind of formal agreement with the mathematical notion of the continuum, although there is no corresponding agreement between the idea of the continuum and the physical objects at the basis of our perceptions.[3] We want in psychology, says Köhler, by the multiplication of observed correlations between terms, to get rid of the terms and interpolate continua.[4]

(γ) Sometimes attempts are made to clarify the problem of continuity by distinctions between mind and world. Thus Dedekind says that a real number is generated by the power of the mind to

[1] B. Russell, *Introduction,* p. 104.

[2] A. Fraenkel, *Einleitung in die Mengenlehre,* p. 222.

[3] E. Rubin, *Visuell wahrgenommene Figuren,* Teil 1, p. 163.

[4] W. Köhler, in C. Murchison, ed., *Psychologies of 1930,* p. 121.

classify rational numbers.[5] According to Hilbert, the infinite† divisibility of a continuum is an operation which exists in thought only.[6] But for the realist Alexander, the mathematical notion of continuity contains no dreaded infinite regress. The infinitude is of the essence of the datum and expresses no repetition of steps on our part.[7]

(δ) Russell once wrote that it would be found possible† to give a genuine definition† of continuity in which no appeal is made to the mass of unanalyzed prejudice which Kantians call intuition†. He proposed to do this by the notion of the Dedekind section.[8] But later he wrote that we must feel the mathematical theory of continuity, as well as reason about it.[9]

The intuitive view of the continuum which is characteristic of Brouwer and his followers will concern us in Chapter XVI, E (vi).

C. (a) Sameness is employed when Russell holds that a function is continuous at a point where its value is the same as the limit† of its value for approaches either from above or from below.[10] Equality is employed when he says that Cantor has shown that every possible† way of dividing a period into two continuous portions divides it into two portions having the same number of terms; but Russell seeks to avoid this in certain cases by establishing a property of "magnitude of divisibility."[11]

(b) In a problem like that of continuity, one must be prepared for negations. Bridgman says that the mind is not even able† to talk about continuity except in negative terms. In terms of operations, continuity has only a sort of negative meaning.[12] For us, the essentially indeterminate character of negatives indicates that what is here involved is open, enotative or innotative reference.

(c) According to Russell, no definite† contradiction can be proved concerning continuity. In fact, among numbers the various forms of continuity which occur can† not† be denied without positive contradiction.[13] Doctrines of the axiomatists concerning freedom from contradiction will be considered in Chapter XVI.

[5] T. Dantzig, *Number, the Language of Science,* p. 171. [6] *ibid.,* p. 238.

[7] S. Alexander, *Space, Time, and Deity,* Vol. 1, p. 148.

[8] B. Russell, *Principles,* p. 260.

[9] *idem, Our Knowledge of the External World as a Field for Scientific Method in Philosophy,* 1915, p. 130. This work will hereafter be cited as *Scientific Method in Philosophy.*

[10] *idem, Introduction,* p. 110.

[11] *idem, Principles,* p. 151.

[12] P. W. Bridgman, *The Logic of Modern Physics,* 1927, p. 94.

[13] B. Russell, *Principles,* pp. 353, 368.

(d) Continuity is explained in terms of a class and its members by Russell, who says that many terms considered as having a cardinal number must be all† members of one† class† and indistinguishable† as far as each is an instance of the class concept†. In this aspect, the whole† of which they are composed is continuous. But in the order of their maniness, they must be different† instances of the class concept and so discrete.[14] Here the lack of distinction or difference characteristic of generality is arbitrarily interpreted as continuity.

Whitehead appeals to universals in his treatment of the continuity of a function. An "interval" of values of the argument x of a function $f(x)$ is all† the values lying between some two values of the argument. A function $f(x)$ is continuous at a value a in its argument, when in the neighborhood of a its values approximate to $f(a)$ —*i.e.*, to its value at a—within every† standard of approximation. An interval can be produced for any† standard we like to try.[15]

(f) Hocking makes the problem involve zeros. To the intellect, according to Hocking, the continuity of the line is to be defined in terms of its points, of which there are an infinite† number. The question is, how shall we describe the setting of these points, each of which occupies a zero length of the line, so that together they will constitute the whole† line?[16] We should say that the question as thus stated involves a hopeless paradox. The proper answer is that zero length in relationship to the whole line is an innotative term and should not be described as an element of the line. The whole line should be taken as including its zero lengths, but those lengths are left innotative.

(h) Some of the most conspicuous problems of continuity appear in connection with rational numbers, real numbers, and other number series. Since these involve limits, they may be considered here, as if problems of continuity were being answered in terms of limits. Thus in a compact series there is a term between any two, and every† terminated† segment has its defining term as a limit.[17] Again, according to the Cantor-Dedekind theories of the arithmetical continuum, to any† partition in the rational domain there corresponds a limiting point of an infinite† sequence; thus a limiting

[14] B. Russell, *Principles*, p. 346.

[15] A. N. Whitehead, *Introduction to Mathematics*, pp. 158-62.

[16] W. E. Hocking, *Types of Philosophy*, p. 199. This work will hereafter be cited as *Types*.

[17] B. Russell, *Principles*, pp. 271, 276.

point of any infinite sequence can† be used as an agent in partitioning the rational domain.[18]

But doubts concerning such limits begin to appear when the attempt is made to define irrational numbers. Broad says that irrational numbers sometimes have been defined as the limits of certain series of rationals; but we can† not† be certain† that there is anything answering to the definition.[19] Russell takes the negative†, maintaining that irrational numbers ought to be defined as those segments of rationals which have no limits.[20] The concept of limits is, however, retained and otherwise applied. In the cases where we naturally supposed that an irrational number must be the limit of a set of ratios, it is the limit of the corresponding set of rational real numbers in the series of segments ordered by whole† and part†. $\sqrt{2}$ is the upper limit of all† those segments of the series of ratios that correspond to ratios whose square is less than two. $\sqrt{2}$ is the segment consisting of all those ratios whose square is less than 2.[21] A real number, which would be commonly identified with a rational, is a segment which does have a rational limit.[20] Thus real numbers involve innotative reference.

The concept of limits is important in the consideration of a closed series, a series condensed in itself, and a perfect series, but the concept is complicated by universals† which introduce fresh paradoxes. Thus it is said that a series is closed when all† the limiting points of a series belong to it—or when every† progression or regression contained in the series has a limit in the series. A series "condensed in itself" is a series where every† member is the limit of a progression or a regression. Cantor defines a series as perfect when all† its points are limiting points and all its limiting points belong to it.[22] In all these cases, we have the concept of limit, with its innotative reference, and along with it some mention of "all" the members of the series, or every member of the series, as if the members left innotative could be described like the others.

(j) Russell combines the problems of the continuum and the infinite in his statement that in the case of abstract objects such as fractions, it is perhaps not very difficult to raise the logical possibility† of their forming a compact series. The difficulties that might be felt are those of infinity.[23]

[18] T. Dantzig, *op. cit.*, p. 174. [19] C. D. Broad, *Scientific Thought*, p. 41.
[20] B. Russell, *Principles*, pp. 286, 368. [21] B. Russell, *Introduction*, p. 73.
[22] *ibid.*, pp. 102 *f.*
[23] *idem*, *Scientific Method in Philosophy*, 1915, p. 133.

It is noteworthy that with all his reliance upon infinites, Russell spurns infinitesimals. He holds that definitions of continuity do not involve infinitesimals. They involve infinite classes of intervals, growing smaller without any limit† short of zero†, but they do not involve any† intervals that are not finite.[24] He says that the denial of infinitesimal segments resolves the antinomy† that the continuum both does and does not consist of elements. Both statements may be made, though in different senses. Every† continuum is a series consisting of terms, and the terms, if not indivis*ible*†, are at any rate not divisible into new terms of the continuum. But, he goes on, if we take consecutive terms together with their asymmetrical relation as constituting what may be called in a certain sense an ordinal element, then in this sense our continuum has no† elements. If we take a stretch to be essentially serial, so that it must consist of at least two terms, then there are no elementary stretches. If our continuum be one in which there is distance, then likewise there are no elementary distances. The demand for consecutive terms springs from an illegitimate use of mathematical induction.[25] This, however, to us is like the procedure of the theory of types. To take the elementary elements of a continuum as merged in certain combinations amounts to an assumption† regarding them, and may lead us to lose sight of the problem of continuity between those elementary elements and leave it innotative. This procedure not merely thus leaves a problem behind it, but it does not at all prevent the recurrence of the same problem at a new level, or sublevel, with reference to the more inclusive units formed by the elementary units in combinations.

(k) Notions of beginning and ending are employed in work on well ordered series. A well ordered series is one which has a beginning, and has consecutive terms, and has a term next after any selection of its terms, provided there are any terms after the selection.[26] Fraenkel says that to the assertion that the continuum is well ordered there are objections, involving questions of perfect induction. It is not that we can† make the continuum well ordered, but only that such an order† in it is think*able*† as free from contradiction†.[27] On the other hand, according to Russell, the continuum or some still more dense series might† be proved to be incap*able*† of having its terms well ordered.[28] The reason for the

[24] B. Russell, *Introduction*, p. 116.
[25] *idem, Principles*, pp. 353 *f.*
[26] *idem, Introduction*, p. 92.
[27] A. Fraenkel, *op. cit.*, pp. 119 *f.*, 304.
[28] B. Russell, *Introduction*, p. 130.

uncertainty is that it involves so much which must be left enotative and innotative. This will appear in more detail in connection with the Dedekind cut, to be studied in E (v), below.

(l) According to Russell, continuity in mathematics is a property only possible† to a series of terms, *i.e.*, to terms arranged in an order, so that we can say of any† two that one comes before the other.[29] But the difficulty here is that any order involves a correspondingly enotative or innotative non-order, which must be reckoned with in the interpretation of continuity.

(n) The theory of continuity has made some rather large drafts upon the resources of possibility. Whitehead says that continuity concerns what is potential.[30] Russell says that, properly speaking, a collection† to be continuous must be as concentrated as possible in every† neighborhood containing any of the terms of the collection.[31]

It should be remembered that recourse to possibilities is sometimes half concealed by the use of the words "can" or "may," and by words ending in "-ible" or "-able."

(o) We shall discuss in Chapter XVI, E (vi), the view of Brouwer, according to whom the continuum as a whole† is only the medium or the field for a process of free becoming.[32]

(q) Fraenkel says that for the continuum, it is not the relationship of aggregate and indivisible† elements which is definitive†, but the relationship of whole and part. It is precisely a fundamental characteristic of the continuum, that it has parts which can† be further partitioned without limit†.[32] Russell says that we need the whole series of ratios in the order of magnitude in order to be able to define real numbers as segments.[33] But for us the difficulty is precisely in the determination of the whole. That the problem is an involved one appears when Russell goes on to say that this definition of real numbers will not give the desired result unless the series of ratios is compact, and it can† not† be compact if the total number of ratios at the stage concerned is finite†.[33]

(s) Continuity is frequently illustrated in terms of geometrical elements in space. Bergson's criticism of Zeno that he tries mistakenly to describe time in terms of space, making the parts of continuous time external to one another, could be amended by saying that Zeno in his treatment of time fails to allow for innotation.

[29] B. Russell, *Scientific Method in Philosophy,* 1915, p. 131.
[30] A. N. Whitehead, *Process and Reality,* p. 95.
[31] B. Russell, *Principles,* p. 324.
[32] A. Fraenkel, *op. cit.,* p. 239.
[33] B. Russell, *Introduction,* p. 134.

Innotation is apparent, although not recognized, when, for Russell, the continuity of a motion is shown by the fact that however near together we take two positions and two instants, there are an infinite† number of positions still nearer together which are occupied at instants that are also nearer together.[34]

Alexander finds that instants of time supply continuity for points of space. Points are continuous because they are not mere points, but are instants as well. Since geometry omits time from space, there is a certain artificiality in the reconstruction of continuity within space on purely spatial terms, and similarly in continuity of time without reference to space.[35] While this seems quite true, the question of continuity here, for us, becomes that of the elemental or irreducible character of the instants. However small they be taken, they mask intervals which have to be left innotative.

(t) For Whitehead, the continuity of nature arises from extension. Every† event extends over other events and every event is extended over by other events.[36] The continuity of nature is to be found in events; the atomic properties of nature reside in objects. We apprehend nature as continuous, but we recognize nature as atomic.[37] But we shall see that Whitehead's events, event particles, etc., have their own problems of horizons.

D. (1) Affirmative descriptions of continuity are common, as is evidenced by passages cited above. Hocking says that the continuity of a line is likely to be described positively by saying that a moving point covers in its journey all† the points of the line.[38]

(2) On the other hand, the continuity of a line is likely to be described negatively by saying there are no gaps in it.[38] For Alexander, in a continuous series there is no next term to any term.[39] Obviously these descriptions fail to leave the issues of innotation open.

E. (1) When we ask how we can legitimately proceed in thinking about the problem of continuity, we find that it is more likely to involve problems of innotation than of enotation. But a continuous series, with its innotative reference between its members, unless specially restricted, will be enotative as regards its beginnings†

[34] B. Russell, *Scientific Method in Philosophy*, 1915, p. 136.
[35] S. Alexander, *Space, Time, and Deity*, Vol. 1, pp. 148 *f.*
[36] A. N. Whitehead, *Concept of Nature*, p. 59.
[37] *idem*, *Principles of Natural Knowledge*, pp. 66 *f.*
[38] W. E. Hocking, *Types*, p. 199.
[39] S. Alexander, *Space, Time, and Deity*, Vol. 1, p. 283.

and endings†, and also enotative as regards the *number* of its members.

Both enotation and innotation are involved when the name "continuum" is given to any segment of the Grand Continuum.[40]

(II) According to Russell, mathematicians have distinguished different† degrees of continuity, and have confined the word "continuous" for technical purposes to series having a certain high degree of continuity.[41] We shall deal here with the continuity of numbers in series, and discuss the continuity of points of a line in Chapter XIX. Dantzig says that our intuitive† idea is that the rational number domain is more compact than the natural; the algebraic numbers are arranged in still denser formation; and finally the real number domain, the arithmetical continuum, is the ultra-dense medium, a medium without gaps, a network with mesh zero†.[42] For Keyser, the system of real numbers is the basal instance of the Grand Continuum, for other continua essentially like† it are derived from it as a model.[40] Russell says that real numbers possess the most complete† continuity known.[43]

There are said to be different orders of continuity. Thus according to Russell, since there are more segments of rational numbers than there are rational numbers, the series of segments has continuity of a higher order than the rationals.[44]

This assurance that there are more segments of rational numbers than rational numbers is open to question, in view of the fact that the number of rational numbers must be left enotative. We shall see another example of different orders of continuity in connection with the Dedekind cut, considered at v, below.

(III, IV) As ordinarily conceived, continuity is hardly associated with any activity of selection; but, according to Russell, the arithmetical continuum is an object selected by definition†, consisting of elements in virtue of the definition, and known to be embodied in at least one instance—the segments of the rational numbers.[45] If the axiom† of selection is made to apply not to the acts of a selector or manipulator, but to the processes characteristic of the numerical or other subsistent realms, we may envisage a series of numbers as itself operating selectively. According to Fraenkel, no one has yet succeeded in setting up by selection a well ordered aggregate in

[40] C. J. Keyser, *Mathematical Philosophy*, p. 394.
[41] B. Russell, *Scientific Method in Philosophy*, 1915, p. 132.
[42] T. Dantzig, *op. cit.*, p. 207.
[43] B. Russell, *Principles*, p. 193.
[44] *ibid.*, p. 272.
[45] *ibid.*, p. 347.

the continuum, although the existence of such an aggregate is demanded by the axiom of selection and seems plausible enough.[46] We find, however, the old difficulty; Fraenkel says that the axiom does not necessarily† carry with it the possibility† of actual† formulation of such an aggregate.

(IV) The axiom of continuity first proposed by Russell is as follows: If A and B be any two magnitudes of a kind, and A is greater than B, there is always a third magnitude C, such that A is greater than C, and C is greater than B. In stricter form, Russell's axiom is that there is a position p to which all† of the positions in a series approximate indefinitely. But we shall find, Russell says, that his axiom can† be denied† with perfect impunity.[47] Hobson admits that the legitimacy of regarding the continuum as forming a determinate whole†, or as constituting a single object of thought, is really a fundamental postulate, the validity of such postulation being subject to the law of contradiction†.[48] In fact, by defining an irrational number as a segment of rationals which does not have a limit†, analysis is able to dispense with any special axiom of continuity.[20] This changes the problem to that of limits, considered negatively†.

(V) The questions regarding boundaries which occur in problems of continuity are often stated in terms of limits†. We have considered some of these statements above, and found that the limits involve innotative references. These innotative references, for example, in the definitions of dense and perfect series, are so indefinite that any limit or boundary associated with them must also be left indefinite. We may say that any limit may function as a boundary, even though we are not able to locate the boundary precisely.

Boundaries do seem to be located in the so-called upper or lower bounds of the segments involved in the Dedekind cut, dividing all the numbers less than $\sqrt{2}$, for example, from all the numbers greater than $\sqrt{2}$. The initial difficulty is the universal "all"†, where the number of points involved is infinite.

Russell says there are four possibilities† in a Dedekind cut: (1) there may be a maximum to the lower section and a minimum to the upper, as in the case of the integers n, $n + 1$; (2) there may be a maximum to the one, but no minimum to the other, as in a series

[46] A. Fraenkel, *op. cit.*, p. 301.
[47] B. Russell, *Principles*, pp. 190, 294 *f.*
[48] See C. W. Morris, in *Jour. Phil.*, 26, 1929, p. 455.

of ratios, the lower up to and including 1, and the upper all ratios greater than 1; (3) there may be no maximum to the one but a minimum to the other, as in the case of all ratios less than 1, and then all ratios from 1 up and including 1; (4) there may be neither a maximum to the one nor a minimum to the other, as in the case of all ratios whose square is less than 2, and all ratios whose square is greater than 2.[49] Each of these possibilities results when a member or members of the number series are selected and the operation involving the questionable universal which we have indicated is performed upon the series with reference to that selected number. In the first case, that of the integers, if the series is to be called continuous, it must be noted that the continuity is arbitrarily restricted to exclude the fractions, etc., or to leave them innotative. In the second case, the ratios on both sides of 1 are left innotative, although the language conceals the fact, and with so much innotation on both sides it is arbitrary to call the number 1 a boundary, or to restrict the boundary conditions, whatever they are, to it. The same statement holds for the third case. In the fourth and most significant case, the innotative region is still more indefinite.

Other difficulties of the Dedekind cut are treated by distinctions between various series of numbers involved. These series may be of different orders of continuity, as in (11), above.[50] According to Russell, Dedekind's axiom should really mean that if from among the terms of a series some can† be chosen out to form a compact series, which is distributed throughout the previous series, and if this new series can always† be divided in Dedekind's manner into two portions, between which lies no† term in the new series, but one and only one term in the original series, then the original series is continuous in the Dedekindian sense of the word.[51]

But since a compact series is one between any two terms of which there are other terms, whose description must be left innotative and whose number must be left enotative, we can not say definitely that such a compact series can be chosen out, or, if it can, that any members of the original series will be left unchosen. According to Stebbing, the theory of types renders invalid the theorem that any† aggregate of numbers has an upper bound.[52]

[49] B. Russell, *Introduction,* p. 69.
[50] *cf.* C. J. Keyser, *op. cit.,* pp. 392 *f.*
[51] B. Russell, *Principles,* p. 279.
[52] L. S. Stebbing, *Modern Introduction to Logic,* p. 462, n. 1.

The uncertainty about boundaries appears also from the statement that the Dedekind cut, applied to a line, does not provide for the point marking the section.[53]

Whitehead says that in series of fractions, the absence of incommensurables leaves an absence of end†-points to certain classes. Hence in the series of fractions there is a quasi-gap where $\sqrt{2}$ ought to come. The possible† absence of limits† or maxima to a class of numbers which does not spread over the whole† series of numbers is no small evil. To avoid this, mathematicians have recourse to incommensurables, so as to obtain a complete† series with no gaps.[54] But in the case of the Dedekind cut, there are still questions about these incommensurables. "How do you know," asks Broad, "that $\sqrt{2}$ is the limit† of the series of rationals whose squares are less than 2? Roughly speaking, how do you know that there exists a number which the series continually approaches but never† reaches? We do not know it and can† not† prove it."[55] With these considerations in mind the concept of a boundary can hardly be used to define rational numbers. Rational numbers, says Russell, correspond to ratios. Taking the case (3) where the lower section of the Dedekind cut has no maximum, and calling this lower section (all ratios less than 1) a segment, those segments which correspond to ratios are those which consist of all† ratios less than the ratios they correspond to, which is their boundary. Those which represent irrationals are those which have no† boundary. But in a series which must be left innotative, we can not specify all ratios which are less than the ratios to which rational numbers correspond, and any boundary (except the number 1) to such a set is so arbitrary that we have no right to say whether a rational number corresponds to it or not, and hence whether, for such a reason, a given number is rational or irrational.

(VI) The appeal to a boundary is superseded by another procedure, where, instead of defining† $\sqrt{2}$ as the limit† of the series of rational numbers whose squares are less than 2, it is defined as this series itself. The series may be regarded as generated in a process analogous to growth. This emphasis on the series has one advantage: as Broad says, there is no doubt that† there is such a thing as $\sqrt{2}$, thus defined.[56] But of course the series is enotative or

[53] B. Russell, *Principles*, p. 279.

[54] A. N. Whitehead, *Introduction to Mathematics*, pp. 76 *f.*

[55] C. D. Broad, *op. cit.*, p. 41.

[56] *ibid.*, pp. 42 *f.*

innotative as regards some of its members, and enotative as regards the number of its members.

In work on continuity, the process of becoming or growth has been insisted upon by Brouwer. For him, in spite of the fact that it contains infinitely† many regularly determined points, the continuum as a whole is still only the medium or field for a process of free becoming. Individual points fall within it, but it is by no means an aggregate of fixed points.[57]

Sometimes the work on problems of continuity contrasts a process and its progressively resultant structure, all of which can be interpreted in terms of growth. There is a kind of "leapfrog" game, where stations once set up are passed, in a process which in turn is halted to furnish a new station. This appears, for example, in the difference between the continuity of a series and the continuity of a function in the neighborhood of a given argument, and in some of Russell's statements about compact series.[58]

The same principle is involved in the diagonal procedure. We may say that all† real numbers belong in a hierarchy; then show by the diagonal procedure that it is possible to exhibit other numbers which, while real, are not among those which have been enumerated.[59] It should be obvious here that if, as is legitimate, the process of growth is used to interpret continuity, it must be with ample enotative and innotative reference.

(IX) We have said repeatedly that continuity involves innotation. This appears in the statement that between any two real numbers an infinite† number of other real numbers can be inserted. The old difficulties reappear in the statement that a definition of continuity in terms of compactness is inadequate because of gaps in some series, such as series of ratios.[60]

The mathematician, says Dantzig, views the aggregate of rational numbers as a compact, continuous mass, seemingly without gaps. But this absence of gaps, especially when rational numbers are mapped on a line, is only a delusion; there are gaps infinite† in

[57] A. Fraenkel, *op. cit.*, p. 239.

[58] B. Russell, *Introduction*, pp. 104, 110; *cf. Principles*, p. 295.

[59] T. Dantzig, *op. cit.*, pp. 220 *f.* The matter is put in terms of goals rather than limits when Dantzig (p. 169) says that any number is conceived as the goal of an infinite† succession of jumps, and the continuum is regarded as comprising not only all† possible† resting stations but all possible goals as well.

[60] B. Russell, *Introduction*, p. 100.

number and in variety, each variety comprising an infinite number of irrational points.[61]

The continuity of a function, too, should be treated innotatively. Thus, according to Russell, the neighborhood of a number x will be all the numbers from $x - \varepsilon$ to $x + \varepsilon$, where ε is some number which in important cases will be very small. Continuity has to do with what happens in *any*† neighborhood of that point, however small. The function $f(x)$ is said to be "continuous" for the argument a if for every positive number, σ, different from zero but as small as we please, there exists a positive number ε different from zero, such that for all values of δ which are numerically less than ε the difference $f(a + \delta) - f(a)$ is numerically less than σ.[62]

[61] T. Dantzig, *op. cit.*, pp. 105-7.
[62] B. Russell, *Introduction*, pp. 109 *f.*

CHAPTER XVI

INFINITES

"Man darf aber trotzdem wohl der sicheren Hoffnung Raum geben, dass im Lauf von, wenn nicht Jahren, so doch Jahrzehnten, sich allmählich eine Klärung vollziehen wird, und dass sich—vielleicht im Zug einer gewissen Reform der Logik . . . —eine Losung herauskristallisiert, die jeder der beteiligten Schulen in einem gewissen Sinn Recht gibt."

—A. Fraenkel, *Einleitung in die Mengenlehre*, 1928, p. 386.

A. Nowhere is the application of the horizon principles more useful than in dealing with the problems of finites and those non-finites which are in common usage called infinites. In this chapter there is some emphasis on mathematical infinites, but the non-mathematical infinites are amenable to the same treatment.

B. (α) It is readily understandable that there should be much skepticism concerning infinites.

(β) Light on the problem of infinites is sometimes sought from psychology. In many primitive languages, when names for succession are exhausted, a name is suggested by and is applied to that circumstance. The word is a mark, either of the exhaustion of the system or of the fatigue of him who is using it.[1]

Bosanquet considers it most probable that the scientific use of the conception of infinity rests in every case on a neglect, justifiable because that which is neglected has a known nature and may be set down as insignificant either altogether or from the point of view of a specific purpose.[2]

(γ) In various ways problems of infinites are made to turn upon alleged relationships of mind and world. Thus, for Hilbert, the idea of the infinite is not the basis of mathematical thought, but is an idea in the Kantian sense, in that while passing beyond all† empirical experience, it still furnishes an ideal of mathematical completeness† and totality†.[3]

In the theory of aggregates, Fraenkel says that it is misleading to suppose that ideas such as aggregate and partial aggregate are in

[1] W. Parkhurst and W. J. Kingsland, Jr., in *Monist,* 37, 1927, pp. 145 *f.*

[2] B. Bosanquet, *Logic,* 1911, Vol. 1, p. 162.

[3] See C. W. Morris, in *Jour. Phil.,* 26, 1929, pp. 455 *f.*

some way objectively existent and to be discovered. The definition serves merely to fix the idea and simplify it. It is practically justified, and should be judged by its applicability, rather than any other test of validity.[4]

Sometimes refuge is sought in theories of the difference of intension and extension. Russell holds that we can not enumerate any infinite collection. At some point we must content ourselves with "and so on." Thus our knowledge in regard to all† such collections can only be derived from a definition by intension.[5] We shall see at D (1) that the Cantorian theories of the infinite are made to depend upon the difference between intension and extension and also are criticized on this ground.

(δ) In the face of formidable difficulties, appeal is made to intuitive and quasi-intuitive methods and convictions. Fraenkel says that the theory of aggregates proceeds from a kind of intuitively evident idea.[6] Hilbert, however, thinks that the infinite has no intuitive (*anschauliche*) meaning.[7]

In fields other than the mathematical, we are said to have intuitive knowledge of infinites. Thus beauty, according to A. C. Bradley, is the image of the total† presence of the Infinite within any limits† it may choose to assume.[8] Claims of intuitive knowledge of the infinites of theology are familiar.

C. (a) Categories of identity, sameness, difference, equivalence, etc., have been frequently employed in work on infinite collections. Ramsey says that in *Principia*, owing to the definition of identity† which is used, the axiom of† infinity means that there are an infinite number of distinguish*able*† individuals, which is an empirical proposition, since, even supposing there to be an infinity of individuals, logic can† not determine whether there is an infinity of which no two have all† their properties in common. We do not have to assume† that any particular set of things, for example atoms, is infinite, but merely that† there is some infinite type which we can take to be the type of individuals.[9] The "that-ness"† and indefiniteness of this assumption remove it safely from the connotative field, and from *Principia's* alleged attempted description in terms of identity and difference.

[4] A. Fraenkel, *Einleitung in die Mengenlehre*, p. 21.
[5] B. Russell, *Introduction*, p. 13.
[6] A. Fraenkel, *op. cit.*, p. 118.
[7] D. Hilbert, in *Die Naturwissenschaften*, 18, 1930, p. 960.
[8] A. C. Bradley, *Oxford Lectures on Poetry*, 1909, p. 62.
[9] F. P. Ramsey, *Foundations of Mathematics*, pp. 59, 61.

As regards equivalence of infinite aggregates, Bolzano was the first to occupy himself with it; Cantor drew the consequences.[10] Thus it is said that the aggregate of all† transcendent† numbers between any† two given real numbers is infinite and not count-*able*†, but rather is equivalent to the aggregate *c*. And it is said that the same holds for the aggregate of all transcendent numbers.[11]

To meet the difficulties various procedures have been proposed by Russell. He says that the assumptions† that up to and including *n*, no number is equal to its successor, will not be always† true if the axiom† of infinity is false. That $n = n + 1$ might be true for a finite number if *n* exceeded the total† number of individuals in the world.[12] Earlier, he had found it necessary, in the theory of extensive quantity, to assume† that aggregates, even when they are infinite, have what may be called magnitude of divis*ibility*†, and that two infinite aggregates may have the same number of terms without having the same magnitude of divisibility. Once more, to prove that all† the terms of one class† are correlated with all of another, we might think a complete† enumeration was necessary†; but we may† substitute "any† term" for "all† terms."[13]

According to Fraenkel, strictly speaking, in dealing with infinite aggregates, we do not need any definition of a cardinal number, but only of the relations of equality and inequality.[14] In an infinite collection, however, it is hard to keep either equivalence or inequivalence clear. Sheldon grasps both horns of the dilemma, arguing that the completed infinite is not contradictory†, at all, if once we grant that sameness and difference do not belie each other. The sameness runs undiminished through all† the infinite list of qualities, whatever their difference.[15]

(b) According to J. N. Keynes, "infinite" and "indefinite" are designations applied to negative names, interpreted as not involving restriction to a limited† universe† of discourse. Without such restriction, explicit or implicit, a negative name, for example "not-white" must be understood as denoting the whole† infinite or indefinite class of things of which "white" can not truly be affirmed,

[10] H. Hahn, in B. Bolzano, *Paradoxien des Unendlichen*, 1920 edition,, p. 140, n. to § 21. The Cantorian theories involve affirmative descriptions, and are discussed at D (1), below.

[11] A. Fraenkel, *op. cit.*, p. 54.

[12] B. Russell, *Introduction*, p. 138.

[13] B. Russell, *Principles*, pp. 141, 305.

[14] A. Fraenkel, *op. cit.*, p. 59. *cf.* p. 56.

[15] W. H. Sheldon, *Strife of Systems*, p. 462.

including such entities as a dream, time, etc.[16] According to our view, it is better to contrast to the finite the enotative non-finite, as a negative of suspension. The term "infinite," as contrasted with this "non-finite," carries the suggestion of a negative statement concerning an enotative region as in D (2), or, especially since Cantor, of an affirmative statement, as in D (1).

Sorley defines the infinity of God negatively†, in terms of the not limited†; but he thinks that this (negative) freedom from limitation is not therefore insignificant. We should say that its significance can not be affirmatively described. Sorley agrees that the problems of the infinite and absolute† here are problems of form rather than content; their solution does not supply us with a positive idea of God.[17]

(c) In the work on infinites, contradictions are often encountered. For example, counting zero as a number, there can not be a greatest ordinal number, since every ordinal is increased by the addition of 1. From this contradiction, Burali-Forti infers that of two different ordinals, as of two different cardinals, it is not necessary that one should be greater and the other less. In this, however, he consciously contradicts a theorem of Cantor which affirms the opposite.[18] Such contradictions are just what we should expect from attempts to describe infinites which must be left enotative.

(d) Sometimes statements about infinites are made to follow from inductions, or at least from statements about classes, especially universal classes. The question often concerns the proper use of the word "all." Bertrand Russell once wrote that "all" enables propositions of finite complexity to deal with infinite classes of terms. Infinite collections, owing to the notion of denoting, can be manipulated without introducing any concepts of infinite complexity.[19] But recourse to denotation here, even at best, means loss of connotation. Elsewhere Russell says that the notion of infinity is a property of classes, and only derivatively applicable to a series;[20] but, as we saw in B above, he takes these classes in intension rather than extension. Even with the notion of class to help with the problem of infinites, he finds that "any" is generally preferable to "all," when infinite classes are concerned.[21] This is because

[16] J. N. Keynes, *Studies and Exercises in Formal Logic*, 1906, p. 59.
[17] W. R. Sorley, *Moral Values and the Idea of God*, 1921, pp. 482, 486 *f*.
[18] B. Russell, *Principles*, p. 323. [19] *ibid.*, p. 73.
[20] *idem*, *Scientific Method in Philosophy*, 1915, p. 156.
[21] *idem*, *Principles*, p. 305; *cf.* p. 188.

"any" allows more to be left enotative or innotative. Ramsey virtually points to enotation or innotation in his recognition of inde-fin*able*† classes. He says that although an infinite indefinable class can† not be mentioned by itself, it is, nevertheless, involved in any† statement beginning "all classes" or "there is a class such that," and if indefinable classes are excluded† the meaning of all† such statements will be fundamentally altered.[22]

(f) Zero has been enlisted here in varying capacities. In considering the problem of infinity to be not a problem of quantity but of order†, Russell finds it convenient to ignore† the absolute† zero and to mean by any kind of magnitude all† the magnitudes of the kind except zero.[23] But, as we might expect, things work the other way, too. Instead of being ignored, zero may be counted among numbers, and may make it look as if the number of numbers is not finite. If n be any finite number, the number of numbers from 0 up to and including n is $n + 1$, whence it follows that n is not the number of numbers.[23]

G. H. Hardy declares that ∞ by itself means nothing, although phrases containing it sometimes mean something because of special definition†.[24]

(h) It is commonly understood that a so-called "sum to infinity," for example in the series $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \ldots$, is not a sum, but a limit. Whitehead says that we are here symbolizing a process of approximation to the limit of the summation of the series.[25]

In a similar problem, Russell says that the deduction of infinity is, when correctly performed, a mere fiction to facilitate compression in the statement of results obtained by the method of limits.[26] This is another way of recognizing the fact that unless one is to accept fictions, the infinite number of terms must be left enotative or innotative, and not given affirmative description.

(k) The concept of ending is sometimes used, in negative fashion, in work on problems of infinites. According to Parkhurst and Kingsland, the concept of unendingness originated in experiences limited† in themselves, but the succession of them has fathered a vague notion of the intermin*able*†.[27]

[22] F. P. Ramsey, *Foundations of Mathematics . . .*, p. 22.
[23] B. Russell, *Principles*, pp. 189, 357.
[24] G. H. Hardy, *A Course of Pure Mathematics*, 1925, p. 112.
[25] A. N. Whitehead, *Introduction to Mathematics*, pp. 197, 199.
[26] B. Russell, *Principles*, pp. 188 *f*.
[27] W. Parkhurst and W. J. Kingsland, Jr., in *Monist*, 36, 1926, pp. 531 *f*.

(l) According to Russell, the problem of infinity is not properly a quantitative problem, but one concerning order.[28] But order also must involve enotative reference.

(n) Problems concerning infinites are often considered in the light of possibilities, but in a series or aggregate it is necessary to distinguish between (1) members actually counted or denumerated, (2) members non-counted, etc., but regarded as countable or denumerable, and (3) members non-counted or denumerated, but regarded as not countable nor denumerable.

The horizon principles help us to distinguish these three cases. The members of the first are known denotatively, or connotatively; those of the second which involve the notion of possibility should be left enotative or innotative; and those of the third, which involve the notion of impossibility, are still more remotely enotative. Between the second and the third, no definite† boundary† can be drawn. They involve different sublevels of possibility.

The second case is the one covered by Uchenko, according to whom all that actuality exhibits, besides a meeting of finite entities, is an indefinite possibility of proceeding towards new finite entities. This, he says, is exactly the meaning of the indefinite or negative† infinite—*i.e.*, the possibility of advancing beyond any† assigned entity to other entities.[29]

Sometimes the issue is more puzzling. Fraenkel says that the explicit representation of the aggregate of all† transcendent numbers is altogether impossible.[30] We should say non-possible, leaving the possibility of explicit representation enotative, to match the enotation of the term "all." Again, every† denumerated aggregate is well ordered, but a denumerable aggregate does not need to be ordered.[31] We should amend this by substituting "non-denumerated" for "denumerable."

The third case is illustrated when Fraenkel declares that the aggregate c of all† infinite decimal fractions is not denumerable. This aggregate in comparison with the aggregate of all natural numbers, or even all algebraic numbers, is evidently so incomparably† more comprehensive that a counting off of its elements and comparison (*Abbildung*) with those of the aggregate of natural numbers will be impossible.[32]

[28] B. Russell, *Principles*, p. 189.
[29] A. Uchenko, in *Monist*, 40, 1930, p. 402.
[30] A. Fraenkel, *op. cit.*, p. 12.
[31] *ibid.*, p. 167.
[32] *ibid.*, p. 45.

The kind of procedure offered is called a "proof by impossibility." According to it, all†, infinitely many conceiv*able*† attempts at the construction of such a comparison necessarily† lead to failure.[33] But, according to our view, this is not a proof by impossibility, but an argument from non-possibility. For each term actually denumerated, we have to recognize an infinite number of other non-denumerated intermediate terms. But these intermediate terms have to be left innotative, and it is an open question what their possibilities are—*i.e.*, whether they are denumerable or not.

(q) Sometimes the attempt is made to grasp the infinite as a totality, or in terms of whole and part. According to Russell, in infinite wholes we are dealing with entities which would not be at all unless their constituents were. Thus there seems to be a special reason for completing the infinite regress in the case of infinite wholes, which does not exist where other asymmetrical transitive relations are concerned.[34] But this takes for granted that infinite "wholes" can be analyzed and the relationships of their parts exhibited, whereas they must always carry some enotative and innotative reference. Russell in fact admits that his argument is less conclusive than could be wished.[34] According to Uchenko, the possibility† of an indefinite advance is the only actuality† so far as infinity is concerned. There is no actual infinite totality; the latter is an illusion arising from the illegitimate confusion between "any"† actually assign*able*† number of an indefinite series and "all"† its members. The "all" is an imaginary entity.[35]

(r) Infinity and absoluteness, or the Infinite and the Absolute, are often associated, especially in the more theological parts of absolute idealism. The term "absolute," however, is as unmanage-ageably enotative as the term "infinite," and we gain nothing by translating from one to the other.

(s) It is said that the infinitists frequently turn to space as the court of appeals for their client.[36] But we shall see that space also carries its enotative reference.

(t) Infinity can hardly be clarified by citing the physical universe as an illustration; the statement that the universe is infinite is open to question.

[33] *ibid.*, pp. 49*f.*, 291.

[34] B. Russell, *Principles*, pp. 147 *f.*; *cf.* R. M. Eaton, *Symbolism and Truth*, p. 143 and n. 1.

[35] A. Uchenko, *op. cit.*, p. 396.

[36] W. Parkhurst and W. J. Kingsland, Jr., *op. cit.*, 37, 1927, p. 139, n. 71.

D. We have noted that when the enotative non-finite is called infinite, it brings the suggestion of unwarranted descriptions, sometimes affirmative and sometimes negative.

(1) Some writers are led to make affirmative statements in connection with estimates of probability†. Thus Bridgman says that in an infinite series of deals of a pack of cards, somewhere there will be deals in which the cards are distributed among the hands by suit and the arrangement in each hand is by rank.[37] But there is no necessity† of supposing that any given arrangement will turn up. Affirmative description of even a part† of an enotative series is subject to the limitations of the partial. The statement that an infinite includes the finite as a constituent, or appears in or through the finite, may be allowed if it is taken merely as a restatement of enotation, but its supposed definitive description of an infinite is, after all, only definitive.

Examples might be multiplied, but we consider only some which occur in the work of Dedekind, Cantor, and others who maintain that an infinite may be described as similar to a proper part† of itself. This statement comes to involve many others. The discussion has gone to such lengths that we find attempts to attack or defend the Cantorian view appealing to several of the methods and the horizon concepts which we have studied. The complexity of the subject makes it advisable to arrange the material somewhat as under B, C, and E of other chapters, using B′, etc., to indicate the subordination of this topic.

B′. (α′) Uncertainty regarding such infinites begins to appear in the statement of Russell, who, defining a reflexive class as one which is similar to a proper part of itself, says that all† reflexive classes and cardinals are infinite, but it is not known whether all infinite classes and cardinals are reflexive.[38]

(β′) Affirmative descriptions of the infinite have sometimes been grounded in psychology. Dedekind held that his own realm of thoughts, *i.e.*, the totality† S of all† things which can be objects of thought, is infinite. For if S signifies an element of S, then the thought S', that S can be an object of his thought, is itself an element of S.[39]

[37] P. W. Bridgman, in *Science*, 75, 1932, p. 422.

[38] B. Russell, *Introduction*, pp. 80, 88.

[39] R. Dedekind, *Essays on the Theory of Number*, transl. W. W. Beman, 1909, p. 64.

(γ') The Cantorian infinite is frequently made to involve distinctions between intension and extension. Russell says that a finite whole† may be defined by enumeration and also by intension. But infinite wholes may be defined only by intension. The latter definition, however, depends upon the notion of "all."[40] He says that the theorem that A and B are similar†, though A is part of B, can not be proved impossible†, for the impossibility could only be proved by enumerations, and there is no reason to suppose enumeration possible†.[41] It is equally true, we should say, that the enumeration need not be supposed impossible; we must leave the bounds of possibility open and enotative. Russell goes on to reiterate that as soon as we rid ourselves of the definition of whole and part by enumeration, the whole contradiction† vanishes.[41] The contradiction vanishes because Russell's definition without enumeration really masks an open enotative reference, where any contradiction can be concealed.

(δ') Fraenkel thinks that in the range of Cantor's definition of an aggregate, one will probably be inclined to see less a definition in the strict sense of the word, than an intuitive indication of an elementary act of logic.[42]

C′. (a′) The equivalence, or similarity, of whole† and proper part† is essential to Cantor's infinite. Difficulties with the Cantorian view appear when Bolzano says that the idea of the equality of two aggregates comes from dealing with finite aggregates, but that there is not the same† ground for it in the infinite aggregates, where there is no last term.[43] Again, Parkhurst and Kingsland argue that with the admitted failure of mathematical induction to apply to infinite classes, all hope of proving two such classes similar vanishes. For if no two infinite classes can be proved similar, they can not have a number, for the number of a class is the class of all classes similar to it.[44]

(b′) Uchenko maintains that the Cantorian definition in its appeal to endlessness still tacitly involves the old negative definition of infinity.[45] For Parkhurst and Kingsland, those consequences of the theory of transfinite numbers which are of actual value to mathematics appear to be the negative consequences—namely those

[40] B. Russell, *Principles*, p. 113.
[41] *ibid.*, pp. 360 *f.* *cf.* A. Uchenko, *op. cit.*, pp. 396, 408 *ff.*
[42] A. Fraenkel, *op. cit.*, p. 13.
[43] B. Bolzano, *op. cit.*, pp. 31 *f.*
[44] W. Parkhurst and W. J. Kingsland, Jr., *op. cit.*, 35, 1925, p. 663. *cf.* A. Uchenko, *op. cit.*, pp. 399 *f.*
[45] *ibid.*, pp. 395 *f.*, 401.

asserting that certain processes can† not be applied to the infinite.[46] With regard to the latter statement, we should say that the negatives of denial are too strong. Whether or not certain processes can be applied to the infinite must be left open, not decided.

(c′) Russell says that Cantor showed that the supposed contradictions of the infinite all depend on extending to the infinite results which, while they can be proved concerning finite numbers, are in no sense necessarily† true of all† numbers.[47] But, according to our view, the Cantorian freedom from contradiction is purchased at the cost of an initial confusion, when the attempt is made to describe an enotative infinite in connotative terms.

(d′) The old difficulties concerning the class of all classes reappear. Dedekind in his argument that infinite aggregates can be put in one-one correspondence with a subset uses the set of all things, and is thus open to the objection of Burali-Forti's paradox.[48]

(i′) It is not difficult to involve Cantor's infinite in difficulties concerning continuity. Uchenko points out that infinity has no immediate predecessor, because there is no greatest finite number. Hence infinity is free from mathematical induction, but then can not have one-one correspondence.[49] This criticism, however, contains negative statements which are as unwarranted as Cantor's affirmative statements.

(i′, j′) Russell says that Achilles may be the victor over the tortoise if it be held that the paths and the times are continuous series, and so contain an infinite number of elements, of which a part is similar to the whole.[50] But it might be replied that one can not specify *what* part is similar to the whole, and if every part is similar, there is no point in starting the race at all, or, at least, when it is once started, of continuing it.

(n′) Cantor's work and criticisms of it involve considerations of possibilities. According to Poincaré, there is no actual infinite. The Cantorians forgot this and so fell into contradictions†.[51] In other words, infinite series are to be left descriptively enotative, with possibilities as to various correlations left open.

(q′) We noted that the concepts of whole and part are essential for Cantor. For Dedekind, an aggregate M is infinite or transfinite,

[46] W. Parkhurst and W. J. Kingsland, Jr., *op. cit.*, p. 659.
[47] B. Russell, *Principles*, p. 304.
[48] J. Pierpont, in *Am. Math. Soc., Bulletin*, 34, 1928, p. 42.
[49] A. Uchenko, *op. cit.*, p. 406.
[50] See C. W. Morris, in *Jour. Phil.*, 26, 1929, p. 453.
[51] H. Poincaré, *Science and Method*, transl. F. Maitland, p. 195.

if there is a proper part-aggregate of *M* which is equivalent† to *M*. If there is no proper part-aggregate of *M* equivalent to *M*, then *M* is regarded as a finite aggregate.[52]

In order to interpret his absolutist universe, Royce adapted the Cantorian infinite in his "self-representing systems"—for example, a map of England in England[53]—and held that we can show an actual† infinity in one sense of the word, in the character of a whole with parts which embody and reproduce the spirit of the whole.[54] Such a "whole" is, of course, like the part said to represent it, an isolated system, deprived of its enotative reference.

(s′) In criticism of Russell's solution of the problem of Achilles and the tortoise, Morris points out that the *spatial* proper part† is never† equal† to the whole†.[55]

An application in terms of time rather than space is also offered by Russell. If it takes Tristram Shandy a year to write down the events of a day of his biography, and we give him time enough, it may be said that no part of his biography will be unwritten. Morris objects to the obverse, "All† parts of the biography will be written" as an illegitimate totality†, since it is expressly said that the series of days and years has no last term.[56] But the original negative statement also involves an illegitimate totality. All that one can say is that there is no end assigned to the years in each of which Tristram Shandy writes the events of a day of his life.

E′. (1′) So the Cantorian theory, when critically examined, is seen itself to involve enotative and innotative references. In this connection we may note Fraenkel's remark that recent work on the foundation for the theory of aggregates is frankly more or less at the expense of the outmost reach of Cantor's definitions, and in part of their importance.[57]

(2) Returning from our examination of the Cantorian infinite to our main discussion, we note again that negatives of denial con-

[52] A. Fraenkel, *op. cit.*, pp. 23-5. Fraenkel maintains that here the infinite is a primary and positive idea, while the other is negative. But an idea does not become primary by being mentioned first in a paragraph, nor in any true sense positive by an affirmative statement about a term which should be left enotative. To speak of an infinite first is like the inversion of perceived figure and ground; if enotation is avoided, innotation is encountered.

[53] J. Royce, *The World and the Individual,* Vol. 1, 1900, p. 504.

[54] B. Bosanquet, *Logic,* 1911, Vol. 1, pp. 164 *f.*

[55] C. W. Morris, in *Jour. Phil.,* 26, 1929, p. 452.

[56] *ibid.*, p. 453.

[57] A. Fraenkel, *op. cit.*, p. 219.

cerning infinites are as much to be avoided as are affirmative descriptions. Here are two more examples: According to Russell, methods of thinning out a progression do not diminish the number of terms. Conversely, we can add terms to the inductive numbers without increasing their number.[58] Consecutiveness does not exist among infinite integers, since these are unchanged by the addition of 1.[59]

E. (1) It should appear from what has been said above that work on finites and infinites involves enotation. In Whitehead's phrase, the "infinitude of irrelevance" is kept out of thought.[60] Even a relevant infinite can not be brought wholly within it.

(11) Russell says that two divis*ibilities*† of which one is infinitesimal with regard to the other are regarded usually as different kinds of magnitude. But on the problem of orders of the infinity and infinitesimality of functions, the greatest authorities are divided.[61]

Associated with the name of Cantor there has been much work on various orders of infinite or "transfinite" numbers. In this respect the familiar series of inductive numbers is said to be an instance of the smallest of infinites.[62]

According to Russell, domains of progressions form a cardinal number, since every class† similar to the domain of a progression is easily shown to be itself the domain of a progression. This cardinal is the smallest of infinite cardinals, $\aleph_0$.[63] Here there is dependence upon virtual closing of the series, indicated by the words "every" and "class."[64]

According to Cantor's definition by mathematical induction, $\aleph_0$ is the number of any† class† *u*, which is the domain of a one-one

[58] B. Russell, *Introduction*, p. 84.

[59] *idem*, *Principles*, p. 150. First negative and then affirmative treatments of infinites are evident in the two definitions of infinites on p. 260. Both should be left enotative.

[60] A. N. Whitehead, *Concept of Nature*, p. 12. A suggestion of this may be had from grammar, where an infinitive indicates something *to* experience, to enter, etc., but something as yet "in-finished." Another hint may be had from infinite judgments (B. Bosanquet, *Logic*, 1911, Vol. 1, p. 282), where denial, though true enough, is unmeaning, as in "Virtue is not square."

[61] B. Russell, *Principles*, pp. 333-6.

[62] *idem*, *Introduction*, p. 28.

[63] *ibid.*, p. 83.

[64] *cf.* B. Russell, *Scientific Method in Philosophy*, 1915, pp. 180 *f.*; W. Parkhurst and W. J. Kingsland, Jr., *op. cit.*, pp. 643, 645, 651. It appears from p. 651 that the difficulty is traceable to what we have called the leapfrog procedure.

relation R whose converse domain is contained in but not coextensive with u, and which is such that, calling the term to which x has the relation R the successor of x, if S be any class to which belongs a term of u which is not a successor of any other term of u, and to which belongs the successor of every† term of u which belongs to S, then every† term of u belongs to S.[65] In other words, we have three essential statements: (1) S contains a term of u which is not a successor of any other term of u; (2) S contains the successor of every term of u which belongs to S; and (3) every term of u belongs to S. But the first statement shows that S is innotative as regards terms of u; the second shows that S is enotative as regards terms of u; and the third (which with its "every term of u" virtually closes the series which must be left open) is altogether too definite to rest upon any such uncertain basis.

Russell once assumes the axiom† of infinity, which we consider at IV, below, because as he says we need the whole† class† of inductive cardinals to establish the existence of $\aleph_0$.[66] But even the axiom of infinity does not allow for the completed or closed infinite class.

Attempts to exhibit the numerical properties of transfinite "numbers" have shown just such bizarre results as we might expect. For instance, in the case of infinite cardinals, addition and multiplication are always possible†, but subtraction and division no longer give definite results. From the ambiguity of subtraction and division it results that negative† numbers and ratios can not be extended to infinite numbers.[67]

There is an imposing theorem that the number of the continuum† is $2^{\aleph_0}$. This, according to Russell, results from the proposition that infinite classes of finite integers form a continuous series,[68] which, we might add, involves an affirmative statement about an infinite. Russell goes on to say that the number of all† classes of finite integers is $2^{\aleph_0}$, and the number of finite classes is $\aleph_0$. Hence the number of all infinite classes of finite integers is $2^{\aleph_0}$, for the subtraction of $\aleph_0$ does not diminish any number greater than $\aleph_0$. $2^{\aleph_0}$ is therefore the number of the continuum.[68] But here are both affirmative and negative statements, which disregard the enotative reference proper to an infinite. No wonder, then, that it is not known whether $2^{\aleph_0}$ is equal† to any of the cardinals in the series of Alephs, nor even whether it is comparable with them in magnitude.[69]

[65] B. Russell, *Principles*, p. 122.
[66] *idem, Introduction*, p. 134.
[67] *ibid.*, p. 87.
[68] *idem, Principles*, p. 311, n.
[69] *idem, Introduction*, p. 92.

The logical function which gives the number ω Cantor calls the second principle of generation of real integers. He defines it more closely as follows: If there is defined any definite† succession of real integers of which there is not† greatest, on the basis of this second principle a new number is created, which is defined as the next greatest number to them all†.[70] But the new entity, ω, the smallest of infinite serial numbers, is formed from an infinite "class" of real integers and defined with reference to them "all." The fact that there is no greatest number in the series suggests open enotation; but the word "all" indicates the virtual closing of a series, regarding it as a single whole.

According to Parkhurst and Kingsland, Cantor eludes the difficulty by an ambiguity. He will not admit that the new non-finite number is to follow the greatest finite number. To avoid this, recourse is had to the ambiguous phrase "first after all† the finite numbers." The phrase chosen can mean nothing†, exactly, and contradiction† is apparently but speciously avoided.[71]

Again, Cantor says that in the series 1, 2, 3, . . . ν, ω may be regarded as the limit† toward which the variable ν tends.[72] But the concept of limits, as we saw, involves enotation and innotation.

Russell says that we may take 1, 2, 3, . . . ν, an infinite series, and get a new number, ω, to express the fact that the whole† collection is given by its law in its natural order† of succession. ω is merely the name of the class† progression or of the generating relations of series of this class. It is the class of all† such transitive generators of progressions.[73] But if there is any law to be invoked, it either is or includes enotative reference. And ω, especially if taken as merely a name, can not be more than denotative in its reference to "all" transitive generators of progressions.

If, with Cantor and Russell, we take ω as a number, we may go on to get $\omega + 1$, $\omega + 2$, . . . $\omega + \omega$ ($= 2\omega$).[73] With ω and 2ω, we may first after all positive numbers ν, take $\omega + \nu$, then 3ω, $3\omega + \nu$, $\mu\omega + \nu$, ω^2, and so on indefinitely.[74] Fraenkel carries the series to ω^ω and beyond; there are an infinite number of transfinite num-

[70] P. E. B. Jourdain, in G. Cantor, *Contributions to the Theory of Transfinite Numbers*, 1915, p. 57.

[71] W. Parkhurst and W. J. Kingsland, Jr., *op. cit.*, pp. 648, 650 *f.*

[72] P. E. B. Jourdain, in G. Cantor, *op. cit.*, p. 77

[73] B. Russell, *Principles*, p. 313; *Introduction*, p. 83.

[74] W. Parkhurst and W. J. Kingsland, Jr., *op. cit.*, p. 651.

bers.[75] But all this is built upon the fundamental uncertainties and, as might be expected, difficulties appear. It is, for instance, proved that $\omega + 1 = \omega$. If this is true, it necessarily invalidates the process by which transfinite numbers of higher orders are reached.[74]

Fraenkel says that Cantor and his successors had misgivings about applying the honored name of numbers to mathematical objects which in respect to their magnitude seemed not to follow a rank order without exception.[76]

There are fresh constructions and fresh difficulties with respect to infinite ordinal numbers.[77]

The upshot is that the theory of transfinite numbers is not constructed upon secure foundations. The supposed foundations rest upon too many enotative and innotative references. What is said about such numbers may be true, but it also may not be true. And this question simply must be left open, enotative. Statements about different powers, levels, sublevels, etc., of transfinite numbers should be understood in this light. In such statements, enotations and innotations are simply piled upon other enotations and innotations.

(III) The "equations" of transfinite arithmetic do not look very imposing after one examines their foundations, but a better case for selective interactions of infinite numbers may be made out among the known members of any infinite series. Since the elements which are said to belong to certain infinite series are sometimes represented as progressively selected, we may, in accordance with our views of subsistents, think of an infinite series of numbers as interacting selectively with its milieu. Fraenkel, as we might expect, finds a number of problems connected with the principle of selection which are as yet unsolved. For instance, he says that one has the impression that a successively increasing length of time is demanded for the act of selection, and that probably the timeless† character to be attributed to all mathematical conclusions and operations is not taken into account.[78] This is an attempt to answer the problem of infinity in terms of timelessness, but both terms involve enotation. For all the time taken by any process of selection, by or from an infinite, future time is still open.

Selection is important in the diagonal procedure, where we are bidden to imagine a single, altogether arbitrary element selected,

[75] A. Fraenkel, *op. cit.*, pp. 189*f.* [76] *ibid.*, p. 195.
[77] See B. Russell, *Principles*, p. 314; A. Fraenkel, *op. cit.*, pp. 166, 190, 192*f.*
[78] A. Fraenkel, *op. cit.*, p. 197.

but "from now on fixed and designated" as the distinguished element of the aggregate in question. Moreover, it is sufficient if we can think of the selection as possible†, and as in any way accomplished.[78] But a given selection, whenever it is *accomplished,* marks the change from an enotative possibility to a denotative or connotative term in a denumer*ated* or finite series.

(IV) In dealing with infinites, especially in mathematics and physics, the procedure frequently is on a basis of assumptions, restrictions, postulates, or axioms.

The so-called axiom of infinity assumes the existence† of infinite collections. Russell admits that although various ways suggest themselves by which we might hope to prove this axiom, there is reason to fear that they are all fallacious and that there is no conclusive logical reason for believing it to be true†. The axiom of infinity will be true in some possible† worlds, false† in others. What is true or false in this world we can not tell. But, as he says elsewhere, there is certainly no logical reason against infinite collections, and we are therefore justified in logic in investigating the hypothesis that there are such collections.[79]

In *Principia's* calculus of propositional functions of one variable, Lewis says that we may assume that any law of the algebra which holds, whatever finite number of elements be involved, holds for any number of elements whatever. In a note, Lewis adds that this procedure, though not invalid, is far from ideal, as are many other details of this general method. But it is a fact that in spite of the many defects of the method, the results which it gives are without exception valid.[80] We would note that the exceptions where it is not valid are ruled out by preemption.

Dependence upon axioms, some of which are used in connection with infinites, has been of fundamental importance in the work of Hilbert and his followers, sometimes called formalists. Hilbert, insisting that no one shall drive us out of the paradise which Cantor has created,[81] says that he wishes to restore to mathematics its old reputation for unassailable truth which it seems to have lost in the paradoxes of the theory of aggregates.[82] The present state of affairs is intolerable, but there is an entirely satisfactory way to escape the paradoxes. To reach this, Hilbert takes symbolic logic, but reinter-

[79] B. Russell, *Introduction,* pp. 77, 141.
[80] C. I. Lewis, *Survey,* 235 *f.,* and 236, n. 7. [81] J. Pierpont, *op. cit.,* p. 48.
[82] D. Hilbert, in Hamburg Universität, *Abhandlungen aus dem Mathematischen Seminar,* 1922, p. 160.

prets it.[83] The foundation of mathematical reasoning is discovered in some immediate knowledge of concrete marks.[84] Signs and symbols of operations are freed from their significance with respect to content. The axioms of mathematics merely express the rules by which formulas follow one another.[83] Thus he can demand that we consider the formal structure of all propositions in such a way as to see that there is no contradiction† and to gain the insight† that contradictions can not occur.[85] He regards the mathematical infinite as arising from the "ideal elements" in mathematics, which are not capable of concrete representation but which are generated from the definitions or postulates of the formal phase of mathematical deduction.[86]

Operations on the infinite can be rendered certain† by an appeal to the finite. So the first axioms laid down by Hilbert relate to the finite. For them the laws of ordinary logic hold. Their freedom from contradiction† is intuitive†.[87] Hilbert maintains that when we consider the "aggregate of real numbers," we must not think of the totality of all possible† laws, according to which the elements of a fundamental series can progress, but rather think of a system† of things whose reciprocal relations are given through a finite and closed system of axioms, and concerning which new propositions are valid only if one can derive them from those axioms by means of a finite number of logical conclusions.[88] Thus a system develops with freedom from contradiction as the characteristic of its processes and the guarantee of its validity. Aggregates which have led to antinomies are excluded, and in such a general sense that even the occurrence of further contradictions within the new structure will be thoroughly improbable†.[89]

For us, this work of Hilbert involves no essential principles not already considered. His reduction of the data of mathematics to marks on paper, etc., confuses symbols and entities symbolized, but is of more importance for problems of realism than for relativism. Symbols and things symbolized, for us, follow the same laws of selection. His selection is axiomatic and involves arbitrary choice of some axioms. Weyl says that the formalist who abides by his prin-

[83] J. Pierpont, *op. cit.*, p. 49. [84] E. Nagel, in *Jour. Phil.*, 26, 1929, p. 481.

[85] H. Weyl, in Rice Institute, *Pamphlet*, 16, 1929, p. 250.

[86] C. W. Morris, *op. cit.*, p. 455. [87] J. Pierpont, *op. cit.*, pp. 48 *f.*

[88] D. Hilbert, in Deutscher Mathematiker Vereinigung, *Jahresbericht*, 8, 1900, p. 184.

[89] A. Fraenkel, *op. cit.*, p. 387.

ciples must leave the question unanswered, why he chooses just these axioms for the starting-point of his proof game.[90] In criticism of Hilbert, Brouwer maintains that it has not been shown that, if even a finite number must satisfy a set of conditions which can be shown to be non-contradictory, then this number actually exists,[91] *i.e.*, is actually constructed or construct*ible*† in thought. Moreover, for Brouwer, even if an axiomatic treatment avoids contradictions, nothing† of mathematical value will be obtained in this manner. A false theory which is not halted by a contradiction is none the less false.[92] We should put it the other way, and say that open freedom from contradiction does not reveal either truth† or falsity in a theory. We should say that any one can make such choices as Hilbert's, dignify them as axioms, and especially in an abstract system, follow them indefinitely without encountering contradiction. Everything contradictory has simply been relegated to the enotative at the start, and it is impossible to say whether an indefinite continuation of a process which has been left with enotative reference encounters the once excluded contradictions out in the enotative or not. "Freedom from contradictions" means that one alternative is accepted rather than another, and even then often asserted as think*able,* or denumer*able,* rather than as definitely thought out or actually† denumerated. Such freedom involves processes analogous to growth, considered in VI, below.

(V) The question of boundaries in infinite series is also best considered in connection with some processes analogous to growth, and with the work of Brouwer and others, to be taken up in the next section.

(VI) Opinions are divided as to whether infinite magnitudes admit of a process of growth. Some, for example, say that the totality† of natural numbers is to be regarded as actually a fixed isolated whole†; others regard it as an open process, continually† becoming.[93] In accordance with the latter view, the value of a variable is often said to be infinite when it is greater than any finite number that can be named—*i.e.*, when it has no maximum.[94]

Sometimes, as we saw, the enotative reference is concealed in an incomplete statement about the law of formation of a series. In this

[90] H. Weyl, *op. cit.*, p. 253.
[91] A. Dresden, in Am. Math. Soc., *Bulletin,* 30, 1924, p. 37.
[92] J. Pierpont, *op. cit.*, p. 52.
[93] A. Fraenkel, *op. cit.*, p. 222.
[94] T. P. Nunn, *Teaching of Algebra,* p. 408.

connection, we note that Sellars regards the transfinite numbers as an operational program.[95]

Sometimes the enotative reference is phrased in terms of possibility.† When an infinite is defined negatively, the situation is essentially indefinite. It indicates a possibility of advance. It does not indicate a numerical series as a totality†, but is, as it were, a series in the making, accompanied by the conviction that it will never† be made.[96] But for us the conviction is rather that no matter what is done, there will still be enotative regions open, where all results are in question.

Again, the enotative reference may lead to negatives† of denial, as when Russell says that mathematical induction applies only to finite systems.[97] Less definitely negative, and therefore more adequately enotative, is the statement of Bolzano, that a magnitude which may become greater than any† assigned finite magnitude can, as likely as not, permanently† remain a merely finite magnitude.[98]

Elements of new growth or development may accrue out of regions left innotative. Fraenkel says that the new development or the self-development of further elements is never to be regarded as out of the question, because an infinite aggregate by its very nature brings with it the continually† unfinished and ever further extens*ible*†.[99]

In an infinite series, the process of depletion is as indeterminate as that of growth.[100]

It is possible, but not necessary, to think of such a process as progress toward a limit†. Weierstrass and Cantor substituted for an irrational number one of its rational approximations—or, what is the same, replaced the limit of an infinite sequence by an advanced term of the sequence.[101]

There may be proximate boundaries, stations, or turning points, as the "leapfrog procedure" shows. According to Parkhurst and Kingsland, in the current theories about infinites, the difficulty is that the word for process has been transferred to the product, from the act of performing to the performance. So we get such short-

[95] R. W. Sellars, *Philosophy of Physical Realism*, p. 344.

[96] A. Uchenko, *op. cit.*, pp. 394 *f.*

[97] B. Russell, *Introduction*, pp. 27 *f.*

[98] B. Bolzano, *op. cit.*, p. 8.

[99] A. Fraenkel, *op. cit.*, pp. 329, 331; *cf.* p. 356.

[100] *ibid.*, pp. 195 *f.*

[101] T. Dantzig, *Number, the Language of Science*, pp. 153 *f.*

sighted statements as that there must be an infinitely great number, since we could† go on enumerating forever.[102]

Since in the work of Brouwer and the intuitionists much emphasis is laid on the nature of a series of numbers as something continually becoming,[103] the Brouwer intuitionism may be considered here. His views on the nature of mathematics,[104] the place of intuition in knowledge, and the function of language[105] are not so important for us, except insofar as he holds that paradoxes arise when regularity in the language which accompanies mathematics is extended so as to apply to mathematical words which do not accompany mathematics.[106]

In particular, the trouble in mathematics begins, when the sequence of natural numbers is declared to be a closed complete† aggregate of elements existing† in themselves. Such a sequence should be conceived, not as closed, but as open to infinity. Certain questions then arise as to whether such a set exists; the answers turn upon various uses of the concept of possibility, [107] sometimes half-concealed in the word "construct*ible*"†.

Brouwer holds that any attempt to establish generalizations about a totality† by means of complete† induction† must after all be limited† to subsets, and must leave outside the subset of the so-called totality an unfinished residue, where one may recognize possibilities which can not be treated definitely enough to yield precise results. He says that the totality of the mathematical propositions and contradictions† deriv*able*† by means of complete induction forms a "denumer*ably*† unfinished set"—*i.e.,* a set of which nothing† but a denumer*able* subset can† ever† be explicitly exhibited, and such that whenever a denumer*able* subset is given, a new element of the set can always† be derived from it by means of a previously defined process.[108] He would be somewhat clearer if he distinguished between a denumer*able* and a denumer*ated* subset, but at any rate he sees that it is uncertain whether for an arbitrary proposition concerning a given infinite system either the "construction" or the "obstruction" can be established.[108] This is what we

[102] W. Parkhurst and W. J. Kingsland, Jr., in *Monist,* 36, 1926, p. 533.
[103] A. Fraenkel, *op. cit.,* p. 233.
[104] See A. Dresden, *op. cit.,* pp. 32-8; E. Nagel, *op. cit.,* p. 481; A. Fraenkel, *op. cit.,* pp. 226-8.
[105] A. Dresden, *op. cit.,* pp. 32 *f.,* 35.
[106] *ibid.,* pp. 32, 38, 51.
[107] H. Weyl, *op. cit.,* pp. 245, 249.
[108] A. Dresden, *op. cit.,* pp. 39 *f.*

express by saying that the possibility must be left open, enotative. According to Brouwer, the unjustified assumption† that one or the other must† be possible† can never be detected; for that would mean that both the hypothesis of construction and that of obstruction would lead to an obstruction in the further process of construction, which conflicts with the law of contradiction.[108]

So, for Brouwer, any enumeration of all† numbers as a genuinely infinite process is neither cap*able*† of being carried out in practice nor think*able*† as having been accomplished. It is therefore foolish to speak of the result of a process which can not be completed.[109] In our terminology, this amounts to saying that if an assumption concerning the enotative could be detected as unjustified, that would mean a negative of denial concerning the enotative, so that no further descriptive statements, affirmative or negative, could be made about any further enotative. The open possibility with respect to *non-A* would be closed. The only workable possibility in dealing with an infinite sequence, for Brouwer, must be that of "running through the totality"† by means of a law which reduces the infinite process to a finite process—in other words, a law which somehow applies to the infinite sequence without taking the series as completed.

The principle or law which does this is declared to be conceptual constructibility (*gedankliche Konstruirbarkeit*).[109] Brouwer proposes, instead of *a posteriori* description of what is given, a kind of conceptual *a priori* construction of the possible running through of such a sequence.[110] Just as Hilbert repulses antinomies with axioms, so Brouwer holds that paradoxes disappear if one restricts one's self to systems explicitly construct*ible* on the basis of the fundamental intuition.[111]

For Brouwer, conceptual constructibility, the criterion of mathematical existence, has great importance for traditional logic. Suppose we say "All men are mortal." This implies that if any men exist, every collection of them is free from immortals. But existence†, for Brouwer, is conceptual construct*ibility;* he would have to say that the proposition means that every† conceptually constructible class "men" is free from immortals. But how about "No men are mortal"? To construct the subject-term, and say "(there are) no men" would mean, for Brouwer, to examine an infinite number of objects and deny that each is a man. And, since this

[109] A. Fraenkel, *op. cit.*, pp. 226-8, 230. [110] *cf.* H. Weyl, *op. cit.*, pp. 248 *f.*
[111] A. Dresden, *op. cit.*, p. 38.

could not be done, "no men are mortal" is not constructible, and need not imply anything about existence; it need not imply that there are any men at all. This non-constructibility and for Brouwer non-existence of a class denoted by a negative† has a bearing upon contradiction†. The negation of a universal† must be so interpreted as not to require the existence of any members of the class, such as men, even though the universal affirmative here does imply that if any men exist, every collection of them must be free from immortals. This robs the contradictory negative of its fixed relation to the affirmative. If the existential import of propositions is so interpreted that "there is" is to mean exhibition or construction, no inference is legitimate from a universal negative proposition which does not imply such existence, to a proposition which does.

Suppose, for example, that we take as the meaning of certain property P of an integer n, greater than 16, the following: "satisfying the relation $2^n + 1$ is prime," and ask, Are there numbers n such that n has the property P? If we can find such a number, we answer yes. If we find proof that P for an n leads to contradiction, no. If we succeed in doing neither, we ordinarily affirm nevertheless that, although at the time we can not decide between them, these alternatives are significant and exhaustive, and that, in some sense of "are," there either are or are not such numbers. Brouwer believes that we must recognize here a third alternative—*i.e.*, that no genuine disjunction is involved, and that the questions must be left open, because, prior to the actual† exhibition of a number having P, the class of such numbers may, as in the case of "no men," be the null†class† for which predication is nonsense. But only enumeration of all† the integers could decide such a question, and since an infinite series can not be examined *seriatim,* the principle of excluded middle must be rejected in the case of a series where no rule is given whereby the examination could be accomplished in a finite number of steps.[112]

Here the appeal is not so much to possibility as to the null class and to "all the integers" in an infinite series. But in any case the question is left open, in what we call a negative of suspension. This gives Brouwer's work resemblance to the principles of enotation and innotation. According to our view, Brouwer's work is an important confirmation of these principles, although at several points he does not sufficiently distinguish the various enotations involved.

[112] See E. Nagel, *op. cit.,* pp. 486 *f.*

He fails, first, to distinguish with sufficient clearness between constructibility and construction. This is like the confusion elsewhere noted between denumer*able* and denumer*ated* sets. Even when both are confined merely to conceptual processes, there is a difference between a construction which, in thought, is actually completed, and a construction which is merely conceived as construct*ible,* or possible.

There may be further confusion between such conceptual constructions, whether actual or possible, in thought, and overt demonstrations or exhibitions in words or other symbols. But overt demonstrations are, for Brouwer, subsidiary; for example, any verbal structures which accompany mathematical construction may be used temporarily to validate parts of it.[118]

Brouwer, quite justifiably, leaves his infinites with open possibilities. But in the interpretation of the open possibilities, he gets away from the negative of suspension into what we take to be a faulty analysis of universal negative propositions. "No men are mortal," he holds, would have to involve examination of an infinite number of objects if it were to imply non-existence of a class "men." But we should say that "no men are mortal" does not mean "no men are." "No men are mortal" involves as much examination of the class "men," and we might add, no more, than does "all men are mortal." Both stop short of examination of an infinite number of objects, and both make statements as regards the appropriation or rejection of certain predicates. The difference between them is not the difference of construction and constructibility; both are constructions in the midst of further constructibility left enotative. The difference between them is that one marks appropriation of a predicate throughout a class, and the other marks the rejection of the predicate into the enotative. We would agree with Brouwer that the laws of contradiction and excluded middle are not as rigid as the traditional logic makes them. But we would not need to reach this statement by way of distinctions between finites and infinites, nor to substantiate it by Brouwer's treatment of a universal negative. We would reach it rather, as in Chapter IX, by drawing consequences from the general principles of enotation and innotation, which go to show the place of the negative of suspension and to exhibit each of the two traditional laws as in important cases exclusive but not exhaustive.

[118] *cf.* A. Dresden, *op. cit.,* pp. 35, 39.

The interpretation in terms of truth-value[113] should be modified in the same way, by adding a non-committal intermediate value. The enotative is, further, interpreted by Brouwer in terms of time†,[114] and by others in terms of the limitations† of our knowledge.[115] This is not unusual, but does not reduce the enotative to description nor solve Brouwer's main problem.

(IX) In connection with infinites, we find the principle of innotation of considerable help in various problems of relations, real numbers, intervals, aggregates, and sets of points. With regard to relations, Russell concludes that since the expression $A\ R\ B$ does not include in its meaning any relation of A or B to R, the endless regress, though undeniable, is logically quite harmless.[116] We should say that it is harmless insofar as it can be left innotative.

Whitehead says that in the calculus the problem is how to keep an interval of length over which to calculate the average increase and at the same time to treat the interval as though it were zero†. He might better say, as though it were innotative. He avoids the difficulty of the infinitely small by emphasizing the importance of the notion of the vari*able*†. We should say that the variable can be pictured as appearing out of the innotative, so that, as he says, corresponding to any standard of approximation, some interval with such and such properties can be found.[117] Innotation explains the paradoxes of the "diagonal procedure," which for Fraenkel has in the theory of aggregates a specially important place on account of its simplicity and transparency. He says that it is theoretically simple, but practically it can not be carried out for transcendent numbers, on account of the time† involved.[118] For us, not merely are the transcendent numbers which can be inserted enotative in their number and innotative as regards their continuity; the instants of time involved are enotative and innotative in the same ways.

Innotation explains the paradoxes of the density of an aggregate of points. An aggregate of points N is said to be dense, when between every† two distinct points of N there is always another point of N—that is, when no point of N has a neighboring point. Between every two points of such an aggregate there are always infinitely many points.[119] We should restate the definition thus: An aggregate of points N is dense, when between any two distinct points of N

[114] A. Fraenkel, *op. cit.*, pp. 328 *f.*
[115] E. Nagel, *op. cit.*, pp. 485-8.
[116] B. Russell, *Principles*, p. 100.
[117] A. N. Whitehead, *Introduction to Mathematics*, pp. 228-34.
[118] A. Fraenkel, *op. cit.*, pp. 49, 55.
[119] *ibid.*, p. 144.

there are always other points of N left innotative. Between any two given points of the aggregate there are always infinitely many points enotative as to their number, and innotative as to their relationship to the two given points.

Innotation in certain magnitudes or series of magnitudes makes it easy to account for the difficulties of infinitesimals. Some of the difficulties are of long standing, and methods which have been devised for dealing with them involve horizon concepts which we have sought to make familiar. We group a brief selection of material as follows:

C′. (c′) Cantor maintains that definite contradictions may be proved concerning the supposed infinitesimals.[120] But contradictions are to be expected where innotative terms are treated connotatively.

(f′) Jourdain says there are great difficulties in trying to determine what infinitesimals are; at one time they are treated like finite numbers, and at another time like zeros.[121] But we must remember that zero is an enotative or innotative term.

(h′) Whitehead shows how Weierstrass avoided the difficulties of infinitely small quantities by a treatment in terms of limits.[122] Russell says that wherever infinitesimals are thought to occur, what really occurs is a set of finite quantities having zero† for their lower limit.[123] But the notion of limit, like that of zero, is enotative or innotative.

(i′) The concept of continuity is employed by Russell, but without sufficient allowance for innotation. He says that though some forms of infinitesimals are admissible, the most usual form, infinitesimal segments in a compact series, is not implied by either compactness or continuity, and is in fact self-contradictory†.[124]

(j′) In attempts to avoid the difficulties of the infinitesimal by the notion of the infinite, Russell says it is easy to fall into an elementary logical blunder. Given any finite distance, we can find a smaller distance. This may be expressed in the ambiguous form, "There is a distance smaller than any finite distance." But if this is then interpreted as meaning that there is a distance such that, whatever finite distance may be chosen, the distance in question is smaller, then the statement is false.[125] His criticism of this interpre-

120 B. Russell, *Principles*, p. 336.

121 See E. Rignano, *Psychology of Reasoning*, p. 173.

122 A. N. Whitehead, *Introduction to Mathematics*, pp. 226 *ff*.

123 B. Russell, *Introduction*, p. 97. 124 *idem*, *Principles*, p. 368.

125 *idem*, . . . *Scientific Method in Philosophy*, 1915, p. 135.

tation, however, holds only when the smaller distance is said to be no longer a finite distance. But the smaller distance beyond the one chosen is enotative or innotative or both, and we should make no attempt to say whether it is finite or not.[126]

(k′) As for beginnings and endings, it is said that an infinitesimal distance would be one so small that any finite distance would be greater; but continual† bisection gives only finite distances. In the end†, the distance will not grow infinitesimal, because there is no† end.[125] Again, no infinitesimal segment can be terminated.[127] But these are negative statements about segments; they disregard the fact that questions about the endings of segments involve either enotation or innotation, or both.

(n′) Sometimes the difficulties appear to be those of actuality and potentiality. Fraenkel says that with regard to the infinitely small, Cantor recognized only the potential infinite of the older mathematics.[128]

D′. (1) Other writers are more favorable to infinitesimal magnitudes. According to Parkhurst and Kingsland, the Cantorians have openly admitted that a number can come after (*i.e.*, be greater than) any† member of an increasing endless series, and thus the chief obstacle to the acceptance of the infinitesimal is banished. For the converse of the operation which yields $\aleph_0$ and its peers to the infinitists will yield the infinitesimals to those who desire them.[129]

In general, the enotative ranges and innotative intervals referred to affirmatively or negatively by the term "infinitesimals" should be referred to non-committally; if a word may be coined, they are "non-finitesimals."

[126] *cf.* A. Fraenkel, *op. cit.*, p. 115.
[127] B. Russell, *Principles*, p. 335.
[128] A. Fraenkel, *op. cit.*, p. 114.
[129] W. Parkhurst and W. J. Kingsland, Jr., in *Monist*, 36, 1926, p. 519.

PART FOUR

Some Problems of Ontology and Cosmology

"The supreme good is knowledge of the union which the mind has with the whole of nature."
—B. Spinoza, *Treatise on the Improvement of the Understanding.*

CHAPTER XVII

Some Problems of Ontology

"All the constructive [continental European] systems . . . feel the necessity of giving the first place to a general discussion of the most universal characteristics which we find ourselves constrained to ascribe in thought to any reality which is to be an intelligible and coherent system and not a mere chaos. This division of the subject is commonly known . . . as Ontology."

—A. E. Taylor, *Elements of Metaphysics*, 1903, p. 42.

Several historic problems of ontology need to be clarified by recognition of enotative and innotative references involved in their discussion. These include beginnings and endings; order and disorder; "whatness" and "thatness;" actuality, possibility, impossibility, and necessity; being, non-being, and becoming (novelty); unity (monism) and contrasted theories; wholes (totalities) and parts; and absolutes and relatives. We shall not discuss them all in detail, nor attempt to cite examples of every kind of treatment of each. Enough examples will be cited and remarks made to illustrate the principles involved.

BEGINNINGS AND ENDINGS

"The world has a beginning in time, and is limited also with regard to space. . . . The world has no beginning and no limits in space, but is infinite in respect both to time and space."

—I. Kant, *Critique of Pure Reason*, transl. F. M. Müller, 1896, pp. 344 *f*.

A. No problem of philosophy is more persistent than that of beginnings and endings, especially the beginning and ending of the universe†.

B. (α) Skepticism here is easy. Brightman, although he is no skeptic, says that if we try to think of a beginning before which there was absolutely† nothing† or of an end after which there is nothing, we are thinking either of an effect without a cause or of a cause without an effect, and either idea is irrational.[1] We should say that such ideas are clarified by recognition of the horizon principles. If in such a series any event is regarded as uncaused or inef-

[1] E. S. Brightman, *The Problem of God*, 1930, p. 99.

fective, the idea is not irrational, but merely enotative. Problems of cause and effect usually involve explicit dualities, but primary causes and ultimate effects must be left enotative.

(β) The problem is treated psychologically by Kempf, who thinks that perhaps the most persistent sources of the belief in a beginning and end of the universe† are ungratified wishes that need comforting fancies.[2] It is not difficult to trace our notions of beginnings and endings to the fact that our processes of perceiving or of thinking, for various reasons, begin and end.[3] But to bring in such considerations multiplies, rather than answers, questions connected with horizons.

(γ) Without doubt, the notions of beginning and ending are categories characteristic of our attempts to reason about the world. But according to our realistic presuppositions, these categories are characteristic not merely of our minds, but also of the objective world.

(δ) It has been rather common for mystics to claim insight concerning beginnings and endings, but their method, too, raises as many questions as it answers.

C. Problems of beginning and ending are studied in the light of other horizon concepts.

(a) For instance, if any beginning or ending is to be detected, it must be on a basis of discerned differences.

(f) Sheldon says that beginning connotes an event such that before it there was nothing† other† than itself.[4] But this merely substitutes one enotative term for another.

(j) Alexander appeals to the notion of the infinite, saying that the world† which is space-time† never† and nowhere came into existence, for the infinite becoming† can not begin to become.[5]

(r) With a suggestion of absoluteness, Urban thinks that the notion of a hierarchy of levels demands that the ideas of beginning and ending shall be brought together in a more ultimate conception.[6]

(s) Beginnings and endings are discussed in terms of time when it is said that any given process can† or can not† go on forever.[7]

[2] E. J. Kempf, *The Autonomic Functions and the Personality*, 1921, p. 144.
[3] See, *e.g.*, W. de Sitter, in *Nature*, 128, 1931, p. 707.
[4] W. H. Sheldon, *Strife of Systems . . .*, p. 479.
[5] S. Alexander, *Space, Time, and Deity*, Vol. 1, p. 338.
[6] W. M. Urban, *Intelligible World . . .*, pp. 453 *f.*
[7] B. Russell, *Scientific Method in Philosophy*, 1915, p. 135.

D. (1, 2) Sheldon says first affirmatively, that a beginning connotes an event which was eternally† accomplished. Then he says, negatively, that it did not have to wait through an eternity before it could happen. Thus he thinks the objection to the endlessness of past time vanishes.[4]

(4) The problem of beginnings lent itself to Kant's antinomies, for the first of which, quoted above, he took the opposing propositions, "The world† had a beginning in time," and "The world had no beginning in time." We may say, on the one hand, that the world must have had a beginning in time, for if it had no† beginning it must be infinite†, and if it is infinite it can† not† be regarded as a given whole†, or world. On the other hand, the world must have had no beginning in time, for if it had a beginning then there must have been empty time beyond it and hence any so-called beginning of the world in time could have been no real beginning at all. The antinomy results from an attempt to treat connotatively what should be left enotative.

E. (1) Strictly speaking, any beginning is selected in the midst of a non-beginning, and any ending in the midst of a non-ending left enotative. Other horizon principles, however, are needed in order to show the intricacy of the relationships. There appears to be a kind of (III) preemption dependent upon factors of order†, in (V) certain boundary regions; but the problem is capable of (VI) expansion toward marginal or limiting† cases.

(II) It is easy to see that, for example, the time-span of periods called by the term "beginning" may vary. 1901 was the first year of the twentieth century, and January 1 was the first day of that year, etc. In this sense there may be beginnings of beginnings, and endings of endings.

(III) We noted that in the problem of beginnings and endings, there is a kind of preemption along a boundary. This appears from the fact that in ordinary problems the terms are correlative; one involves the other. The beginning of anything can be taken to mark the ending of something else, and *vice versa,* as when the beginning of modern history involves the ending of mediaeval history, and the beginning of mediaeval history involves the ending of ancient history.

Parkhurst and Kingsland say that whatever unit we select to determine duration, extent, or repetition is a unit which has both a starting-point and a finishing. Every object clearly terminates. By

what are these objects severally terminated? Surely by nothing† but other objects which succeed them. As regularly as a walking-stick is found to end at all, its termination marks the beginning of some other object, or the inauguration of a novel sensory† stimulus.[8] A given boundary region may be called beginning or ending, according to the order† of events which at the moment is denotatively, or connotatively, considered. Where a region is thus referred to as a beginning, its correlative status as ending should be treated enotatively or innotatively; where the given region is referred to as an ending, its status as beginning should be thus treated.

(IV) Especially in the problem of the world's origins, it is always possible more or less arbitrarily to assign a beginning which lies outside the reach at least of connotative thinking, and can be referred to at most only denotatively. Thus, some of the naturalists assume or insist that the world began at some natural level, say in a distribution of electromagnetic energies; the supernaturalists ascribe the beginning of the world to the creative activity of God. Some of the naturalists take the opposite view that the world never began, but is eternal.

(V) Beginnings and endings are essentially boundaries between regions, especially temporal† or spatial† regions. These regions we know denotatively or connotatively. There is always some feature in a beginning or an ending which is treated enotatively as regards any such region, and innotatively when it is held to belong between two regions.

(VI) The problem of beginnings and endings is capable of expansion analogous to growth. Let us take again for an example the problem of the beginning of the world†. If we say, "The world began in a distribution of electromagnetic energies," the question may be asked, "What began the distribution?" Disregarding further possible analyses, the question may be answered by saying "God began the distribution"; or, in general, whatever be the end-term of an analysis or causal series, the beginning of whatever is represented by the end-term may be ascribed to God. Such ascription, however, appears to be a theological, rather than a logical answer; logically, it may lead to further questions, "What began God?", "What began that which began God?", and so on. It appears that an answer to a problem of beginnings (or endings) is always the beginning of a new problem.

[8] W. Parkhurst and W. J. Kingsland, Jr., *Monist,* 36, 1926, p. 532.

In a marginal problem, like that of the primary beginning, this preemption appears to be fixed. In this, we should leave one order of possible events, the non-world, enotative, and hence not describe its beginning, its ending, or its content. We can not show, except in some proximate or secondary fashion, how the connotatively or denotatively known world arose from the enotatively or innotatively known non-world. Here we have to deal with an outpost of our thinking, outlined against our utmost horizon.

Where the term is left enotative, we should not speak of progress *from* a beginning or *toward* an ending; we should say, rather, progress in a direction which may be indicated by the term "beginning" or "ending."

(IX) The fact of varying time-spans, indicated at (II), is enough to show that within one beginning, etc., another may be left innotative.

ORDER AND DISORDER

"Whenever we try to deny order completely, absolutely, we find that we are leaping from one kind of order to the other indefinitely."
—H. Bergson, *Creative Evolution,* transl. A. Mitchell, 1911, p. 274.

A. The category of order is sometimes regarded as fundamental and indefinable, although closer examination may show that it is non-order or disorder which can not be defined.

B. (α) The category is at any rate so nearly fundamental that ultimate skepticism concerning it is not to be taken seriously.

(β) Whitehead says that we can understand order, because in the recesses of our own experience there is a contrasting† element which is anarchic. But sheer anarchy means the nothingness† of experience.[9] Reasons for the difficulty of definition of disorder appear from the observation of Fraenkel, that as a rule when we try to think of an aggregate which is not ordered, it can† not† be otherwise† than in a sequence.[10]

(γ) Fraenkel goes on to say that any given aggregate is not necessarily ordered, even though we in writing down or speaking of the individual elements are obliged to regard them in a certain sequence.[11]

C. (a) Mrs. Swabey says that the hypothesis of absolute† chaos must be discarded on logical grounds. Even the most rudimentary

[9] A. N. Whitehead, *Function of Reason,* p. 27.
[10] A. Fraenkel, *Einleitung in die Mengenlehre,* p. 122. [11] *ibid.,* p. 126.

conception of mind requires some element of identity, some common essence, that runs through the particles like a thread and unites them.[12] But, she goes on, this identity requires difference. In the notion of the chaos, the common quality that is held to be shared among the particulars is precisely the absence of all† common properties, so the concept in paradoxically assuming as its connecting link the absence of all linkages falls into self-contradiction†.[12] We should say there is no serious contradiction, as long as either the identity or the difference is left enotative or innotative.

(d) Order is made to involve a universal, when Stebbing says that a system is an ordered system only if all† its constituent elements are related by relations having certain logical properties. These logical properties define† order. But elsewhere Stebbing is somewhat more cautious. An order is not a property of the elements regarded as a class. It is a relation which, given the properties of some members of a set of elements, the properties of "other members of the set, or at least some† of them" are thereby determined.[13]

(h) A notion elsewhere associated with that of limits, appears when Koffka says that in psychology the least degree of structuration would give us chaos.[14] But even the *least* degree of structuration, if described, must be described as an order.

(n) Mrs. Swabey appeals to possibility, saying that the notion of cosmos is differentiated† from that of chaos by its consistency and potential interrelations. Presumably cosmos would seem to be a more adequate term to describe the world, since it at least allows for the possibility of a coherent unity† of the aggregate.[15]

(t) Northrop holds that the macroscopic atom imposes order upon the sum of the microscopic particles.[16]

E. (1) Anything which is described is ascribed some order, but any such order carries implicit reference at least to a non-order, if not to a disorder. The relationships of order, disorder, and non-order are somewhat like those of various negatives† to one another. Disorder may be variously interpreted. Sometimes it means only another order, different† from an order given or sought.[17] The difference may be merely one of degree, and the reference may be

[12] M. C. Swabey, *Logic and Nature,* 1930, p. 367.
[13] L. S. Stebbing, *Modern Introduction to Logic,* pp. 201, 227 *f.*
[14] K. Koffka, in *Brit. Jour. Psy.,* 15, 2, 1924, p. 157.
[15] M. C. Swabey, *op. cit.,* pp. 367 *f.*
[16] F. S. C. Northrop, *Science and First Principles,* p. 242.
[17] H. Bergson, *Creative Evolution,* p. 274.

denotative, or even connotative. In this respect it is like a negative of otherness. Or, disorder may mean merely non-order, a condition left without description one way or the other, and thus enotative. In this interpretation it is like a negative of suspension. Finally, disorder may be interpreted, without further specification or description, as the denial and exclusion of a given order. Here it is also left enotative, like the negative of denial and exclusion.

(IV) An order may be made to depend upon an axiom, as when it is said that the existence of a well-ordering of the continuum† follows from the axiom of selection, but not in the slightest does the possibility† follow, that we can actually† put down any given well ordered arrangement of any kind.[18] For us, it would be necessary to leave the possibility, whether of writing down the ordered arrangement or formulating the law, open and enotative.

(IX) Disorder may be in certain respects innotative, lurking within a system. For instance, Stebbing maintains that a jig-saw puzzle is a system, but not a coherent system. The shape of a piece in the right-hand top corner may be independent† of the shapes of any pieces in the left-hand side.[19]

WHATNESS AND THATNESS

"The 'that' is enough to keep you going."

—W. E. Hocking, *Types of Philosophy*, 1929, p. 393.

A. Sometimes a distinction between these highly abstract terms partially reflects the principles which we are investigating.

E. (I) The demonstrative "that" may be used in referring denotatively to anything; later on, that which is known connotatively comes to have the characteristics indicated by "what," as selected in the midst of the "non-what." To refer to the "non-what," we may use the conjunction "that"—we know, for instance, *that* the non-what is to be left enotative, but we do not know *what* its content is. In contrast to "what," the word which best expresses enotation and innotation is "whether." The issues to which it refers are open.

ACTUALITY, POSSIBILITY, IMPOSSIBILITY, AND NECESSITY

"The word 'possible' is slippery and treacherous."

—G. Santayana, *The Realm of Essence*, 1927, p. 26.

A. The problems of actuality, possibility, potentiality, compossibility, non-possibility, impossibility, contingency, and necessity offer

[18] A. Fraenkel, *op. cit.*, pp. 301 *f.*

[19] L. S. Stebbing, *op. cit.*, p. 198.

some of the most striking examples of horizons. It should be recalled that possibilities, etc., are often indicated, or concealed, in suffixes like "-able" and "-ible," and in auxiliary verbs like "may" and "can." In ordinary use, actuality, possibility, etc., have reference to the structure and processes of the world†, or of our knowledge† of it, in time†. The actual is the world, or some part† of it, now; the possible is that object, process, aspect, etc., to which the world is, or will be open; the impossible is, correspondingly, that to which the world is not or will not be, open.

B. (α, β) The concept of possibility is made to cover ignorance, if not skepticism, when Dubs says that the merely possible are those statements which are not impossible, *i.e.,* not contradictory† to any known truth, yet not sufficiently well substantiated to be known with certainty†.[20]

(γ) The connection of possibility with the problem of the relationships of mind and world is most subtle. According to the foregoing view, possibility would pertain to the mind rather than to the objective world of fact. With regard to the work of mind, Dubs says that the impossible can not any longer mean "inconceiv*able*"†; too many things once declared to be inconceivable have been conceived.[21] This is a negative† way of saying that the range of possibilities open to our conception, or imagination, is wider than ordinarily supposed. It does not describe the impossible, but refers only to some things which have been conceived, although it was once thought impossible to conceive them. The realistic view of possibilities adopted in the epitomization hypothesis will be considered in E (1).

As we saw in the discussion of validity, Lewis defines his "strict implication" in terms of impossibility, and takes strict relations to be relations of intension.[22] But his distinction between intension and extension seems to us inadequate.

(δ) Lewis and Langford hold that in propositions expressing possibility, logical insight, rather than proof based on assumptions†, seems to be advantageously employed.[23]

C. (a) For Lewis and Langford, any† two impossible propositions and any two necessary propositions are logically equivalent; but this does not hold concerning any two possible propositions.[24]

20 H. H. Dubs, *Rational Induction,* p. 328.
21 *ibid.,* p. 326.
22 C. I. Lewis, *Survey,* p. 317.
23 *cf.* C. I. Lewis and C. H. Langford, *Symbolic Logic,* p. 355.
24 *ibid.,* p. 176.

The equivalence is involved in the "peculiar propositions," which we considered in Chapter XIII.

(b) Urban says that the region of the possible is negative, but he adds that it is determined by the principle of non-contradiction†.[25] For Lewis, an impossible proposition is one which implies its own negation, and a necessary proposition is one which is implied by its own negative.[26] But, for us, all that this really does is to label one indeterminate enotative term with the name of another.

(c) Possibility is described in terms of contradiction, when Stebbing maintains that since no system can contain *B* and *not-B*, and since either *B* is actual or *not-B* is actual, the system that contains *A* and *B* (if the world contains *not-B*) or the system that contains *A* and *not-B* (if the world contains *B*) is a possible system, and therefore a possible world, but not an actual world.[27] Here the enotative possibility is identified† with the enotative contradictory. In the work of Urban, above, it is determined by the non-contradictory.

(e) Lewis offers a solution when he says that the meaning of possible varies with the universe of discourse;[28] but the term "universe" also involves a horizon.

(g) Some interpretations of possibility link it with truth. Johnson says that "the possible" may mean what is epistemically possible, or that which is not† known† to be false; formally possible, or that which does not conflict with any formally certified proposition; or nomically possible, or that which does not conflict with any law of nature†.[29] But other notions akin to possibility are linked with falsehood. For Whitehead, every actual occasion is set within a realm of alternative interconnected entities. This realm is disclosed by all the untrue propositions which can be predicated significantly of that occasion.[30]

(h) Uchenko says that actuality may be defined as a limitation of the realm of possibilities.[31]

(j) Adler regards actuality as a finite class† of entities internally related to one another, and excluding possibilities which are contradictory† to it.[32]

[25] W. M. Urban, *Valuation, Its Nature and Laws,* 1909, p. 411.
[26] C. I. Lewis, *Survey,* p. 338; C. I. Lewis and C. H. Langford, *op. cit.,* p. 143.
[27] L. S. Stebbing, *op. cit.,* p. 199. [28] C. I. Lewis, *Survey,* p. 329.
[29] W. E. Johnson, *Logic,* Part 1, p. 61.
[30] A. N. Whitehead, *Science and the Modern World,* 1925, pp. 220 *f.*
[31] A. Uchenko, in *Monist,* 40, 1930, p. 402.
[32] M. J. Adler, *Dialectic,* pp. 205 *f.,* 210.

(n) Actuality is linked with monism by Stebbing. Of possible systems, only one (if any†) can be actual. Which is actual, if any, it may be impossible for us to ascertain.[33] Here the word "impossible" refers to a possibility of another kind, the possibility of our ascertaining. It is a case for E (11), below.

(q) Bradley says that wherever you have a whole which is viewed as grounded internally and throughout, anything within that whole which is yet short of it will, because of the whole, be the real† possibility of anything else in the whole, and will thus and so far be, even itself, real and actual.[34] Buchanan says that possibility is the regulative idea for the analysis of wholes into parts, like a spectroscope which analyzes sunlight into its components.[35] But the "wholes" must be treated enotatively and the possibilities innotatively.

(r) Absolutes are employed, when Dubs says that if we are seeking a criterion of absolute impossibility, we must declare nothing† is absolutely impossible. There is no† criterion of absolute possibility or impossibility. These terms have no absolute meaning. Absolutely speaking, anything† is possible.[36] It will be noted that the absolute property is invoked both negatively† and affirmatively†.

(s) Dewey suggests considerations of the future. The possible denotes ends or consequences not now existing†, but which the actual may through its use bring into existence.[37]

D. (1) The view of Wittgenstein that logical truth is tautological, in that the propositions of logic exhaust the truth-possibilities,[38] amounts to an attempt to reduce the enotative or innotative references of possibilities to affirmative description.

(2) Once a possibility has been specified, one may speak of the impossibility of anything else; but since both terms carry enotative reference, a good deal of confusion results. This is one part of the difficulty in what Eddington says concerning the principle of least action. The principle that action is a minimum (subject to constant boundary conditions) means that it is impossible for action to have any† value other than its least possible value. But if no† other† values are possible, it would be equally true to say that it has the greatest possible value. He explains the difficulty by saying that

[33] L. S. Stebbing, *op. cit.*, p. 201.
[34] F. H. Bradley, *Principles of Logic*, 1922, Vol. 2, p. 707.
[35] S. Buchanan, *Possibility*, 1927, p. 81. [36] H. H. Dubs, *op. cit.*, pp. 327 *f*.
[37] J. Dewey, *The Quest for Certainty*, 1930, p. 284.
[38] I. Wittgenstein, *Tractatus*, 4.46 (p. 97), 6.1 (p. 155).

"possible" is here given two different meanings, the one rather technical, namely, "not incompat*ible* with certain mathematical equations."[39] The statement is another case for E (II).

E. (I) We should amend the ordinary view of actuality and possibility, mentioned in A, by making possibilities subsistently real, constituting a monadic level in the realm of pure logic. All possibilities are "actual," inasmuch as they all interact in the usual monadic fashion. What is ordinarily called the actual world† is a temporal† cross-section of the various realms at a time called contemporary, or now. This contemporary "actual" of ordinary usage is a cross-section selected in the midst of a non-actual milieu. Since the latter is enotative, the precise distinction between it and the not-actual can‡ not‡ be indicated. The fact that it can not introduces (II) possibilities, or impossibilities, concerning possibilities, which it is best for the moment to disregard. Within this non-actual, *i.e.,* not contemporary, still with its boundaries not precisely fixed, is the possible, or as we should say, the level of possibilities, as well as its outstanding non-possibilities.[40] (All possibilities, etc., are not in time contemporary or otherwise, just as all radiations or atoms are not in biological organisms; we view the outstanding realm of possibilities only as refracted through time, just as we detect radiations or atoms by means of the physiological apparatus of our sense organs.) Somewhat more definite than the non-possible, and still, as above, excluded from the contemporary actual, and also excluded from all known realms and levels, is the not-possible, or impossible. Lewis takes the notion of impossibility as a fundamental idea; Lewis and Langford define it in terms of negation and possibility†.[41]

By comparison with possibility and impossibility, the necessary appears to be secondary; it will be discussed at (V), below.

(II) It is quite "possible" to speak of the possibility of a possibility.[42] In recent logical discussions, there is even mention of propositions "necessarily-necessary."[43] It is doubtful, however, if such compounding marks any great gain for metaphysics, or even for logic.

(IV) The essential dependence of several theories of possibility and impossibility upon axioms and assumptions is illustrated when

[39] A. S. Eddington, in Arist. Soc. Lond., *Proc.*, Suppl. Vol. 10, 1931, p. 166.

[40] On the non-possible, see G. P. Conger, *Epitomizations*, pp. 371 *ff*.

[41] C. I. Lewis, *Survey*, p. 292; Lewis and Langford, *op. cit.*, p. 160.

[42] See G. P. Conger, *Epitomizations*, p. 374.

[43] C. I. Lewis and C. H. Langford, *op. cit.*, p. 499.

Whitehead says that if x stands for a primitive proposition, xj means that the predicate is a possible predicate of a subject, insofar as the self-evident propositions regulate our knowledge of possibilities.[44] Lewis and Langford assume "self-consistency or possibility" as a primitive, undefined idea; "p is self-consistent" is said to mean "it is false that p implies its own negation."[45] In his consideration of the relationships of strict implication and material implication, Lewis says that the transformation of "p is impossible" into "p is false" is arbitrarily considered to be complete† when no material relations remain in the expression.[46]

In criticism of Lewis's fundamental axioms and their development, it may be doubted whether it has been definitely established that Lewis's strict implication with its "impossible" is not simply formal implication between propositional functions whose variability is suppressed.[47]

(v) Since, in our view, the non-possible is the enotative milieu of the possible, the possible may be viewed as an intermediate zone between the actual and the non-possible, and might conceivably form a boundary between them. But on almost any view, the range of the possibles is so great and so indescribable that this interpretation must seem quite inadequate.

The most striking instance of an attempt virtually to set a boundary to possibilities is in the concept of necessity, which appears to be only a restriction upon the range of the possible, and often to be a restriction indefinite as to its precise content or description.

A′. The place of necessity in the logic of Lewis and Langford has been considered in Chapter XIII, C (n). Differences between logical and physical necessity are not of primary importance for us, since both involve horizons.

C′. (b′) The essential indefiniteness of necessity is reflected in the negatives used to define it. For Lewis and Langford, if p is necessary, it is not possible that p should be false.[48]

(c′) Another enotative term is used when Lewis and Langford call a proposition necessary, if and only if in point of fact it does not have a contradictory.[49]

[44] A. N. Whitehead, *Universal Algebra*, p. 113.
[45] C. I. Lewis and C. H. Langford, *op. cit.*, p. 123.
[46] C. I. Lewis, *Survey*, pp. 317 *f.*
[47] N. Wiener, in *Jour. Phil., etc.*, 17, 1920, p. 78.
[48] C. I. Lewis and C. H. Langford, *op. cit.*, p. 160.
[49] *ibid.*, p. 478.

(e′) Stebbing disposes of necessity by considering the actual universe in the light of its possibilities. What is logically possible does not suffice to determine the system of the actual world. Consequently the system of the actual world can† not† be logically necessary, and anything that is the case might have been other† than it is, so there are an infinite† number of incomp*atible*† systems.[50]

(g′) Dewey finds necessity in the relationship of implication. Wherever we can say, If so and so, then something else, there is necessity, because partialities are implied which are not just parts of a whole. A world of "ifs" is alone a world of "musts."[51] The difficulty, we should say, is not met by translating it into terms of implication or validity. Validity has the same difficulty on its own account.

(m′) Again, necessity appears to be a matter of "thatness" rather than "whatness." Let us grant, says Whitehead, that we can† not† hope to be able to discern the laws of nature† to be necessary. But we can hope to see that it is necessary *that* there should be an order† of nature.[52]

(q′) Dewey maintains that the common failure to note the fact that a world† of complete† being† would be a world in which necessity is meaningless is due to a rapid shift from one universe† of discourse to another. First we postulate† a whole of Being; then we shift to a part. Now since a part is as such logically dependent in its existence† and its properties, it is necessitated by other parts. But we have unwittingly introduced contingency in the very fact of marking off something as a part.[51]

D′. (1′, 2′) According to Stebbing, there is no meaning in the assertion that the system of the actual† world is a necessary system. One system will not be necessary as contrasted with other systems, though both may be coherent systems.[53] Such denials and affirmations, for us, are too sweeping; the bounds of possibility are too indefinite to indicate whether there would be meaning in them or not.

(vi) Possibility may be associated with the notion of a progressive process. Fraenkel distinguishes between the actual writing down of a well ordered series and the mere possibility of it. Brouwer and the intuitionists use as their criterion *constructibility*,[54] a term which,

[50] L. S. Stebbing, *op. cit.*, p. 201.
[51] J. Dewey, *Experience and Nature*, pp. 64-5.
[52] A. N. Whitehead, *Science and the Modern World*, 1925, p. 104.
[53] L. S. Stebbing, *op. cit.*, p. 201. [54] A. Fraenkel, *op. cit.*, pp. 199, 226-8.

as we saw in Chapter XVI, carries with it the notion of possibility progressively actualized.

(VIII) Possibilities may be aggregated in compossibilities or integrated into relations;[55] in either case the description of them must be left innotative.

(IX) An innotative interpretation may be used for Northrop's theory of possibilities within the physical universe. Contrasting what he calls the "macroscopic" and the microscopic principles, he says that the possibles are changing relational forms introduced by the microscopic principle. In this difference between the actuality of the necessary and the actuality of the possibles, truth† and error† has its basis.[56] Wittgenstein leaves possibilities virtually innotative in objects as thought of. We can† not† think of any object apart from the possibility of its connection with other things. If I can think of an object in the context of an atomic fact, I can not think of it apart from the possibility of this context.[57] For us, subsistent possibilities are enotative, but possibilities persistent within the later levels and realms are innotative.

As distinct from possibility, potentiality suggests the projection of psychological qualities into nature. Potentialities are most easily described as of a past time†, *i.e.*, after they have become actual. Viewed as within objects now, they are not easy to describe. Is the boy before me potentially a man, or only a youth? Is the atom potentially a part of a molecule, or only the unit from which electrons and protons can be disengaged? Such difficulties of definite description and prediction serve to bring out the point that potentialities, as long as they are inherent or implicit, are to be left innotative.

BEING (REALITY), NON-BEING, AND BECOMING (NOVELTY)

"All judgment is of reality, and that means that it makes its idea the adjective of the real universe. . . . In judgment, the reality to which in fact we refer is always something distinguished. It *is* reality as our whole world, but at the same time and none the less it is also *this* reality. It is a limited aspect and portion of the universe. . . . Reality as the subject of our judgment is always a selected reality. . . . This two-fold nature of reality . . . is, if you please, inexplicable. But none the less I must insist it is a fundamental fact, the ignoring of which brings certain ruin to any theory of judgment."

—F. H. Bradley, *The Principles of Logic*, 1922, Vol. 2, pp. 628 *f.*

[55] G. P. Conger, *Epitomizations*, p. 377.
[56] F. S. C. Northrop, *Science and First Principles*, p. 244.
[57] L. Wittgenstein, *Tractatus*, p. 33.

A. Much of the classical philosophy has been written in terms of doctrines of Being. This is the most abstract of several related terms—so abstract that it practically defies characterization and may be taken to signify no more than the fact that† an object, not otherwise described, is to be reckoned with. In this sense "being" is one of the most abstract denotative terms. According to our view, the object or objects thus denoted need not be existent, because being includes also the subsistent. In the epitomization hypothesis, the cosmogonic, biological, and neuropsychological realms are regarded as existent, and the logical, numerical, and geometric-kinematical realms as subsistent. Being may be taken as synonymous with "reality;" it is usual to give the latter term rather more concrete content, although the description of it need not be connotative. Another related term is "substance."

B. (α) McTaggart holds that thoroughgoing skepticism about existence is out of the question. If one denies that something exists, this denial involves the existence of the doubter, or at the very least, of the doubt. And if the doubt itself be an illusion, the existence of the illusion still remains.[58]

With a view which is not fundamentally skeptical, but which questions the adequacy of analysis, Whitehead doubts whether there is any term sufficiently comprehensive to embrace the ultimate† concrete fact. It seems impossible† to obtain a term with positive† content which does not thereby exclude†. But in speaking of ultimate fact there is nothing† to exclude. Against the background of the becomingness† of existence we can only project the various abstractions which are the product of the different modes of analysis.[59]

(γ) The problem has for centuries been conceived in terms of the relationships of our minds and the world. Here, stated in its most abstract form, is the question at issue between idealists, for whom experience is the same† as reality,[60] and realists, for whom external reality is independent of our minds.

(δ) Mystics of all ages have held that they have peculiar ways of access to what they have called Being. According to Underhill, the true mystic is intensely aware of, and knows himself to be one† with

[58] J. E. McTaggart, *Nature of Existence*, Vol. 1, pp. 57 *f*.

[59] A. N. Whitehead, in Arist. Soc. Lond., *Proc.*, 22, 1922, pp. 133 *f*.

[60] F. H. Bradley, *Appearance and Reality*, p. 145.

the active work of becoming, and apprehends the Absolute†, pure Being, the utterly transcendent†.[61]

C. The abstractness of the concept of being makes it easy to discuss the problem almost interchangeably, in terms of other abstract concepts, which have horizons.

(a) For example, identity throughout being results in a monistic† universe†.

(b) Demos notes that non-being is not given to mind, and all reasoning which restricts itself to the criterion of givenness is bound to deny non-being.[62]

(c) For Bradley, ultimate reality is such that it does not contradict itself; here is an absolute† criterion.[63] It has sometimes been held that becoming is essentially a contradiction of pure being.

(d) The concept of being is perhaps the highest generalization.

(e) Hegel regards pure† being as the beginning† of his universe.[64]

(f) For Bradley, being, which is the same† as the most general† sense of experience, differs† from nothing, because in being the emphasis falls on the positive side.[65] McTaggart uses the enotative or innotative concept "nothing" in another way, when he says that if we stop with existence and refuse to go any further, the existent is a perfect† and absolute† blank, and to say that only this exists is equivalent† to saying that nothing exists.[66]

(g) Being is often regarded as the object dealt with in valid or true propositions.

(j) For the neo-scholastic metaphysics, self-subsistent being is necessarily† infinite being. . . . For a finite nature to be the source† of its own existence† is in the nature of things a contradiction†.[67]

(k) In a passage replete with horizon concepts, Taylor argues that there can be no final evolutionary explanation of being as a whole†, any more than there can be a manufacture of manifold and different† objects out of raw material which has no† character of its own and is thus pure nothing†. Somewhere behind all† evolu-

[61] E. Underhill, *Mysticism*, 1918, pp. 42-9.
[62] R. Demos, in *Jour. Phil.*, 30, 1933, p. 86.
[63] F. H. Bradley, *Appearance and Reality*, p. 136.
[64] G. W. F. Hegel, *Enzyclopädie, Werke*, Vol. 6, 1843, p. 165.
[65] F. H. Bradley, *Appearance and Reality*, p. 243; *Logic*, 1922, Vol. 2, p. 670.
[66] J. E. McTaggart, *The Nature of Existence*, 1921, Vol. 1, p. 60.
[67] G. H. Joyce, *Principles of Natural Theology*, 1923, p. 424.

tions and supplying all with material and driving force there must be the strictly eternal†.[68]

(n) Bergson says that judgments which posit† the non-existence† of a thing formulate a contrast† between the possible and the actual.[69]

(p, q, r) For Bradley, the real is individual; it is one in the sense that its positive character embraces all differences† in an inclusive harmony.[70] This harmony characterizes a whole which is regarded as the absolute reality.

D. (1, 2) With such a rarified term, some curious statements can be made, as when Cohen says that non-being has being of a sort.[71] We should say that it can not "be" of the same sort. The difference may be seen from the point of view of E (II).

E. (I) Demos has stated the essential point, in saying that being is in the foreground, and non-being is in the background. The latter belongs to the beyondness of the given. To drag non-being into the field of presentation is to destroy its nature and turn it into being[72] —in our terminology, to treat the neglected enotative region as if it were selected and connotatively known. In spite of the high abstraction, we should keep to our horizon principles, and distinguish between the denotatively known being; the perhaps somewhat more connotatively known reality, the enotatively known non-being, which is left noncommittal, and the enotatively known not-being which is more definitely excluded from being and left as an object of enotative reference.

(II) Difference of sublevel or type appears when Demos says that the whole† concrete entity, defined as being with non-being, is itself a being. But to grant this is to posit† non-being at a further remove, with which the stated being is contrasted. Non-being turns into being which itself is contrasted with more non-being, and so forth.[73]

(VI) "Becoming" is most easily interpreted as coming out of non-being into being by reason of the growth of the latter. Thus for Broad, when an event becomes it comes into existence and it was not anything† at all until it had become.[74] Whitehead says the

[68] A. E. Taylor, in collective work, *Evolution . . .*, 1925, p. 454.
[69] H. Bergson, *Creative Evolution,* p. 290.
[70] F. H. Bradley, *Appearance and Reality,* p. 140.
[71] M. R. Cohen, *Reason and Nature,* p. 165, n. 4.
[72] R. Demos, in *Jour. Phil.,* 30, 1933, p. 86.
[73] *ibid.,* p. 98.
[74] C. D. Broad, *Scientific Thought,* pp. 67 *f.*

future† is a synthesis in α of eternal† objects as not-being and as requiring the passage from α to other† individualizations in which not-being becomes being.[75] Sheldon's view of novelty was indicated in Chapter VII, E (VI). Considerations of becoming also afford a point for the interpretation of novelty. The new can† not† be completely or "absolutely" new, else we should not be able to recognize it. Insofar as it is new, it is new with certain relationships to the old left enotative or innotative. Hocking seeks to exhibit the logic of novelty, albeit "in simple, barren fashion," thus: Assume† a point, *A*, which shall be outside every† particular system of thought or character; and adopt the general† principle that any† such system, *B*, when seen from the standpoint of *A*, changes its character, becoming for experience, say *B′*. With these two assumptions we have defined at once the conditions for an infinite† progress in *B*. For as *B* by reflexion from *A* becomes *B′* so *B′* by reflexion from the same† *A* becomes *B″*, and so on. Thus endless novelty springs from recurrent contact with that which is eternally† the same.[76] The point is that the initial step is an assumption with regard to an *A* which should be left enotative, and that the succeeding statements about this enotative term are gratuitous. A process akin to depletion seems to be illustrated when existence is reduced to essence, or the existent to the essential, with the neglect of non-essentials, which are left enotative.

(IX) Some of the difficulties encountered in attempts to describe the traditional "substance"[77] of metaphysics seem to have been due to failure to recognize innotation. Innotation also helps in dealing with gaps in evolutionary series, and with other more subtle difficulties encountered in philosophies of becoming.

UNITY (MONISM) AND CONTRASTED THEORIES

"And therefore the one can neither be other than other, nor the same with itself. Impossible."

—Plato, *Parmenides*, 139 (translation by Jowett).

A. Other classical ways of construing the world† are seen in the categories of unity, duality, and plurality, and the metaphysical

[75] A. N. Whitehead, *Science and the Modern World*, 1925, p. 246.

[76] W. E. Hocking, *Meaning of God in Human Experience*, 1912, pp. 474, n. 1, 475.

[77] For example, when substance is said to be indivisible, something is left innotative. *cf.* Spinoza, *Ethics*, I, XIII, and J. E. McTaggart, *Nature of Existence*, Vol. 1, pp. 174 *ff*.

systems of monism, dualism, and pluralism. Strictly speaking, dualism means a complete and irreconcilable dichotomy between two principles, while duality means a difference† of aspects which is not so fundamental or complete.

B. (α) A pluralist may easily be a skeptic and regard the "world," whether actually known or not, as too unrelated and chaotic to be unified, except in his doubts or his illusions.

(β) Part One of the present book may be cited to show the psychological side of at least some problems of monism and dualism. This commits us to an implicit duality, although not necessarily to a dualism in the work of minds.

(γ) With our realistic presuppositions, we regard the duality of selected and neglected objects of perception and thought as grounded in the relative individuations of the real world.

(δ) The one and the many are traditionally regarded as synthesized in the mystical experience and consequent doctrines of mysticism.[78]

C. The problem of monism is frequently discussed in terms of other problems which should carry enotative reference.

(a) Identity, if magnified into a metaphysics, results in a monism; difference, similarly magnified, yields dualism or pluralism.

(b, c) Monism, of course, denies any view which makes it self-contradictory. But the contradictions in which it can be involved have been famous ever since Plato's *Parmenides*.

(d) Monism is either a narrower or higher generalization than dualism or pluralism.

(e, f) Monism attempts to include a universe and exclude nothing.

(h) For Hobhouse, monism encounters a limitation; dualism is a final† truth†, limiting but not invalidating efforts towards unity.[79]

(j) The monistic universe is almost certain to be described as infinite.

(m) Hocking holds that it is more important to believe that the One exists† than to know what it is like†.[80] We should say that the first is denotative, the second connotative, and that both select the One in the midst of conditions which must be left enotative or innotative.

[78] E. Underhill, *op. cit.*, p. 49.
[79] L. T. Hobhouse, *Development and Purpose*, 1913, pp. 340 *f.*
[80] W. E. Hocking, *Types*, p. 393.

(q) Bradley is the great champion of the view that the universe† is one, in the sense that its differences† exist† harmoniously within one whole, beyond which there is nothing†.[81]

(r) Sorley criticizes monism in terms of the monistic doctrine of the Absolute and its beginnings†. The monist is, after all, the essential dualist. His Absolute is indeterminate. He never succeeds in showing how the Absolute comes to appear or express itself in phenomenal modes or any† particular† modes at all.[82] We should say that the monist is, indeed, a dualist, but the duality is left enotative.

D. (1) Hocking sees the difficulty of affirmative statements of monism, when he says that if we are consistent in our logic, to describe the One as neutral, cosmic, or even as One in the ordinary numerical sense would be to exclude from it the characteristics of the non-neutral and the like. In all consistency, we should find ourselves reduced to silence.[83]

(2) It should be noted that statements that reality† is *not* one are, for our view, also overdrawn. The question whether reality is ultimately one or not must be left open.

(3, 4) Sheldon offers a treatment of monism and dualism which may be interpreted first in terms of "either-or," and then in terms of "both-and." His discussion involves a number of problems which we have considered. He makes fundamental his interpretation of negation†, in terms of otherness†. But this difference, he thinks, does not deny or exclude sameness†. The really fruitful principle for understanding, sameness-in-difference, is illustrated in its applications to several traditional problems. That we are permitted to say either *A* is *A* or *A* is *B* is the truth of ultimate monism. Reality† is dualistic, or monistic, or both; but since duality is what permits the alternation, the primary trait is duality. We shall see, at E (11), that the duality which permits the alternation between monism and dualism is a duality of another order. This is why, for Sheldon, duality and unity are not of quite equal† rank, and why the way is opened for some puzzling statements.

Reality lends itself, for Sheldon, to either interpretation; this means that reality itself is both. The principle by which we rid ourselves of exclusion, he says, is not an exclusive inclusion, but a free

[81] F. H. Bradley, *Appearance and Reality*, p. 144.

[82] W. R. Sorley, in J. H. Muirhead, ed., *Contemporary British Philosophy*, 1925, Series 2, p. 264.

[83] W. E. Hocking, *Types*, p. 391.

inclusion. This differs from any remedy proposed by partisan or synthetist. But again, dualism would not be dualism if it were dualism alone; for then it would be single and exclusive. It also permits monism, but once more in no exclusive manner. Having distinguished two aspects of a fact, we are metaphysically correct in choosing to regard only their distinctness, and to say that they are two entirely different sides of it. Or, on the other hand, we may choose to regard their ultimate identity. When we make this latter choice we are telling nothing† but the absolute† truth†.[84] According to our view, Sheldon's fundamental principles of otherness and identity, as well as the combination of them, sameness-in-difference, and his principle of creativity, productive duality, all seem to be provided for in ways which the work on epitomizations shows to be repeatedly exemplified in the empirical sciences, and which do not lead to the difficulties which he encounters. Of particular value in avoiding these difficulties is the recognition of different orders of duality, as at E (II), and of their enotative reference.

E. (I) It is our contention that any monistic view means that the selected world is unified merely denotatively, or at most connotatively, but that the conditions of such unity, and such a world, are left enotative or innotative. The facility with which such considerations can be left out or left open helps to account for the strength of the monistic philosophies. The dualistic and pluralistic views of the world are likely to be explicit rather than implicit, and accordingly do not concern us so much here. But dualism and pluralism, too, exemplify the principles of enotation and innotation.

(II) The reason for Sheldon's shifting between monism and dualism appears when we find that monism and dualism are open to treatment in terms of successive and even alternating sublevels. Sheldon says that if the axioms† of monism and dualism are ultimate†, they ought to apply to themselves as well as to everything† else. If the two things are two, he goes on, they can† be both two and one. Then he takes the principle which applies to the things and makes it apply to his principles; unity, he maintains, is other than duality, but there is no excuse for saying that it is opposed to or denies it. The parts† of reality† are independent and they are interpenetrated, and these properties of independence and interpenetration are again independent and interpenetrative, *and so on*.[85] The

[84] W. H. Sheldon, *Strife of Systems* . . . , pp. 456, 474 *ff.*, 493 *f.*
[85] *ibid.*, pp. 472, 474, 476 (italics ours).

point is that when we choose between monism and dualism, the duality presented for our choice is of another order than the dualism originally opposed to monism. But the totality† of monism and dualism in such a choice is also of another order than the original monism, and so the game goes on. It is another leapfrog procedure, except that the leaping is from one level or sublevel to another, and another. In any case, the result must be indicated by "and so on"—that is, it must be left enotative.

(IV) Monism or its contrasted views may be treated either as axioms or with reference to axioms. In logic, says Ouspensky, let us dwell on the idea of monism—the fundamental unity of everything† which exists†—and consequently recognize the impossibility† of constructing any† axioms which involve ideas of opposites†.[86]

Or, monism may be made to represent a kind of cosmic preemption, an ousting of characteristics opposed to it. For Northrop, the macroscopic atom imposes a unity upon the sum of the microscopic particles.[87] Preemption is suggested when Hocking maintains that when faced with the apparent contradictions† of monism and dualism, the mystic does not pretend to be neutral. One contradictory may be nearer the truth† than the other.[88]

(IX) Bradley says that in pluralism internal states give rise to hopeless differences.[89] Perhaps one may say that a monistic world carries or "packs" away in itself a pluralism left innotative, and *vice versa*. This last would show the strength and the weakness of the classical refutation of pluralism, to the effect that the pluralist speaks of the many elements, but unifies them in the very act of speaking or thinking of them. The pluralist really leaves the unity innotative.

WHOLES (TOTALITIES) AND PARTS

"It is possible to say that the utmost our thought can postulate is a system completely intelligible in itself but not exhaustive of reality."

—L. T. Hobhouse, *Development and Purpose*, 1913, p. 337.

A. The correlative concepts of whole and part are among those most frequently employed in attempts to interpret the universe†. They are closely related to other horizon concepts. A class† may be spoken of as a whole class, and a universal† term refers to a whole

[86] P. D. Ouspensky, *Tertium Organum*, p. 265.
[87] F. S. C. Northrop, *op. cit.*, p. 242.
[88] W. E. Hocking, *Types*, p. 392.
[89] F. H. Bradley, *Appearance and Reality*, p. 141.

class. If a whole is limited†, it is a part. If a whole is not thus limited, it becomes the universe; the latter may be called the whole universe. Wholes which differ in their internal relationships may be referred to as aggregates, totalities, or integrates.[90]

B. (α) Cohen maintains that the absolute† totality of things is too indefinite, as well as the object of too much ignorance, to explain anything in it.[91] For Wittgenstein, any method that may be suggested for speaking of the totality of things is fallacious.[92]

(β) That the nature of our minds imposes certain limitations upon our concept of totality is often realized. It has been said that to think is to take great risks. But the mutilation begins not with the act of selection and abstraction, but with the attempt to take the selected aspect as the whole and to interpret the concrete thing in terms of what is at best but a part of it.[93] Hocking insists that it is not knowledge that is relative†; it is the temporal act of knowing. It is one's momentary position as a being in time† and space†, which determines that at any moment one may see but one side of a shield. One must by alternating between parts and whole transcend relativities as they make themselves felt.[94] But, for us, at each step further wholes and parts remain enotative.

(δ) Since the whole must include both subject and object, problems of this kind are often phrased in terms of the more inclusive ways of knowing summed up in intuitionism. The totalities concerning which Wittgenstein holds that it is impossible† to speak logically are, however, thought by him to exist† and are the subject-matter of his mysticism.[95] According to Hocking, the mystical experience finds the idea of the whole, as love finds the idea of a person.[96] Whitehead says that faith in reason is the trust that the ultimate† natures of things lie together in a harmony which excludes† mere arbitrariness. Faith in the order of nature springs from direct inspection of the nature of things as disclosed in our own immediate present experience.[97] That the complete† rationalization, the complete fusion of totality and progress is possible†, Urban says he

90 See G. P. Conger, *Epitomizations*, pp. 15 *ff*.

91 M. R. Cohen, in *Jour. Phil.*, 24, 1927, p. 681.

92 B. Russell, in L. Wittgenstein, *Tractatus*, pp. 16 *f*.

93 H. J. W. Hetherington and J. H. Muirhead, *Social Purpose*, 1918, p. 33.

94 W. E. Hocking, *Meaning of God in Human Experience*, p. 411, n. 1.

95 B. Russell, in L. Wittgenstein, *op. cit.*, p. 23.

96 W. E. Hocking, *Meaning of God in Human Experience*, p. 433.

97 A. N. Whitehead, *Science and the Modern World*, 1925, p. 26.

would be the last to affirm. An element of the mystical and ineffable can no more be excluded from this idea than from any of the great interpretative ideas.[98]

C. The difficulties of the concept of the whole, or totality, are sometimes illumined from other horizons. Many of the views cited in our studies of universals and the logical universe might be mentioned again here.

(a) It would ordinarily be held that the whole of things must be identical with itself, but Wittgenstein maintains that of the totality of things that can be named, or that there is in the world, we can not even assert that it is self-identical.[99]

One of the most important questions of metaphysics is whether or not a (subordinate) whole is in all cases identical with, or equivalent to the sum of its parts taken severally. This is the problem of integration, of great importance in evolutionism and in the epitomization hypothesis.

(b) Bennett says that knowledge of the whole, even when inarticulate, is a fruitful source of negations and exclusions.[100]

(d) The universal involved in the totality of the class of all classes was considered in Chapter X, E (11), in connection with the theory of types.

(f) There are ways in which attempts to describe totality land us close to nothing. According to Harkness, if we make God everything, we make Him nothing. As Spinoza put it, all† determination is negation†. The whole has no specific character of its own.[101]

(g) It has been a cardinal principle of idealism that the real is the whole. Hocking holds that in the ultimate questions anything short of the whole truth is untrue.[102]

(j) In the theory of number, Russell says that it is when we wish to deal with the whole class† or series of inductive cardinals or of ratios that the axiom† of infinity is required.[103]

D. (1) The literature abounds in affirmative statements about totalities. For Eaton, the idea of totality once being given, as it is in the restricted totalities of presentation, the idea can† be extended to totalities which lie beyond presentation and to infinite† totali-

[98] W. M. Urban, *Intelligible World*, p. 468.
[99] B. Russell, in L. Wittgenstein, *op. cit.*, pp. 16 *f.*
[100] C. A. Bennett, *Philosophical Study of Mysticism*, p. 97.
[101] G. S. Harkness, *Conflicts in Religious Thought*, 1929, pp. 178 *f.*
[102] W. E. Hocking, *Meaning of God in Human Experience*, p. 577.
[103] B. Russell, *Introduction*, p. 134.

ties.[104] For Whitehead, every event is a factor in a larger whole, and significant of that whole.[105]

(2) On the other hand, commenting on evolutionism, Taylor insists that there can† be no† final† evolutionary explanation of being as a whole.[106] There is a similar treatment of creationism, when Alexander says that we can not ask the whole how it came into being, how it was created, for such concepts as production, causation, etc., belong to the parts of the world and not to the world as a whole.[107]

E. (I) That our thinking of the whole proceeds enotatively has been recognized in principle by various writers. Wittgenstein's fundamental thesis is said by Russell to be that it is impossible† to say anything about the world† as a whole, and that whatever can be said has to be about bounded portions of the world. We could only say things about the world as a whole if we could get outside the world—if, that is to say, it ceased to be for us the whole world.[108]

(II) As we saw in connection with the theory of types, some of the difficulties about totality are helped by the recognition of sublevels. Wittgenstein holds that propositions can† not† represent the logical form which they must have in common with reality† in order to represent reality. In face of this difficulty, Russell suggests a "hierarchy of languages," but sees fresh difficulties as regards the totality of such languages. Wittgenstein might reply that his whole theory is applicable unchanged to the totality of such languages. The only retort, Russell thinks, would be to deny that there is any such totality. In this way the totality resulting from such a hierarchy would be a delusion, and thus the subject-matter of Wittgenstein's mysticism would be abolished. Russell says that such a hypothesis is very difficult and he can see objections to it which at the moment he does not know how to answer. Yet he does not see how any easier hypothesis can account for Wittgenstein's conclusions.[109]

Recognition of sublevels of totality with enotative reference appears to answer the question. Each outstanding difficulty can be accommodated within a fresh totality of languages, but each succes-

[104] R. M. Eaton, *Symbolism and Truth,* p. 143.
[105] *cf.* A. N. Whitehead, *Principle of Relativity* . . . , pp. 21, 26.
[106] A. E. Taylor, in collective work, *Evolution* . . . , p. 454.
[107] S. Alexander, in Brit. Acad., *Proc.,* 13, 1927, p. 248.
[108] B. Russell, in L. Wittgenstein, *op. cit.*, pp. 16 *ff.*
[109] *ibid.*, p. 23; *cf.* p. 79.

sive totality carries with it an enotative reference, which extends beyond the range of descriptive formulation.

(VI) In the problem of infinites the relationships of whole and part are sometimes treated in terms of growth. One imagines, says Uchenko, a diagram of a whole-part relationship with a smaller circle within a larger circle. The endlessness† of the part precludes its having rigid or permanent boundaries. The radius of its circle is perpetually increasing, so that its swelled boundary may, so to speak, catch up with the outside circle of the whole. Hence the equality† of the part to the whole is proclaimed.[110] But, we would say, if the part is endless it should not be represented as a circle with a boundary. This is the fundamental difficulty, which is overlooked when the questionable affirmative statement about equality is made. To be sure, says Uchenko, the whole can† swell up in its turn, yet it can be overtaken again by its growing part. There is then a process of shifting boundaries. The part is equal† to the whole in the making, but not to the whole.[110] We would agree that wholes and parts may be regarded as growing, but such growth should provide for enotative reference. The same confusion of product and process is evident when some say that the totality of the natural numbers is really a sharply distinguishable whole; but others—according to our view, with more adequacy—think that it is to be regarded as an open process, continually in development.[111]

Cohen says that absolute† totality is an ideal† limit in some respects never† actually† attained. It is a necessary† ideal to indicate the direction of our scientific effort, but it can† not† serve as the explanation of any particular thing in it.[112]

(VIII, IX) Parts are combined or at least found together in aggregates and integrates. The resemblances† and differences† of the two are important, and are difficult to describe with precision.[90] It may be observed here that when attempts are made to scale down an integrate, like an atom or a cell, and in a reductive theory make it amount essentially to nothing other than the sum of its parts taken severally, the internal or combining relationships of the integrate are left out of account, and innotative.

(IX) Keynes suggests innotation when he says that atomic uniformity is in no way implied by the principle of the uniformity of

[110] A. Uchenko, in *Monist,* 40, 1930, p. 408.

[111] A. Fraenkel, *op. cit.,* p. 222.

[112] M. R. Cohen, in G. P. Adams and W. P. Montague, editors, *Contemporary American Philosophy,* Vol. 1, p. 231.

nature. Yet, says he, there might† well be quite different laws for wholes of different degrees of complexity and laws of connection between complexes which could† not† be stated in terms of laws connecting individual parts. In this case, natural law would be organic, and not as is generally supposed, atomic.[113] But even in its organic character, natural law could not be regarded as self-enclosed without innotative and enotative reference.

ABSOLUTES AND RELATIVES

"We have neither the vocabulary nor the imagination for a description of absolute properties as such. . . . To gain an understanding of the absolute it is necessary to approach it through the relative."

—R. B. Haldane, *The Reign of Relativity,* 1921, p. 124.

A. One of the favorite descriptions of the totality† of things, or the universe† of being†, has been in terms of qualities regarded as absolute. This has been true in philosophy and physics alike; absolute goodness, absolute truth, the Absolute Mind have been more familiar than an absolute motion or frame of reference. In this chapter we are concerned with philosophical absolutes and relatives. The theory of relativity in physics will be considered in Chapter XIX. In philosophy the adjective "absolute" has been given the status of a noun, and there has been much discussion about "The Absolute." What is said in terms of absolutes and relatives may also, by slight changes of meaning, apply to the "unconditioned" and the "conditioned";[114] to theories of ultimates and proximates; to things perfect and imperfect; to things pure and impure; and to things complete and incomplete. It is our contention that descriptions in terms of absolutes encounter the horizon conditions.

B. (α) Both skeptics and gnostics of all schools have appealed to absolutes; but the fact that views so opposed can make the same appeal should make us wary of it.

(β) The natural conditions of the psychological processes are envisaged as absolute by Northrop, according to whom knowledge must have both an absolute and a relative character, absolute insofar as man and his nervous system has its basis in the whole† of nature†.[115]

[113] J. M. Keynes, *Treatise on Probability,* p. 249.

[114] Sir W. Hamilton, in *Discussions on Philosophy and Literature,* 1866, pp. 12, 15, etc.

[115] F. S. C. Northrop, *op. cit.,* p. 223.

(γ) Absolutism in various forms has often been phrased so as to involve questions of the relationships of mind and world. Hocking, for instance, holds that the reflexive turn reveals never alone the Absolute within, but always the Absolute within in conjunction with the Absolute without.[116]

(δ) Like the monists, the absolutists have regarded intuition and mysticism as their own. As William James put it, whoever calls the Absolute anything in particular or says that it is *this* seems implicitly to shut it off from being *that*. It is as if he lessened it. So they deny the this, negating† the negation which it seems to imply, in the interest of the higher affirmative attitude by which they are possessed.[117]

C. In a problem so unmanageable as that of the Absolute, it is not strange that help has been sought from other concepts which turn out to be similarly conditioned by horizons.

(a) Mrs. Swabey finds that there must be some degree of identity; an absolute chaos, if meant to stand for an aggregate of disparates totally† devoid of common characters, would seem to be inconceiv*able*†.[118] According to Eddington, the absolute may be defined as a relative which is always† the same, no matter to what it is relative.[119]

(c) Dewey criticizes doctrines of an Absolute Experience because of their failure to allow for inherent logical contradictions. Although absolute, eternal†, all-comprehensive†, and pervasively integrated† into a whole† so logically perfect† that no separate patterns, to say nothing† of seams and holes, can exist† in it, it proceeds to play a tragic joke upon itself by appearing in rags in the garb of temporal†, partial†, and conflicting† things. That a doctrine which avowedly takes logical consistence for its method and criterion, should terminate in such thoroughgoing contradictions shows it is engaged in an arbitrary† sorting out of characters of things which in nature† belong together.[120] It is noteworthy that the Absolute also allows just the opposite interpretation, to the effect that its one supreme attribute is non-contradiction.[121] Our study

[116] W. E. Hocking, *Meaning of God in Human Experience,* pp. 191-5, 202, 206.

[117] W. James, *Varieties of Religious Experience,* 1902, p. 416.

[118] M. C. Swabey, *Logic and Nature,* p. 367.

[119] A. S. Eddington, *Space, Time, and Gravitation,* 1921, p. 82.

[120] J. Dewey, *Experience and Nature,* p. 61.

[121] B. Bosanquet, *The Principle of Individuality and Value,* 1927, pp. 267 *f.*

shows that contradiction is to be expected, although it may be mitigated by leaving one member of it enotative.

(f) An absolute may be described as "nothing but" some specified property or characteristic. But this indicates only that other features are somewhere neglected. On the other hand, Russell says that if everything† were relative, there would be nothing for it to be relative to.[122] But when everything is regarded as relative, this means its non-relative features are neglected; the neglect is what is indicated by the term "nothing."

(h) Keyserling says that all spiritual values are characterized by their absolute quality. For every being, there is an extreme limit or degree of self-realization. Once this has been reached, then, as if by magic, absolute values seem to be manifested.[123]

(j) Dubs brings in considerations of infinites and arrives at a negative statement. He says that to have absolute truth† in deduction, we must have absolutely true premises, and so on *ad infinitum.* Such an infinite regress makes absolute truth impossible†.[124]

(n) For Creighton, the category of the absolute is justified only when it is shown to be a necessary† standpoint in order to enable reason to overcome actual† difficulties that present themselves within experience† itself.[125] The enotation involved in the last few statements is evident.

(o) Thinkers who have followed Parmenides describe their absolute in terms of being rather than becoming.

D. (1) The literature of absolutism abounds in affirmative descriptions. Haldane, however, remarks that he does not think that Hegel ever meant his Absolute to be accepted as more than an ideal† to be worked towards, and that it was not to be regarded as capable† of description, excepting in abstract terms which were therefore insufficient for the concrete ideas of human beings.[126] Santayana says that transcendentalism forgets itself when it assigns conditions to experience.[127]

[122] B. Russell, *The A B C of Relativity,* 1915, p. 14.

[123] H. Keyserling, *The Travel Diary of a Philosopher,* 1925, Vol. 1, pp. 134 *f.*

[124] H. H. Dubs, *op. cit.,* p. 350.

[125] G. W. Cunningham, *The Idealistic Argument in Recent British and American Philosophy,* 1933, p. 345.

[126] R. B. Haldane, in G. W. F. Hegel, *Science of Logic,* transl. W. H. Johnston and L. G. Struthers, 1929, Vol. 1, p. 8.

[127] G. Santayana, *Scepticism and Animal Faith,* p. 23.

(2) In negatives, Dubs says that absolute truth can† not† be literally undefinable, for we can not think or speak of something of which we know nothing†.[128] But according to our view, we can think or speak of an Absolute enotatively, and of "nothing" in the same way. This may answer for definition, although not description in connotative terms.

E. (1) In criticism of reasoning in absolute terms, we have dealt elsewhere in some detail with attempts which have been made to describe the universe of reality in terms of absolute idealism, disregarding the principle of enotation, and with the difficulties which follow this disregard.[129] According to R. Otto, our understanding can† only compass the relative. That which is in contrast absolute, though it may in a sense be thought, can not be thought home, thought out. It is within the reach of our conceiving, but beyond the grasp of our comprehension.[130] This is a fine way of expressing denotation, if not enotation; Otto, however, goes on to add a more affirmative description. The absolute is that which surpasses the limits† of our understanding, not through its actual qualitative character, for that is familiar to us, but through its formal character.[130]

An unintended recognition of the enotative reference in absolutism comes when Sorley, in the passage above cited, says that in spite of his protestations, the monist is the essential dualist. His absolute is indeterminate. He never succeeds in showing how the Absolute comes to appear or express itself in phenomenal modes or any† particular† modes at all.[131] The point is that any absolute, when described, tends to become relative to something else, even if only to the terms used in the description, and their opposites. Any absolute, if it is to be other than relative, must forego description sufficiently to include enotative or innotative reference.

(11) It seems proper to recognize differences of level of absoluteness; they help to clear up the difficulty in the view of Bradley, that every† proposition, true† or false†, ascribes a predicate to a subject, and there is only one† subject, the Absolute. Bertrand Russell has a tortuous argument to show that Bradley's view is self-contradictory†. If the Absolute has predicates, then there are predicates; but the proposition "there are predicates" can† not† be

[128] H. H. Dubs, *op. cit.*, p. 350.

[129] G. P. Conger, *Course in Philosophy,* Chap. XXVIII and pp. 475 *ff.*

[130] R. Otto, *Idea of the Holy,* pp. 145 *f.*

[131] W. R. Sorley, in J. Muirhead, ed., *Contemporary British Philosophy,* Vol. 2, p. 264.

admitted, because it has not the Absolute as subject. Moreover, if we can not admit the proposition, the Absolute, qualified by predicates, is qualified by that which can not be admitted, and the Absolute can not be qualified by nothing†. Moreover, the proposition, "There are predicates," is logically prior to the proposition, "the Absolute has predicates."[182] We should say that the propositions, "The Absolute has predicates" and "There are predicates" (if indeed they are not to be best understood as saying the same thing, once with the suggestion of more connotation rather than with bare denotation), belong to different sublevels, orders, or types of generalization. Each type is included within one which follows, but until thus included is to be left enotative.

(IV) For Whitehead, certain propositions may be arbitrarily enumerated by sheer convention, and to them may be assigned absolute validity.[183]

(VI) Even a denotative reference to an Absolute, with allowances for enotation and innotation, may have its uses. Cohen maintains that although the so-called vision into the Absolute is either into a fathomless depth in which no† distinctions† are visible, or else into a fulness of being† that exceeds our human comprehension, we need the idea of it to characterize our actual knowledge at any time as incomplete and fragmentary†.[184]

(IX) Allowance for innotation within the field of an absolute offers a solution for the paradoxical view that an absolute exhibits degrees of truth† and reality†.

[182] B. Russell, *Principles*, p. 448.
[183] A. N. Whitehead, *Universal Algebra*, p. 109.
[184] M. R. Cohen, *Reason and Nature*, p. 146.

CHAPTER XVIII

Space-Time

"[Space and time] get shaken from each other in thought."
—S. Alexander, *Space, Time, and Deity*, Vol. 1, p. 82.

A. Under the general title of space-time, we include consideration of geometrical-kinematical elements, primarily points and instants. When time is considered separately, the divisions into past, present, and future are important. When space and time are considered together, the relationships which constitute motion are important.

B. (α) Skepticism as regards our knowledge of time, at least, is not unusual. Whitehead says that our ignorance is so abysmal that our judgments of the likeness† or unlikeness† of future events hardly count.[1]

(β) Sometimes the difficulties of space and time are traced to the differences between perception and thinking. Russell admits that the mathematical account of motion is perhaps artificially simplified when regarded as describing what actually occurs in the physical world.[2] We should say, however, that both perception and conception proceed with enotative references.

(γ) Problems of space and time have long been discussed in terms of the reality or unreality of the objective world or some apparent features of it. Since Kant, a prominent part in our notions of space and time has been assigned to consciousness, the mind, and the self. Montague says there is only one thing conceiv*able*† which can qualify to give present actuality† to the future and the past, and that is consciousness.[3] But it is obvious that much in this statement depends on whether the "actuality" is presentation or representation. Among the realists, Sellars holds that movement is a physical change which must be thought of ontologically.[4] Northrop insists that a disembodied conceptual form can not determine the motion

[1] A. N. Whitehead, *Concept of Nature*, p. 35.
[2] B. Russell, *Scientific Method in Philosophy*, 1915, p. 133.
[3] W. P. Montague, *Belief Unbound*, 1930, p. 80.
[4] R. W. Sellars, *Philosophy of Physical Realism*, p. 352.

of a particle. Nature exhibits a metric in every frame, and the metric is conditioned by matter.[5]

(δ) Particular facts connected with space and time are often held to be known intuitively. Robb recognizes that what we call "now" singles itself out in the mind in a manner which is perhaps indescrib*able*†. Simultaneousness, or the lack of it, is an ultimate† fact, and must† be regarded as absolute†.[6]

As an example of intuitional treatment of problems of time and space, no recent work has been more prominent than that of Bergson. He says that to know an object as it really is, we must "place ourselves inside" it—that is, we must think of it as if we were living the career of that object. We must, in other words, try to get at it internally rather than externally, in terms of time rather than space. Ordinarily, our intellects deal with the world in spatial terms, and as if it were in parts† external to one another. The truer way is to deal with it as "real duration," in which the parts interpenetrate one another. This interpenetration which, it may be noted, is still a spatial term, is to be apprehended inwardly, directly, intuitionally—not as geometry is apprehended, but as life is. We make great mistakes when we try to answer the problems of time in terms of space; we should treat them in accordance with our intuition of duration, or growing old, living through a course of experience.[7] We shall see in Chapter XXI the great importance of intuitional approaches to the fact that our experience of the world of things and persons tends to be organized in time. But we should remember that our experience is organized in space, too. It is doubtful if Bergson has done justice to spatial relationships; our intuition of taking up room is as real as any intuition of living through time.

C. (a) Remembering that motion and rest may be interpreted in terms of the relationships of space and time, we find that they can be reduced to identities and differences. For Broad, sensible motion and rest are absolute†, but they seem to depend on relations of identity and difference, respectively, between physical processes in the body which appears and the body of the observer.[8]

[5] F. S. C. Northrop, *Science and First Principles,* p. 111.

[6] A. A. Robb, in collective work, *Evolution* . . . , 1925, p. 408. G. H. Mead, in his *Philosophy of the Present,* 1932, holds that in each new present the past is continually reconstituted.

[7] H. Bergson, *Time and Free Will,* trans. F. L. Pogson, 1910, pp. 100, 128; *Introduction to Metaphysics,* transl. T. E. Hulme, 1912, pp. 7, 40, 88.

[8] C. D. Broad, *Scientific Thought,* p. 454.

(b) Whitehead assigns to the future a certain quality indicated by a negative of exclusion. He says that the future is a synthesis of eternal objects as not-being, and as requiring the passage in which non-being becomes being.[9] But this involves a description of the enotative not-being.

(d) Nagel says that to be able† to make at least general statements about the future is the condition of intelligible discourse about it.[10] But inductions as regards future events should carry enotative reference, with possibility† of future revision.

(f) According to Broad, the future is nothing, and judgments which profess to be about future events refer to nothing, but they are not about nothing. They are about some characteristic and about becoming†.[11] But Uchenko thinks that Broad fails to maintain the nothingness of the future.[12] The reason, of course, is that he does not recognize "nothing" as essentially an enotative term. Its use as an innotative term is illustrated when Bergson says that where events simultaneous for one observer become successive for another, the interval between the successive events is spoken of in terms of *"néant dilaté."*[13]

(h) The assignment of values of functions in space and time is declared to be dependent upon the concept of limits. Broad points out that it is often forgotten that the value of a certain determin*able*† at a certain point or instant is always a highly artificial and sophisticated one. Whenever a meaning free from contradiction† can† be given to the notion of the curvature of a curve at a point or the velocity of a particle at an instant, it is always in terms of limits, and on the assumption† that certain functions are continuous†.[14]

(i) Space and time are often discussed in terms of continuity. The problem is complex, and is treated in a variety of ways which are brought together at this point, instead of being scattered through this and other chapters. We indicate its subordination by using our notation with primes.

B′. (α′) Skepticism is suggested when Russell says that it can† not† be empirically proved that all space is continuous†, nor that it is not so.[15]

[9] A. N. Whitehead, *Science and the Modern World*, 1925, p. 246.
[10] E. Nagel, in *Jour. Phil.*, 26, 1929, p. 488. [11] C. D. Broad, *op. cit.*, p. 77.
[12] A. Uchenko, in *Monist*, 40, 1930, p. 405.
[13] H. Bergson, *Durée et simultanéité*, 1922, p. 231.
[14] C. D. Broad, in Arist. Soc. Lond., *Proc.*, Suppl. Vol. 10, 1931, p. 158.
[15] B. Russell, *Principles*, p. 442; *Mysticism and Logic*, 1918, p. 118.

(β′) Psychological considerations appear when Russell says we naturally regard space and time as continuous†, or at least as compact, but this is mainly prejudice.[16]

(γ′) Questions of real or ideal existence may easily be raised. According to Russell, the theory of mathematical continuity is an abstract logical theory, not† dependent for its validity† upon any† properties of actual† space and time. What is claimed for it is that when it is understood, certain characteristics of space and time, previously very hard to analyze, are found not to present any logical difficulty.[17] Hobson is careful to say that in calling the segment of a line a continuum†, he is talking of "that straight line which is the ideal† object of geometry and which is the ideal counterpart of the physical straight line of perception†."[18]

Appeal may be made to differences of intension and extension. In the problem of continuity, where there is distance, Russell has maintained that large distances, being intensive magnitudes, may† exist† where there are not† smaller ones.[19]

(δ′) There is even a suggestion of intuitional solutions. Russell says that points and instants have a property, which is easier to feel than to define, and which is called continuity†. With such a theory we need to acquire the kind of intimacy which one acquires with a foreign language. The absence of this kind of intimacy makes many philosophers regard the mathematical theory of continuity as an inadequate explanation of the continuity which we experience in the world of sense†.[20]

Bergson has an immediate, intuitive way of apprehending motion. His answer in a problem like Zeno's Achilles, or that of the movement of the arm and our analysis of the movement, is that although we can† divide at will the trajectory once created, we can† not† divide its creation, which is an act in progress and not a thing.[21]

C′. (a′) The continuity of points on a line may be said to involve or be involved in the identity of the line.

(d′) A universal is used when Hocking says that the mathematician means by a continuum all the points on the line.[22]

[16] B. Russell, *Introduction*, p. 140.
[17] *idem, Scientific Method in Philosophy*, 1915, p. 131.
[18] See C. W. Morris in *Jour. Phil.*, 26, 1929, p. 454.
[19] B. Russell, *Principles*, p. 354.
[20] *idem, Scientific Method in Philosophy*, 1915, pp. 129 *f.*
[21] H. Bergson, *Creative Evolution*, pp. 309 *ff.*
[22] W. E. Hocking, *Types*, p. 200.

(h′) The concept of limits, here as elsewhere, is easily associated with that of continuity. Russell says that we want to be sure that every† point in geometrical space which can† not† be specified by rational coordinates can† be specified as the limit of a progression of points whose coordinates are rational.[23]

The notion of limits is also used when it is said that an ensemble of points is perfect†, provided it be identical† with the ensemble of its limit points, where a limit point is such a point that there are points of the ensemble distant from the given point by an amount less than any† prescribed distance, however small.[24]

(j′) Problems of motion raise questions of continuity which are made to involve infinites. For Russell, motion is the occupation by one† entity, of a continuous series of places at a continuous series of times.[25] According to Russell in later work, the interval between any† two instants and two positions is always finite. But, however near together we take the two positions and the two instants, there are an infinite number of positions still nearer together, which are occupied at instants which are also still nearer together. The moving body never jumps from one position to another, but always passes by gradual transition through an infinite number of intermediaries.[26]

D′. (1′) When space and time are treated as continuous, affirmative descriptive statements in terms of infinites† may be made about them. Thus, Russell says that, assuming† space and time to be continuous, $2^{\aleph_0}$ is the number of instants in time or points in space.[27] But the statement is open to the usual criticisms, that it neglects the enotative and innotative references involved.

(2′) As an example of negative statements about the continuity of a line, we read that Dedekind and Cantor have found ways of defining a series of points in such wise that there are no† gaps in the line, and no chance to cut in between the points.[28] But the views of Dedekind and Cantor are open to criticism; the nature of continuity, as well as the essentially indeterminate character of negatives†, suggests that what is here involved is open, enotative or innotative reference.

E′. (iv′) Euclid's axioms are classical, and many other postulates are useful. Nunn says that the points of a line are a sequence

[23] B. Russell, *Introduction*, pp. 101 *f.*

[24] C. J. Keyser, *Mathematical Philosophy*, pp. 395 *f.*

[25] *cf.* B. Russell, *Principles*, p. 473.

[26] *idem*, *Scientific Method in Philosophy*, 1915, p. 136.

[27] *idem*, *Introduction*, p. 86.

[28] W. E. Hocking, *Types*, p. 200.

obeying the Dedekindian postulate, and the same† is true of real numbers. Both are said to possess continuity. He says, however, that it should be noted that he does not† prove that the scheme provides a real number for every† point on the line. Whether it does so or not is apparently impossible† to say. What can† be said is that mathematicians have never† found it necessary† to postulate any† points to which real numbers do not† correspond.[29]

(vi′) Solutions are offered in terms of processes. Sheldon argues that the tip of Zeno's arrow is at the present moment here, and at another very near moment a little further on, but this occurrence is one† integral existential† thing. But the second moment is already present in the first moment; such is the continuity and transeuncy of time.[30]

(ix′) Finally, innotation is evident when Fraenkel says there are on a line an infinite† number of points represented by rational and algebraic numbers, but infinitely more points not represented by algebraic numbers.[31] Again, in the problem of continuity, Alexander holds that instants supply continuity to points.[32] This may be illustrated by the fact that to mark down a succession of discontinuous points requires a continuous stretch of time.

In general, with regard to the description of space, time, or space-time as continuous, it appears that every description must pay regard to the innotative, if not also the enotative features of the data.

(j) Coming back to the main line of argument about space-time, and considering interpretations in terms of infinites, we find that Alexander holds that the infinite space is the positive† object, of which the finite is the limitation†. Infinite space is positive; finite, negative†. The infinite is not what is not finite, the finite is what is not infinite. Space or time is presented as an infinite thing which is prior to every† finite piece† of it.[33] Whatever there is of value in such an interpretation is a restatement of the selective-neglective procedure, which reverses the fields without evading the essential principle. It is covered by treating the infinite denotatively and the finite innotatively. Uchenko sees, in effect, that we should not make affirmative statements about the infinity of space. He says that the

[29] T. P. Nunn, *Teaching of Algebra,* p. 417.
[30] W. H. Sheldon, *Strife of Systems . . . ,* p. 477.
[31] A. Fraenkel, *Einleitung in die Mengenlehre,* p. 52.
[32] S. Alexander, *Space, Time, and Deity,* Vol. 1, p. 148.
[33] *ibid.,* Vol. 1, p. 42.

identification† of the possibility† of advancing beyond any† actual† limit† of space with the actual infinity of space is by no means necessary†.[34]

(k) Appealing to what is virtually the concept of beginning, Sheldon maintains that there can be a first moment of time—for instance, a dual† moment of present-future, which in accordance with the "externality principle," can be described without in the least presupposing an earlier moment.[35] We should say an earlier moment is presupposed, but left enotative. Sheldon's principle of duality, considered in Chapter XVII, leads him to attempt a description of what just now was left enotative. He says that the internality principle need not be denied, for there may be an infinite† number of preceding moments also. One and the same† event can† be occurring at continuously† successive moments; beginning means an event which always was, an event such that before it there was nothing† other than itself. That event always was.[35] So the infinite, once left enotative, is given a positive content in terms of the event to be accounted for.

(n) The concept of possibility aids in the problem of futures. For Uchenko, the events of the future are in the predicament of the possibility of an entity which can† not† be called nothing†, and yet which is not an actual† being.[36] But we should say that the possibility, after all, must be left enotative, and not limited† by necessity.

(q) Whitehead conceives space in terms of the present and the present in terms of a totality. The germ of space is to be found in the mutual relation of events within the immediate general fact which is all† nature now discern*ible*†, namely within the one† event which is the totality of present nature†. Relations of other† events to this totality of nature form the texture of time.[37] The whole set of time-systems derived from the whole set of space-time abstractions expresses the totality of those properties of the creative advance which are cap*able*† of being rendered explicit in thought.[38] But these totalities, discernible or explicit in thought, are selections, with correlative instances of neglect.

(r) The theory of relativity has made familiar the view that there is no absolute frame of reference for our measurements of

[34] A. Uchenko, *op. cit.*, p. 403.
[35] W. H. Sheldon, *op. cit.*, p. 479.
[36] A. Uchenko, *op. cit.*, p. 404.
[37] A. N. Whitehead, *Concept of Nature*, p. 53.
[38] *idem, Principles of Natural Knowledge*, p. 81.

space and time. Some of the early interpretations of the theory took it almost in Kantian fashion, as if each observer furnished his own space and time, and hence his own system of measurements. But the later interpretations are more objective, making the theory of relativity aim at a system of measurements which will be the same† for all† observers.[39] As is well known, it is easier to replace the absolute features of the older physics by other absolute features than to dispense with absolutes altogether. For example, the velocity of light *in vacuo* may be taken as the same for all observers in all frames of reference, and hence as a new absolute. This identification† is not† necessary†, but it helps to show that, in general, no downright refutation of the absolute theory of motion is possible†.[40] Absoluteness may be given other special meanings. For Einstein, simultaneity is defined in terms of a physical motion of propagation, which must be taken as undefined, and hence as absolute.[41]

According to Eddington, the study of the absolute structure of the world† is based on the "interval" between two events close together, which is an absolute attribute of the events independent of any† mesh system. The geodesic, or track of maximum or minimum length between two distant events, has an absolute significance. Again, a being coextensive† with the world might† well have a special separation of space and time natural to him. It is time for this being that is dignified by the title "absolute."[42]

Whitehead notes that it is not at all obvious that invariance of form in respect to all† time-systems is a requisite in the complete† expression of laws of nature†. The demand for relativistic equations is only of limited† applicability.[43]

(t) Some of the relativists try to explain space in terms of the relationships of the parts of matter. Whitehead maintains that the physical fact of the concrete unity of an event is the foundation of the continuity of nature, from which are derived the precise laws of the mathematical continuity of time and space.[44] Other relativists make space an abstraction of material relationships;[45] we have discussed these views elsewhere in other connections.[46] From our

[39] See G. P. Conger, *Epitomizations*, pp. 31 *ff*.
[40] See C. D. Broad, *op. cit.*, p. 203. [41] *cf.* F. S. C. Northrop, *op. cit.*, p. 108.
[42] A. S. Eddington, *Space, Time, and Gravitation*, pp. 150, 162 *f*.
[43] A. N. Whitehead, *Principles of Natural Knowledge*, p. 161.
[44] *ibid.*, p. 77.
[45] See A. S. Eddington, *op. cit.*, p. 8; F. S. C. Northrop, *op. cit.*, Chap. II.
[46] See G. P. Conger, *Epitomizations*, pp. 25 *ff*., 564, 576.

present point of view, the great difficulty in the views just mentioned is that the material universe, even with space and time somehow composed within it, must still involve enotative and innotative references.

Northrop has made an attempt to solve such problems with the whole† physical universe as referent. He says that Zeno showed that if the referent for motion is continuous† space, then the movement of a body through a finite† distance in a finite time should be an impossibility†. In a continuous space there are an infinite† number of points in any† finite distance. Hence to move through a finite distance means, if motion is in space, that a body must pass from one point to another an infinite number of times. This is impossible† in a finite period of time. Northrop says this proof is simple and valid†, and that obviously the Greeks should have concluded that a referent other than the microscopic particles exists†, and that it is not absolute space.[47]

This solution, however, seems to us to raise more difficulties than it solves; the simpler way is by recognition of enotation and innotation in our concepts with reference to the physical universe.

D. (1) Affirmative descriptions of enotative regions of space and time are often made with confidence. For instance, Taylor holds that the eternal which is at the back of all† development must† contain "in a more eminent manner" all that it bestows and may† contain much more, since we must not infer that what it has hitherto bestowed is all it can have to give.[48]

Sometimes the affirmative descriptions refer to a future, as when Nagel holds that it is possible† to enumerate exhaustive though very general† properties about the future, if that future is not† altogether independent† of the character of the present, or any particular† time.[49]

The term "eternal" is regarded by Dewey as a weak description of what we should call an enotative reference. Individually qualified things, he says, have some qualities which are pervasive, common, stable. They are out of time in the sense that a particular temporal quality is irrelevant to them. If anybody feels relieved by calling them "eternal," let them be called eternal. But let not "eternal" then

[47] F. S. C. Northrop, *op. cit.*, p. 10.
[48] A. E. Taylor, in collective work, *Evolution* . . . , 1925, p. 460.
[49] E. Nagel, *op. cit.*, p. 488.

be conceived as a kind of absolute† perduring existence† or Being†.[50]

(2) Broad maintains that when an event becomes, it comes into existence†, and it was not† anything† at all until it had become. There is, however, no such thing as ceasing to exist; what has become exists henceforth forever.[51]

(4) Antinomies and paradoxes are not difficult to find in attempted descriptions of space and time. Bolzano notes the paradox that extended time, space, etc., are made of elements which have no extension.[52] This is due to an attempt to render connotative what has to be left enotative and innotative. Other paradoxes occur in connection with Whitehead's method of extensive abstraction, considered at the close of this chapter.

E. (I) It seems natural to say that the present is connotatively known in the midst of an enotative past and future, but in some respects it is better to regard the past as connotatively known, with innotation where there are gaps in our knowledge, and the future as enotative and open. The present then becomes (V) a boundary between them. For Broad, no judgment about the future is absolutely† certain† (with the possible† exception of the judgment that† there will always† be events of some† kind or other).[53]

If space is described as made up of points, a point in space is a point in the midst of other cognate points, which for the time being are left enotative. Similar considerations apply to instants, and to point-instants. The negative† reference in such enotation is conspicuous in the words "im-mense" and "in-terminable."

(II) Space-time is characteristic of most levels of the geometrical-kinematical realm, not of any one level. Even if there are spaces of spaces, etc., they can be analyzed to show that lines and figures in space exhibit differences of level and sublevel in the usual ways.

(III) Some features of space-time systems suggest selective processes. For Whitehead, a duration is essentially related to one space-time system, and omits those aspects of the passage of nature† which find expression in other† space-time systems.[54]

(IV) From the days of Euclid, the traditional treatments of space and time abound in axioms, postulates, and assumptions. If

[50] J. Dewey, *Experience and Nature,* p. 148.
[51] C. D. Broad, *op. cit.,* pp. 68 *f.*
[52] B. Bolzano, *Paradoxien des Unendlichen,* edition of 1920, p. 72.
[53] C. D. Broad, *op. cit.,* p. 70.
[54] A. N. Whitehead, *Principles of Natural Knowledge,* p. 81.

there is difficulty about fundamental conceptions or definitions, Hilbert says that instead of trying to define a point, or a straight line, or plane, one may introduce three sets of things and subject them to certain relations called axioms. One has no mental picture of these things; the reasoning makes no call on geometrical intuition†, but is purely formal.[55] According to W. H. and G. C. Young, we can† not† assert that to every† point of a straight line there corresponds a rational number, and, with proper assumptions as to the nature of the straight line, it can be proved† that this is not† the case.[56] On the other hand, Nunn says that no argument has ever been produced which contradicts† the common assumption that the real numbers form a number scheme adequate for all calculations regarding space.[57] This difference of opinion and ease of making opposite assumptions is explained by the fact that the series of rational, irrational, and real numbers is properly left enotative and innotative.

(v) That figures or objects in space involve boundaries is evident enough. It is not quite so evident in the case of events in time. Whitehead says that duration is in a sense unbounded. It is a "temporal slab" of nature. A temporal slab would be all† nature at any instant.[58] The duration is unbounded as regards space, but not as regards time; it is only a slab, all nature at an instant. In order to have boundaries in time, it is only necessary that some portions of time shall be particularly distinguished as lying between other portions. This seems most clearly to be the case when the past is known connotatively, the future enotatively, and the present is the shifting moment between them, and, with respect to them, is known innotatively. This is somewhat the view of Dantzig, who says that the *now* is the partition which separates all† the past from all the future. The present is irrational, because it belongs to neither the past nor the future.[59] We should say that it is not irrational, but merely innotative.

This accounts for difficulties of definition of a "present." According to Whitehead, there are no† maximum and no minimum durations. The perfect† definition of a duration so as to mark out its

[55] J. Pierpont, Am. Math. Soc., *Bull.*, 34, 1928, p. 41.
[56] W. H. Young and G. C. Young, *The Theory of Sets of Points*, 1906, p. 13.
[57] T. P. Nunn, *Teaching of Algebra*, p. 418.
[58] A. N. Whitehead, *Principles of Natural Knowledge*, p. 69.
[59] T. Dantzig, *Number, The Language of Science*, pp. 175 *f*.

individuation† is an arbitrary postulate† of thought.[60] In fact, the present is itself a duration, and includes directly perceived time relations between events contained within it. In other words, we put the present on the same† footing as the past and the future in respect to the inclusion within it of antecedent and succeeding events, so that past, present, and future are in this respect exactly analogous ideas.[61] That is, as we should say, they all carry innotative reference.

With regard to space-time, considered as a four-dimensional manifold, it has become customary to say that different† systems of coordinate axes or frames of reference represent so many "partitionings" of the manifold. Any three-dimensional spatial axes may be said to mark a boundary in a four-dimensional world.

(vi) Figures in space or events in time may at least appear as if they had boundaries which are moving. Such apparent spatial or temporal boundaries may mark increases or (except perhaps in the case of time) decreases in the figures or events considered. There is no need to develop examples of the growth of figures in space. With regard to time, Sheldon holds that the present no sooner is than it becomes something new. In the very act of occurring the present shows a little bit of futurity. Correspondingly, any† instant just preceding has that fading quality which we call disappearance or pastness.[62]

In the "diagonal procedure,"[63] the attempt is made, by means of a figure with diagonals, to diagram a set of points. The figure can† be elaborated indefinitely, and it is usually said that the points up to and including a given point will be denumer*able*†. Then on the assumption† that we have all† the points, statements and calculations are made concerning their universality† and exhaustiveness—only to find that, as the process goes on once more, fresh points, not hitherto reckoned with, are encountered. According to our view, this procedure illustrates the principle akin to growth, and has the defect that the process which carries enotative or innotative reference is confused with the denotatively or connotatively known

[60] A. N. Whitehead, *Concept of Nature,* p. 59.

[61] *idem, Aims of Education,* p. 190.

[62] W. H. Sheldon, *op. cit.,* pp. 477 *f.*

[63] See A. Fraenkel, *op. cit.*, pp. 31, 46, 49, 55, 197. The confusion between process and product, with lack of recognition of enotative reference, is the root of the difficulty studied by W. Parkhurst and W. J. Kingsland, Jr., in *Monist,* 37, 1927, p. 132, n. 67.

product. At any moment when the statement is made, all the points we have are those which have been appropriated from the once enotative or innotative region and denumer*ated*, "up to that point" and up to that instant, when the act of appropriation is suspended and account is taken of the points thus far appropriated and denumerated. As soon as the "leapfrog" process is resumed, fresh points of course come into the picture.

(IX) Innotation may be recognized in the interpretation of the Boole-Schröder algebra for regions in a plane, where 0† represents the null region contained in every† region.[64]

Innotation offers the only consistent explanation of the doctrine that there are never† any† next points, either in the continuum† or anywhere else. Any pair of points not next one another constitutes the terms of a line or distance. Therefore the non-nextness that holds of every point-pair on the continuum implies that everywhere† on that continuum there are distance relations which are as† numerous and as omnipresent as the points themselves.[65] We should say that a successful and simple way to save the points on the line from antinomies is to leave some of them innotative, without attempting so many conflicting and contradictory† descriptions.

Eddington is content with thinking that if the subdivisions of a plane surface are sufficiently fine, any† point can be specified with all† the accuracy needed.[66]

Whitehead seeks to meet Zeno's difficulty by an "epochal theory of time." Temporalization is an atomic succession, the realization of a complete† organism. This organism is an event, holding in its essence its spatio-temporal relationships, both within itself and beyond† itself, throughout the spatio-temporal continuum†.[67] But detailed description of these relationships must, as in extensive abstraction, allow for enotation and innotation.

In work on the theory of relativity, the principle of innotation would indicate that congruences, coincidences, and simultaneities are at most only approximations, and that intervals left innotative are characteristic features of the world†.

A striking example of reasoning which involves larger and smaller units of space and time is afforded by Whitehead's "method

[64] C. I. Lewis, *Survey*, p. 120.

[65] W. P. Montague, in Columbia University Department of Philosophy, *Studies in the History of Ideas*, Vol. 1, 1918, p. 246, n. 3.

[66] A. S. Eddington, *Space, Time, and Gravitation*, p. 77.

[67] A. N. Whitehead, *Science and the Modern World*, pp. 179 *f*.

of extensive abstraction." This seems to us to be best understood in the light of principles of enotation and innotation.

Broad illustrates the method by taking the problem of analyzing an area or volume into its parts†; such an attempt, he finds, encounters a paradox. We commonly entertain two incompatible notions of points and use them alternately as convenience requires. When we want to talk of an area as analyz*able*† into points, we think of points as little volumes. A point is thought of as the limit† of a volume, although Broad says we are not sure that such limits exist†.[68] According to our view, this is because limits are enotative. But, on the other hand, Broad says that when we want to measure the length of a line and hence to think of points as having exactly† definite† distances, we take them to have position but no magnitude. They do not become manage*able*† until we come to parts† with no size and events with no duration.[68] For us, such negatives† and zeros† are enotative or innotative, and should not be described as either manageable or not manageable.

Broad's and Whitehead's way out of such a paradox might be called a method of approximation. Upon reflection, it appears that it does not in the least matter to science what is the inner nature of a term, provided it will do the work that is required of it. Any† entity that will do the work of a point may be called a point, no matter what its other properties may† be.[68] In other words, certain approximations may be permiss*ible*†, and may give us a definition which amounts to much the same thing as the definition of limits†, but does not leave us in any doubt as to the existence† of something answering to it. The essence of the theory is that it can state the meaning of such phrases as "converging to a point" in terms which involve nothing† but volumes and their relations to each other.[69]

This amounts to an attempt to abandon the notion of no size and duration, and take in its place the notion of no *smallest* size and duration. This does not escape enotation and innotation; it merely substitutes an infinite† series, with or without a limit†, for a negative† or a zero†. Later on, Broad says that in our analysis of objects or events, we can† not† sense† fields of no† duration. But we can sense events of shorter and shorter duration. We can thus conceive any slab of a sense-history as cut into thinner and thinner slabs. In the end†, we can† conceive† slabs of no† duration, and can imagine† the whole† sense-history analyzed into an infinite† series of such

[68] C. D. Broad, *op. cit.*, pp. 37-9.

[69] *ibid.*, pp. 39, 41, 46.

instantaneous slices.[70] The enotative and innotative terms of the last sentence are enough to leave it open to half a dozen lines of criticism, indicated by our daggers. Broad goes on to describe his results by saying that such momentary slices are not† of course existents†, and they are not literally parts† of the sense-history. But they can be defined† by extensive abstraction, and a Pickwickian meaning can be given to the statement that the sense-history is composed of them.[70]

Whether we consider perceptual fields or points, the same principles are involved. According to Whitehead, the sole use of points is to facilitate the employment of the principle of convergence to simplicity. The introduction of points enables† this principle to be carried through to its ideal† limit†.[71]

The approximations are elaborate and complicated. There are different† types of extrinsic character of convergence which lead to the approximation to different types of intrinsic character as limits†.[72] Once he says that extensive abstraction, like differential calculus, converts a process of approximation into an instrument of exact† thought.[73] But it should be clear, in the light of the principle of enotation, that the word "exact" here is used in a special sense. If allowed at all, it must be taken denotatively, not connotatively.

Consider, says Whitehead, a set of enclosure objects. Now (1) of any† two of its members, one encloses the other.[74] This, let us say, provides a serial arrangement. He goes on to say that (2) no† member is enclosed by all† the others.[74] This, for us, amounts to saying that there are members smaller than any given member; but between (1) and (2) there is a shift from "any" to "all." Choice of "any" leaves something innotative as regards "all," especially when, as in an infinite† series, the "all" can† not† properly be said to mark a closed totality†, but is to be left enotative. Again, for Whitehead, (3) there is no† enclosure object not† a member of the set which is enclosed by every† member of the set.[74] This secures, somewhere along the line, the exclusion of objects which do not belong to the set. In this way, says Whitehead, we converge toward an ideal† simplicity to any† degree of approximation to which we like† to proceed. The series as a whole† embodies the complete† ideal along that route of

[70] *ibid.*, p. 459.
[71] A. N. Whitehead, *Aims of Education*, p. 215.
[72] *idem, Concept of Nature*, p. 83.
[73] *idem, Principles of Natural Knowledge*, p. 76.
[74] *idem, Aims of Education*, p. 210.

approximation.[74] Here the statements about the series as a "whole" and the "complete" ideal attempt to close what should be left enotative and non-closed, *i.e.*, without description as to whether it is closed or not. For Whitehead, systematic use of these abstractive classes is the method of extensive abstraction. All† the spatial and temporal concepts can† be defined† by means of them.[75]

Now a series with no smallest member is defined as an abstractive set. Consider, says Whitehead, a set of durations all† taken from the same family. Let the two conditions be that of any† two members one contains the other as a part†, and there is no† duration which is a common part of every† member of the set. The durations can have no smallest duration, nor converge toward a duration as its limit†. For the parts, either of the end† duration or the limit, would be parts of all† the durations of the set, so the second condition would be violated. Such a set is an abstractive set.[76] But the terms "all" and "every" should not be applied to an infinite series, without reservations such as enotation and innotation supply.

Now Whitehead brings in the notion of zero† size and duration. As we pass along an abstractive set, he says, we finally† reach the ideal† of an event so restricted in its extension as to be without extension in space and extension in time. Such an event is a mere spatial point-flash of instantaneous duration. He calls such an ideal event an event-particle.[76] He says that it is evident that an abstractive set as we pass along it converges to the ideal of all† nature with no† temporal extension—the ideal of all nature at an instant. But this ideal is in fact the ideal of a non-entity†.[76] In other words, recognition of the enotation of the process leads to denial of the existence of the product. But all such descriptions of enotation, whether in terms of negatives or zeros, or even in positive terms, whether of point-flashes or all nature, are gratuitous.

Once Whitehead makes a more conservative statement. What the abstractive set is in fact doing, he says, is to guide thought to the consideration of the progressive simplicity of natural relations, as we progressively diminish the temporal extension of the duration considered.[76] But he does not continue this type of statement; to save the situation the quantities involved are even separated from the entities concerned. The whole point of the procedure, he says, is

[75] A. N. Whitehead, *Principles of Natural Knowledge*, p. 104.
[76] *idem, Concept of Nature*, pp. 60 *f.*, 172 *f.*

that the quantitative expressions of these natural properties do converge to limits†, though the abstractive set does not† converge to any† limiting duration.[76] Again, this explanation seems not quite satisfactory; he says that the laws relating these quantitative limits are the laws of nature "at an instant," though in truth there is not† nature at an instant and there is only the abstractive set.[76] But, for us, both quantities and entities must be left somewhere enotative and innotative.

Appealing to a concept clearly enotative, Whitehead at length admits that it is the infinite† series which is important. We can† exclude any† set of events at the big end† without the loss of any important property.[77]

How, we may ask, can we "make anything out of" entities which tend to ravel out at the small ends as these abstractive sets do? Whitehead finds an answer in a new paradox, that of mutual covering. He says that the required character of the abstractive sets which form event-particles would be secured, if we could define them as having the property of being covered by any† event-particle which they cover.[77] An abstractive set *p* covers an abstractive set *q*, when every† member of *p* contains as parts† some members of *q*. If, of two abstractive sets, each covers the other, we say they are equal† in abstractive force.[77] This, for us, is like saying of two infinites that each must be greater than the other. It involves a kind of back-and-forth leapfrog procedure; Whitehead interprets it in terms of other enotative concepts when he says that the possibility† of this equality of abstractive sets arises from the fact that both sets *p* and *q* are infinite† series toward their small ends†.[77] But at their small ends the two sets are indefinite, and enotative. Still, Whitehead says that the importance of the equality of abstractive sets arises from the assumption† that the intrinsic characters of the two sets are identical†.[77] Elsewhere he says that there will be relationships and characteristics with respect to which the two sets are not equivalent†.[78] In other words, Whitehead's method ought to be called, not the method of extensive abstraction, but the method of "extens*ible*† analysis." The suffix would make clear its enotative or innotative reference. The series with which it deals should be treated neither as closed nor as not-closed, but as *non*-closed.

[77] A. N. Whitehead, *Concept of Nature*, pp. 81-4, 86.

[78] *idem, Process and Reality*, pp. 454 *ff*. For Whitehead, an abstractive element is the whole† group of abstractive sets which are equal† to any† one of themselves (*Concept of Nature*, p. 84). Here the neglect of differences and the innotation are obvious.

CHAPTER XIX

The Physical Universe

"Possibly other universes palpitate beyond these [spaces] . . . . Our glances are confined forever within this giant—yet too small—monad."

—C. Nordmann, *Einstein and the Universe,* 1922, p. 202.

A. The logical universe, or universe of discourse, was considered in Chapter XI. As a special case, we may now notice how the physical universe is subject to the principles involved in the horizons of thought. In the root senses of the words, the universe, or that which "turns around one"† point or center, might be understood to provide enotative and innotative references, and the term "cosmos" might be restricted to any denotative or connotative "order"† selected. We shall be concerned in this chapter merely with a brief treatment of some horizon principles involved in the study of the physical universe. A detailed study of the data may be found in Part One of *A World of Epitomizations.* Although since its publication there have of course been changes in our knowledge of the data, the essential principles and features of the argument seem to us to be unaffected.

B. (α) Bridgman has stoutly maintained that if we sufficiently extend our range we shall find that nature is intrinsically and in its elements neither understandable nor subject to law.[1] Sometimes the emphasis is not on the weakness of our partial knowledge, but its strength. Eddington says that physics is an exact science because the chief essentials of a problem are limited† to a few conditions, and that it draws near to truth with ever-increasing† approximations as it widens its purview. Physics must formulate the laws which are approximately† true† for the limited data of sense and extend them into the unknown.[2]

(β) Whitehead, as we saw, recognizes some difference between nature which is perceived and unperceived, but holds that as regards the unperceived we have dim and inferential knowledge, which is the basis of the scientific doctrine of externality.[3]

[1] P. W. Bridgman, in *Harper's Monthly Magazine,* 158, 1929, p. 444.

[2] A. S. Eddington, *Space, Time, and Gravitation,* p. 154.

[3] A. N. Whitehead, *Principles of Natural Knowledge,* pp. 58 *f.*, 72.

(γ) Doctrines of realism and idealism are sometimes suggested to help us with the problem of the universe. For Cohen, the universe is neither given in experience, nor is it a mental construction, yet it is certainly in some sense given.[4] Eddington says that without reckoning with our consciousness we do not quite attain the thought of the unity† of the whole† which is essential to a complete† theory.[5]

(δ) Santayana links our intuitions with the realm of matter. He says that there is a spirit in us living creatures and this spirit, already actual†, is the gift of intuition, feeling, apprehension, an overtone of animal life, a realization on a hypostatic plane of certain moving unities† in matter.[6]

C. (a) Especially where a process is going on, the identity of the physical universe, if asserted, must be asserted with allowances for differences left innotative.

(c) Problems of the physical universe are sometimes virtually abandoned to their contradictions. Thus de Sitter thinks that we must be prepared to allow the universe to have contradictory properties, although the apparent contradictions must be perhaps hidden from view.[7] Nothing† serves so well for hiding contradictions as an enotative or innotative region.

According to Northrop, a principle of opposition and contradiction is inherent in nature, and any† principle except that which defines† the macroscopic atomic theory gives rise to its negate when pressed far enough.[8] But the macroscopic atom, if it exists, appears, as we shall see, to include features which need to be left enotative.

(f) Lloyd Morgan seeks to sharpen the concept of the universe by the aid of that of "nothing." There being *ex hypothesi*† nothing beyond it, the universe is just a gigantic whole† of intrinsic† relatedness, with no opportunity of external† relatedness.[9] But "nothing" is an enotative term; it is not to be interpreted in terms of denial of the existence of anything outside the universe, but to be taken noncommittally.

(h) In our theories of nature we may need to proceed within certain limitations. Eddington holds that all† properties attributed

[4] M. R. Cohen, in G. P. Adams and W. P. Montague, editors, *Contemporary American Philosophy*, Vol. 1, p. 234.

[5] A. S. Eddington, *The Nature of the Physical World*, 1929, p. 330.

[6] G. Santayana, *Platonism and the Spiritual Life*, 1927, p. 47.

[7] W. de Sitter, *Nature*, 128, 1931, pp. 708 *f*.

[8] F. S. C. Northrop, *Science and First Principles*, p. 143.

[9] C. Lloyd Morgan, *Emergent Evolution*, 1923, p. 71.

to objects in the physical universe are inferred†. Unless we set limits† to this retrospective† description of properties, we are accepting determinism† uncritically, since we could† always† ascribe retrospective properties sufficient to determine whatever† we observe.[10]

(i) In dealing with the physical world, appeal is sometimes made to the concept of continuity. In the neo-scholastic philosophy, extended things are regarded as divis*ible*†, but the parts of a continuous whole† are potential†, not actual†.[11]

There are, perhaps, some epistemological difficulties. According to Bridgman, the mind seems essentially incapable of dealing with continuity as a property of physical things. But to ask whether nature is really known in its smallest ultimate† units, or only appears† so because of the inability† of our minds to treat with continuity, is a meaningless question.[12]

As might be expected in dealing with a concept which carries innotative reference, the view opposed to continuity has its advantages, too. Russell remarks that perhaps if the relativity theory dispenses with the assumption† of continuity it may be able† to account for electrons, protons, and quanta.[13] Elsewhere he says that the theory of quanta, whether true† or false†, illustrates the fact that physics can† never† afford proof of continuity, although it might possibly† afford disproof.[14]

(j) In recent years there has been much discussion about the finitude or infinity of the universe. The inconclusive results of scientific investigations seem to reflect the enotative character of the problem. The difficulty of describing the content of any infinite term with reference to the universe is illustrated when Boodin recalls the familiar statement concerning "heat death," that if this were the law of the universe, then in infinite past time† the universe should have run its course and reached the dead level.[15] But, according to our view, we have no right to say what a process in infinite time, nor what the content of an infinite term, would be.

[10] A. S. Eddington, Arist. Soc. London, *Proc.,* Suppl. Vol. 10, 1931, p. 169.

[11] D. F. F. J., Card. Mercier, *A Manual of Modern Scholastic Philosophy,* transl. T. L. and S. A. Parker, Vol. 1, 1916, pp. 88 *f.*

[12] P. W. Bridgman, *Logic of Modern Physics,* 1927, pp. 94 *f.*

[13] B. Russell, *A B C of Relativity,* p. 188.

[14] *idem, Introduction,* p. 140.

[15] J. E. Boodin, *Cosmic Evolution,* 1925, p. 108.

In mathematical physics, the work of Heisenberg affords another example of the way in which enotation lurks in calculations in which space-time or anything in it is regarded as infinite. G. P. Thomson says that on the Heisenberg theory we can† be perfectly certain† of the velocity of an electron when it is associated with an infinite wave train, but then its position is quite indeterminate†, for we can† not† proceed to determine the direction without cutting the train.[16] This, in effect, reduces an enotatively known element to one connotatively known. On the other hand, Thomson goes on, if we have an infinite wave train whose direction can be determined with perfect† exactness†, the position in the wave front is quite indefinite†. By contracting the wave front the position can be made more certain, but the direction then becomes uncertain, for the waves will spread by diffraction. If the wave front is reduced to a point†, the direction becomes completely undetermined.[16]

(k) The most ancient and widespread way of dealing with problems of the beginning (or ending) of the physical universe consists in ascribing it directly or indirectly to a supernatural Being, regarded as essentially distinct from it. From the point of view of the present chapter, we may see one reason why the cosmological and the teleological arguments for the existence† of God so often turn out to be equivocal. It is because the supernatural is in some sense beyond the natural universe, and *so far as the process of thinking reflects the nature of that universe and is confined to it, the supernatural must be left enotative,* without final statement or definite description. This is by no means our last word concerning philosophy of religion, which is to be discussed in a volume in preparation.

Some supernaturalist views of the beginnings of the universe raise specific questions of enotative reference. Taylor's alternative to supernaturalism is, in short, "nothing†." He says that we can† not† intelligibly think of an evolution from featureless plasticity. The purely plastic and featureless would be only another name for just nothing at all, and no environment can be supposed to elicit a response from a mere nothing.[17] But, as an indication of the equivocal character of enotative terms, we find, on the other hand, that the term "nothing," besides its classical use in doctrines of *creatio ex nihilo,* has even been used to describe God. Seth Pringle-Pattison, commenting on Scotus Erigena's statement that "on

[16] G. P. Thomson, *Wave Mechanics of Free Electrons,* p. 124.
[17] A. E. Taylor, in collective work, *Evolution . . . ,* 1925, p. 452.

account of His supereminence, God may not improperly be called Nothing," says that this thought, grasping at the transcendent† and seeking something more real than reality†, overleaps itself and falls into the abyss of absolute† nothingness."[18] Of course the term "infinite"† has been used time and again. Joyce maintains that self-subsistent† being is of necessity† infinite being. For a finite nature to be the source of its own existence† is, in the nature of things, a contradiction†.[19] Again, to create at all demands infinite power.[19] Whenever the supernatural is admitted to be infinite, there is no difficulty in supplying it with descriptive qualities. Joyce holds that if it be granted that God possesses infinite power, there is no apparent reason why a created thing could† not† be a spirit, and if spiritual, free. Infinite power can† effect† whatever† is not† self-contradictory†.[19] But such descriptions are gratuitous. The difficulty is that the opposite statements also hold; all of them together are readily accommodated within an infinite, which is ever open for more.

At the opposite extreme from supernaturalism, we find the naturalistic statement that science regards the ultimate† scientific object as being spatio-temporally homogeneous† and assumes† that those objects never† begin or end.[20] In other words, problems of beginning and ending are given negative answers.

In terms of the theory of relativity, according to Eddington, one may† picture the curvature of the universe either as due to some cause at infinity† working from outside†, and thus ruled outside the scope of experiment, or as due to a force acting at every† point of space† and time† and curving the world to a sphere.[21]

In connection with his metaphysics of organic mechanism, Whitehead has argued, first, that we must not be too free with assumptions†. He cautions us that any easy assumption that there is an ultimate† reality, which in some unexplained way is to be appealed to for the removal of complexity, constitutes the great refusal of rationality to assert its rights. We have to search whether nature does not in its very being† show itself as self-explanatory. Yet in a sense all† explanation must† end† in an ultimate† arbitrariness†.

[18] A. S. Pringle-Pattison, *The Idea of God in the Light of Recent Philosophy*, 1917, p. 314.

[19] G. H. Joyce, *Principles of Natural Theology*, 1923, pp. 424 *f.*

[20] C. D. Broad, *Scientific Thought*, p. 403. *cf.* R. W. Sellars, *Philosophy of Physical Realism*, p. 370.

[21] A. S. Eddington, *Space, Time, and Gravitation*, p. 158.

His demand is that the ultimate arbitrariness of matter of fact from which our formulation starts should disclose the same† principle of reality† which we dimly discern as stretching away into regions beyond† our explicit powers of discernment.[22] According to our view, the dim discernment here is because of the stretching away into the enotative. Whitehead says, in short, that there is required a principle of limitation†. If we reject the reality† behind the scene, we must provide a ground for limitations which stands among the attributes of the substantial activity. This attribute provides the limitation for which no† reason can† be given, for all† reason flows from it. God is the ultimate† limitation and his existence† is the ultimate irrationality†. No reason can be given for just the limitation which it stands in His nature to impose. God is not concrete, but He is the ground for concrete actuality† There is a principle of concretion not discover*able*† by abstract† reason.[22] In other words, we are to leave the ground of the world undescribed, and enotative, or innotative, or both. But then Whitehead says if God be conceived as the supreme ground of limitation, it stands in His very nature to divide the evil from the good and to establish reason within her domain as supreme.[22] This, of course, is affirmative description of a term properly enotative or innotative, with no conclusive reason given for adopting this description rather than its opposite.

(n) Sometimes the universe is interpreted in the light of possibilities or necessities. Northrop seeks to strengthen the contingent by the help of the necessary. He maintains that even a partial† amount of elemental contingency is unthink*able*† and inexpress*ible*†. The relatedness produced by the macroscopic atom is necessary in the sense of being changeless†. Contingency occurs in the relations of the atomic parts of the universe, and a meaning for possibility and contradiction† is found here. A meaning for contingency, in the sense of vari*able*† relatedness, exists† within the universe; we can talk about it, because it is, so to speak, framed† in by necessity. Such can† not† be the case in a philosophy which rests on pure† contingency.[23] The necessary, we should say, is here taken as absolute, and lacks its enotative reference. Within it, contingency is left innotative; the innotative always includes something "non-thought" or "non-expressed," although to say that it is unthink*able* or inexpress*ible* is an unwarranted description of an innotative

[22] A. N. Whitehead, *Science and the Modern World,* 1925, pp. 130, 249 *f.*
[23] F. S. C. Northrop, *op. cit.*, p. 144.

term. We can talk about contingency in any way which recognizes the horizon principles.

(p) Cohen at least phrases the problems of the universe in terms of monism, although he can not find that to be the answer. He says that the fact that we can† refer to the universe leads us to emphasize its unity. Whether the universe as the totality† of all† things actual† and possible† has a greater unity than a merely spatial† one we are, in view of the fragmentary† character of our knowledge, in no position to answer.[24]

(q) Cohen thinks that the totality of nature through all† time† and space† is a limit† which we can† never† attain, and yet the idea of it is a necessity† of scientific method. A completely rational system coincides in part† with the Bradleian Absolute†, but it is an ideal† limit rather than an actual† experience.[25]

(r) Northrop interprets the physical world by the aid of his macroscopic absolute. He says that after introducing and recognizing more relativity than even the most imaginative speculative mind has ever conceived, Einstein's discoveries reveal that there is something absolute in nature, remaining objective and invariant through all the relativity which exists†, and this absolute factor is matter and motion. Nature is physical and contains motion; relative motion is presupposed† in defining space-time†. So the theory of relativity necessitates† the existence of absolute motion, and a referent in addition to the moving microscopic particles exists. This referent is the macroscopic atom.[26] It appears to be easier to say *that*† there is an absolute condition involved than it is to describe *what* the absolute condition is. This seems to be borne out by Northrop's descriptions of the macroscopic atom as spherical; physical; changeless†; finite†; the basis of actuality†; a primary† substance† with a determinate† conscious experience; congesting the microscopic atoms; eternal†; the body of God; rational, perfect†, and unequivocally† clear-headed; transcendent† and immanent†; possessed of aesthetic character†, but not† omniscient†.[27]

(s) Whitehead says that, since we limit† contingency by the fixed† conditions† which are the laws of nature, it is evident that

[24] M. R. Cohen, in G. P. Adams and W. P. Montague, editors, *Contemporary American Philosophy,* Vol. 1, p. 233.

[25] M. R. Cohen, *Reason and Nature*, p. 158.

[26] F. S. C. Northrop, *op. cit.*, pp. 108 *f.*, 120 *ff.*

[27] *ibid.*, pp. 120 *f.*, 268, 270 *f.*, 273, 275, 280-3.

a scientific object must qualify future† events.[28] We should say that the word "fixed" provides qualification for future events, but without proper enotative and innotative reference.

D. (1) Affirmative descriptions of the universe are common. One of the easiest affirmations is that of W. K. Clifford, who held that the real† universe extends at least far beyond† the cosmos, the order† of which we actually† know.[29] But strictly speaking, anything beyond the horizon of our knowledge must be left enotative. The difficulty of making affirmative, or even negative, descriptive statements about the universe is well stated by Cohen, who says that attempts to characterize the universe as a whole† as one†, not many; continuous†, not discontinuous; conscious or purposive, and the like, all involve a stretching of the ordinary use of words to include their opposites, and from this, only confusion, rather than determination†, can† result.[30]

(4) If there were any need to increase the literature on the Kantian antinomies, it might appear here that it is easy to argue both that there exists† and that there does not† exist an absolutely† necessary† cause for the world.

E. (1) The principle of selection and neglect as regards the physical universe is familiar in scientific work. Whitehead says that in tracing the antecedents of events, common-sense thought habitually assumes† that the greater number of antecedent events can† be neglected† as irrelevant. In scientific thought, it has been assumed that the events in an arbitrarily† small preceding duration† are sufficient.[31] We have seen repeatedly that scientific thinking, like all other thinking, is selective and "neglective." But if scientific thinking is thoroughly selective-neglective, the universe itself is a selection. According to Russell, Wittgenstein's fundamental thesis is that it is impossible† to say anything† about the world as a whole†. Whatever† can be said has to be said about portions† of the world. Wittgenstein even says that the contemplation of the world *sub specie aeterni*† is its contemplation as a limited† whole†.[32]

The universe is a selection, and there is a correlative neglect. Sometimes it has seemed easy to see this. The older astronomers conceived the physical universe as existing in the midst of a "Beyond," which

[28] A. N. Whitehead, *Principle of Relativity*, p. 34.
[29] W. K. Clifford, *Lectures and Essays*, 1886, pp. 406 *ff*.
[30] M. R. Cohen, *Reason and Nature*, p. 153.
[31] A. N. Whitehead, *Aims of Education*, p. 222.
[32] B. Russell in L. Wittgenstein, *Tractatus*, pp. 17 *f.*; Wittgenstein, p. 187.

they, properly enough, were at a loss accurately to describe. It was easy to think of the milieu as an infinite† aether, but even an infinite aether can† not† be described without enotative and open reference. If we write "space" in place of the word "aether," the statement becomes more abstract, but the principle still holds. Sometimes the relativity theories have construed space as a relationship between material bodies in a universe "finite but unbounded," and left "nothing" outside such a system. But this is precisely the point—"nothing" is an enotative term and means simply that questions of extra-spatial connotative content are left open, while space is treated innotatively as a feature within the system.

(II) There is no reason why cognate "universes" could not constitute a "Universe of universes," but the principles of enotation, etc., would still apply.

(IV) Examples of assumptions and postulates in the physical sciences are easy, perhaps too easy, to find. More fundamental than those of the aether or relativity theories, for example, is the basic assumption of scientific thinking that all† that is the case in the physical world is included† in a system. This assumption may be regarded as a fundamental postulate of science. It leads scientists (III) to reject† some facts and to include other facts in the "system of the physical world."[33] In these and other cases, the scientific assumption leads to the adoption of some concept or theory which properly should carry enotative reference. And whatever the justification which may be found for a given assumption, the enotative reference should always be recognized and retained.

(V) In certain classes of problems connected with the physical universe, there is sometimes an enotative procedure, by which difficulties are relegated not to a milieu, but to supposed boundary conditions of the universe itself. This amounts, in effect, to a positive description of the enotative milieu, ascribing to it the characteristics of a boundary. It has been thought that the difficulties postponed by the theory of relativity reappear in boundary conditions at great distances.[34] According to Northrop, the spherical shell of the macroscopic atom is a tremendous object off at the edge of the whole† physical universe.[35] Some of the problems connected with this view have been noted above. That the physical universe exhibits anything

[33] L. S. Stebbing, *Modern Introduction to Logic,* p. 198.
[34] A. S. E., in Roy. Astr. Soc., *Monthly Notices,* 77, 1917, p. 379.
[35] F. S. C. Northrop, *op. cit.,* p. 280.

like a boundary region has been questioned by Russell, who says that our world may† be bounded for some superior† being who can† survey it from above, but for us, however finite† it may be, it can† not† have a boundary, since it has nothing† outside† it.[86] We have noted several times, however, that the term "nothing," being enotative, is when taken properly, non-committal.

(VI) Can the physical universe as a whole† be said to exhibit any process of growth or depletion? The prevailing doctrine of the "expanding universe" is not free from difficulties, but even with these removed, such a universe must suggest with new insistence the problems of enotative reference.

(IX) Cosmology, too, has its innotations. According to L. T. More, science places man in a middle world of law and order†, and relegates all† perturbing complexities to the incomprehens*ible*† background† of the immeasur*ably*† small or indefinitely† great. Life and matter on the earth are the dance of atoms, and atoms are so small that we can forget their variations. The earth itself is in so vast a universe that its perturbations are negligible†.[87]

When it is said that every particle of matter in the universe interacts with every other particle, or even that every object is related to every other object, the statements should be understood according to the horizon principles which in the cases of particles and objects correspond to the innotations of our thinking about them.

In some of the more nearly ultimate problems of metaphysics, just as recognition of enotation helps us to deal with doctrines of transcendence, recognition of innotation helps with doctrines of immanence.

[86] B. Russell, in L. Wittgenstein, *op. cit.,* p. 18.
[87] L. T. More, *The Dogma of Evolution,* 1925, p. 12.

PART FIVE

Some Problems of Freedom, Values, and Intuition

"In the world of knowledge, the idea of good appears last of all and is seen only with an effort."

—Plato, *Republic,* VII, 517 (translation by Jowett).

CHAPTER XX

Freedom and Value

FREEDOM

"Any positive definition of freedom will ensure the victory of determinism."
—H. Bergson, *Time and Free Will*, transl. F. L. Pogson, 1910, p. 220.

A. The "problem of freedom" is one of the oldest and most confused of all the problems of philosophy.[1] The first difficulty is that of definition; Broad has tried to face it in a careful formulation. Let *S*, he says, be any† substance, ψ any characteristic, and *t* any moment†. Suppose *S* is in the state σ with respect to ψ at *t*. Then, according to determinism, the compound supposition that everything† else in the world† should have been exactly† as it in fact was, *and* that *S* should instead have been at one of the two other† alternative states with respect to ψ is an impossible† one. The alternatives are:

(i) S has not† ψ at *t*.
(ii) S continues† ψ at *t*.
(iii) S changes† ψ at *t*.

The determinist generally does not assert either the necessity† or the impossibility† of the separate items of this. According to indeterminism, there is at least one substance *S*, one characteristic ψ, and one moment *t*, such that, although *S* was in fact in the state σ with respect to ψ at *t*, yet the compound supposition that everything† else in the world should have been exactly† as it in fact was up to this moment, *and* that *S* should instead have been one of the other two alternative states with respect to ψ at that moment is a possible† one.[2]

It will be noted that this definition is in terms of time† and of possibilities and impossibilities. Any of these considerations involves so much enotation that, if this definition or anything like it is to be adopted, the problem of freedom must be considered in the light of this principle. We shall return to the problem of definitions at

[1] See G. P. Conger, *Course in Philosophy*, pp. 332 *ff*. For definitions, H. H. Horne, *Free Will and Human Responsibility*, 1912, pp. 64 *ff*.
[2] C. D. Broad, in Arist. Soc. Lond., *Proc.*, Suppl. Vol. 10, 1931, pp. 136 *ff*.

E (1), below. Next we must distinguish between any alleged contingency of the laws of nature and any alleged freedom of the human will; it is always a question how much bearing the one has on the other. The question is considered at C (d), below. In this chapter we are chiefly concerned with freedom as a property of the human personality, rather than of other portions of nature.

B. (α) It is often said that indeterminism, contingency, or freedom are but names to cover the gaps in our knowledge; and that if we knew all† the facts, fatalism would be the only tenable view. Quite to the opposite effect, Bergson argues that in order to be in possession of all the facts bearing on a man's action, one would have to be that man himself, and that one would then be in position to affirm freedom.[3]

(β) There are numerous theories involving psychology. According to Kempf, the will is the compensatory affective or autonomic striving, which, as a wish, protects the individual from the anticipation or fear of failure; the fear of failure causes a compensatory speeding up of autonomic reactions.[4] Herrick argues that a thing or process with specific organization is able to exhibit the behavior characteristic of or typical for that organization in an appropriate environment. Within a lawfully determined and determining natural system, we recognize in our common speech a natural freedom which, though limited†, is real†.[5] We should say that the limitation is enough to make freedom innotative within nature. According to Broad, it is a question whether voluntary decisions are or are not completely determined; it is an entirely different question whether they do or do not determine effects.[6]

Perhaps the most subtle difficulty connected with the problem of freedom comes from the fact that the very discussion of the problem involves processes which may themselves be interpreted in terms of either position. For example, you and I engage in an argument about the problem, and the argument is conclusive. There are, let us say, four possible cases: (1) I argue for freedom, you argue for fatalism, I convince you; (2) I argue for freedom, you for fatalism, you convince me; (3) I argue for fatalism, you for freedom, I convince you; (4) I argue for fatalism, you for free-

[3] H. Bergson, *Time and Free Will*, pp. 184 *ff*.
[4] E. J. Kempf, *Autonomic Functions and the Personality*, p. 98.
[5] C. J. Herrick, *Fatalism or Freedom: A Biologist's Answer*, 1926, pp. 52, 54.
[6] C. D. Broad, *op. cit.*, p. 139.

dom, you convince me. But as for (1), I, or any one, can say that I was, or am, the inevitable cause of convincing you of freedom. (This is among the reasons why, as Bergson points out, a definition, or description, of freedom tends to turn to the advantage of the fatalist.)[7] In (2), I, or you, or any one, can say that you are, or will in future be, free to change your mind and reverse your opinion. As to (3) and (4), the corresponding statements may be made with changes of pronoun. It appears therefore, that the problem of freedom really belongs with the Epimenides, and other problems in which what may be called the formal conditions of any discussion impose certain limitations† upon the content of the discussion. The effect of the limitations in such cases is to keep the conclusions from being too sweeping.

(δ) There is a certain attractiveness about intuitional treatments of the problem. Bergson's emphasis on intuitive knowledge here accords well with many feelings and convictions of freedom. He has also brought out the fact that, while life is characterized by freedom, articulated and systematic argument about it tends to favor fatalism.[7]

C. (a) Sometimes freedom has been imputed to all† individual entities† because they are individuals. According to Sheldon, freedom really lies in the individual way of acting and reacting which each entity in the universe† displays. The "nature" of any one entity is not determin*able*† from anything† else but just itself.[8]

(d) Some writers, and notably Eddington,[9] call to their aid the process of induction as it is used in recent physical science. According to the Heisenberg principle of uncertainty, our measurements of electrons must be statistical, rather than individually precise†, and our scientific generalizations are subject to this limitation†. This has sometimes been regarded as evidence for a fundamental "indeterminacy" in nature, which might appear in the human nervous system as "indeterminism" in mind. Eddington says that minds, mental events, and psychical causation† are, on any view that is worth a moment's consideration, so extremely unlike† matter, material events, and physical causation that the argument for determinism is not very convincing.[10]

[7] H. Bergson, *Time and Free Will,* transl. F. L. Pogson, 1910, pp. 220, 230.
[8] W. H. Sheldon, *Strife of Systems,* p. 487.
[9] A. S. Eddington, *Nature of the Physical World,* pp. 220 *ff.*, 306 *ff.*
[10] *idem,* Arist. Soc. Lond., *Proc.,* Suppl. Vol. 10, 1931, p. 147.

Here, again, there is no way to give a precise or definite answer to the problem. The gaps between electrons and electives may be due to our ignorance†. The motions of individual electrons, even if undetectable, need not be contingent or arbitrary. Even if they are, the statistical laws of physics may be exact enough to account for everything significant in the behavior of electrons concerned in neuropsychological processes. One thing seems sure; we should not confuse the negative of denial and the negative of suspension. What we actually face in the enotative or innotative is non-committal as regards either fatalism or freedom.

(h) Considerations of our ignorance† and of limits may be combined to make room for freedom. According to Boring, no causal relationship is ever so precisely† established that the determinist does not still believe in the persistence of a probable† error, and a probable error measures the persistence of ignorance. Perhaps† it leaves room for freedom. The problem is one of limits. The probable error gets less and less as the precision of research increases. Is the limit zero† or is it a finite† value?[11] It can not be denied that if we knew more, we should know more. But the argument in terms of ignorance and that in terms of limits alike involve enotative and innotative reference.

(s) Freedom seems to involve factors of time. The principal witness here is Bergson, who has long insisted on the close connection between the two. It is essential that the future be viewed as open. According to Whitehead, future occasions are objectified as incomplete†, without determinate† concretion.[12]

(n) Freedom is, after all, a specific form of possibility. Arguments against it on grounds of supposed necessities must meet the challenge that the bounds of possibility are unknown and that necessity itself is enotative.

(p) Sheldon says that a combination of freedom and determinism verifies his principle of duality. Insofar as the reaction of a body is identically† the nature of the body, it is free. Insofar as the reaction is only an individual† event, while the nature of the body is a permanent† character, the reaction is determined. These two, the individual event and the "nature" of the body are the same†, yet

[11] E. G. Boring, *op. cit.*, p. 117.

[12] See A. N. Whitehead, in International Congress of Philosophy, Sixth, *Proc.*, 1927, p. 61.

distinct†. The sameness-in-difference allows each aspect of the matter to be ultimately† true†.[13]

(r) The absolute enters the argument for Bennett, who says that in our art, our altruism, and our problems of duty, we best hit the relative by aiming at the absolute, and in such recovery of direction for the will lies the hope of attaining a creative freedom.[14] But we should say that an absolute at which we aim is at most treated denotatively, not connotatively.

D. (1, 2) The long history of the problem of freedom has abounded in examples of unwarranted affirmative and negative statements. . . .

E. (1) Enotation and innotation, once they are recognized, can be of great help in the problems of definitions. When the question is that of the human mind or will, there are three views, or kinds of views. They may be called fatalism, determinism, and libertarianism, although they are impossible to distinguish or describe with precision. The best procedure is first to lay out a scale allowing for three variable concepts, two at the uncertain extremes of the scale and the third oscillating uncertainly between them. The scale might be pictured thus:

<...*Fatalism*..>..<..*Determinism*..><..*Libertarianism*..>

Fatalism and libertarianism are extreme, or, as we might say, limiting‡ concepts. Like other concepts involving limits, they may be described to any length that any one wishes to proceed with the analysis. Fatalism, at one extreme, is the view that, as far as any one wishes to proceed with the detailed analysis, our actions are the inevitable result of past actions, or other causes, or both. Libertarianism, at the other extreme, is the view that as far as any one wishes to proceed with the analysis, our actions are *not* related as effects to past actions, or to other causes, or both. Determinism oscillates between these extreme views; none of the three is fixed with precision, but all are relative. The attempt to be precise in their definitions fails, because, as the arrows indicate, in these limiting concepts so much has to be left enotative and innotative. Fatalism is, in fact, an affirmative statement, and libertarianism, defined as above, is a negative statement about a matter properly left enotative.

[13] W. H. Sheldon, *op. cit.*, p. 489.
[14] C. A. Bennett, *Philosophical Study of Mysticism*, p. 170.

Allowance for enotation seems particularly important in dealing with doctrines of freedom; the fact is, we may affirm *that*† we are free, at least in the sense that we are "*non*-bound" and even indicate or refer to our freedom denotatively, but we can not both affirm our freedom and describe it. As soon as we try to describe in detail *what*† act is free, or what the content of a free act would be, we find ourselves inevitably associating the "free" act with the other acts connotatively known, in such a way that, caught in the web of causes† and effects†, it loses its free character and appears to be as much bound as the other acts. This is the reason for the perennial difficulties about defining academic freedom and freedom of speech, and their violations. Freedom is one of the concepts, the precise description of which lies beyond the horizons of thought. As a matter of fact, the same thing is true of fatalism. It is not possible for us, with Aristotle, to carry any analysis of causes and effects unmistakably back to the Prime† Mover or First† Cause. Even if the universe can be analyzed into its primary† elements, the universe is itself a selection, and, moreover, within it, its primary elements can† not† with precision† be detected in their exact relationships to one another. The difficulty of defining determinism appears in the apparently precise, but really vague, statement that self-determination is freedom. In all such attempts at definition, something must be left enotative or innotative, or both.

(ii) It is possible denotatively to conceive systems of determinism within other systems, and free acts within free acts, but their enotative references make it useless to try connotatively to describe them.

(iii) A unit of mental organization characterized by freedom might be expected selectively to appropriate and reject elements suited or unsuited to its progress.

(iv) The great Kantian postulate is enough to show that freedom may be treated in this way; so may determinism or fatalism. But such assumptions are, after all, assumptions. They are regulative and not constitutive.

(v) If fatalism pertains to the past†, and freedom to the future†, a theory might be worked out which would take determinism to be the special characteristic of the present† moment of experience, and thus, as we saw in Chapter XVIII regarding present time, determinism might serve as a boundary condition.

(VI) We forget the enotation and the innotation involved in fatalism, first, because of the number and weight of the facts which we manage connotatively to know. And time†, at least in our part of the universe and on our scale of apprehension, is unidirectional, flowing from past to future. Hence there is not merely a large and important group of facts connotatively known, but there is in our connotative knowledge a kind of momentum due to its connections with the past. We tend to be swept through the thin cross-section or boundary which we call the present, forward into the future. The future is preeminently enotative. So, with freedom enotative and the future enotative, it is easy to associate or even to identify† them. Without either affirming or denying the identity, the one thing we should say is that the future is *non*-fixed; the question whether it is fixed or not remains open.

(VIII) The next important point is one which has been emphasized by Spaulding[15] and others; there are differences of level of integration (or differentiation) in mental life, and freedom is a quality pertaining to some levels rather than to others. This is the modern "mental chemistry"; here a metaphor, once too hastily condemned, comes into its own. Sodium is sodium, and chlorine is chlorine; but salt is salt. Instincts and tropisms, taken severally, are instincts and tropisms. But, taken in their places as elements in larger long-time integrates of mental organization, they may even be indistinguishable, so different is the end-reaction or the sentiment from the elements which enter into it.

In other words, the scale on which degrees of freedom are measured does not need to be, as in our diagram in E (I), a linear, dead-level affair; it may be a spiral, and a scale like that indicated may, while perfectly† continuous†, pass from one level to another of mental organization. This is another reason why, for instance, the dictum that "self-determination is freedom" may be admitted. Determination on the level of self or personality is, and is not like† determination at lesser or lower levels. The different levels of mental organization are characterized by the different lengths of time involved in their integrations or differentiations. A sentiment has typically longer range than a simple end-reaction; a value outlasts a sentiment; a self or a personality requires a lifetime for its full integration and development.

[15] E. G. Spaulding, *The New Rationalism*, 1918, pp. 449 *f.*

(IX) Even with the main lines of such long-time developments fixed, it is possible† to argue for intervals or interstices left innotative and therefore free.

SOME PROBLEMS OF VALUES

"Value reflects a substantial organization of interests and attitudes in organisms."

—R. W. Sellars, *The Philosophy of Physical Realism*, 1932, p. 448.

A. Discussions concerning values are so numerous and intricate that no brief treatment of the subject can hope to be adequate. We merely indicate some points which might be developed in much more detail, especially with more references to the literature.[16] The main questions about values are those of their significance and authenticity as regards the nature of the world—in short, the problem of the evaluation of values.

B. (α) Skepticism as regards our values is familiar enough in pessimism.

(β) R. B. Perry's treatment of the general problem of value in terms of interest and the motor-affective life makes its psychological basis conspicuous.[17] Our own treatment of values is also primarily in terms of psychology. In the first place, value, for us, is a quality characteristic of valuations. The process involved is that of valuation; value, in its relationship to valuation, is like the hereditary qualities characteristic of an organism of a given species. Values, then, ought not to be hypostasized; the entities involved, with their structures and processes, are neuropsychological.

When we look to the neuropsychological units or monads involved, we find that they are units which are at least more inclusive than patterns, and that they may include as constituents such ideas as correspond to language-patterns. The series of monads appears to be reflexes, patterns, end-reaction complexes, sentiments, valuations, selves, and total individual nervous systems at work, or personalities. The details of these structures and processes are studied in Part Three of *A World of Epitomizations*. Of immense importance for present-day thinking about the world is recognition of the natural rights of these larger units among the data of psychology. From being too animistic, psychology has come to be too atomistic.

Distinctions between end-reactions, sentiments, values (or ideals), selves, and personality are not of fundamental importance for us

[16] See G. P. Conger, *Epitomizations*, pp. 313 *ff*.

[17] R. B. Perry, *General Theory of Value*, 1926, 27 *f*., 115 *f*.

here. All these are monads of different levels of complexity and are marked by various types of end-reaction. Although there is no precise distinction, end-reaction complexes usually are directed to objects; sentiments to persons, or things with strong personal attachments; and values to more abstract ideas, or to objects or persons interpreted with the aid of such ideas. All, however, involve one very important basic principle.

This principle is that valuations, etc., are neuropsychological units which conspicuously are *organized in time*. Every nervous process occurs in time; in fact, in its beginnings nervous conduction represents a speeding up of primitive non-nervous or neuroid conductions. Any reflex or pattern involves a certain reaction-time, which on the whole is longer as the pattern is more complex, with more opportunity for inhibitions and refractory states. Comparatively simple reactions of this sort are characteristic of the lower animals. Some animals, however, have an advantage over others in that they have not merely contact-receptors, requiring actual touch for stimulation and the eliciting of response, but also distance-receptors, like eyes and ears, for the reception of stimuli from objects with which the organism is not yet in actual contact. The worm, for example, is in this respect at great disadvantage compared to the bird; that, rather than the early rising, is the reason why the worm gets caught.

As soon as distance-receptors appear, a stimulus from a distance may elicit first precurrent and then consummatory reflexes or pattern-reactions, and, in the resulting end-reaction complex, organization in time becomes more than ever conspicuous and important. With the distance-receptors, actions come to be directed toward objects distant in space and time—in other words, upon objects which are treated as ends. All that is needed from this point to perfect the higher types of mental organization is easily seen from discussions in current psychology. There is, first, let us say, the process of conditioning, so that, for example, the animal will react not toward the original stimulus but toward a sound or cry which represents the original stimulus. Thus the way is prepared for language and for thinking, and for the direction of activity upon ends not actually seen, but signalled, talked about, or thought of—in short, suggested by some substitute stimulus. Again, there is the process apparent in circular or chain reflexes, in which a response to one stimulus either is a new stimulus or modifies the environment or a

former stimulus and thus elicits a new response with reference to a new end. This concatenation of responses, stimuli, new responses, etc., makes possible the differentiation of subordinate units of behavior, and the integration of more inclusive units, and thus makes of many ends mere means to further ends.

Once more, there are varying degrees of emotional content, with tonuses or "sets" variously pitched. And finally, there are varying degrees of abstraction† and generalization† of ideas, and corresponding differences of remoteness in time among the ultimate aims and ideals. All these processes going on together in a personality involve a lifetime of organization and reorganization.

The world presents us with some ends which are remote and toward which we must edge our way through many an intermediate struggle, with the long fidelity demanded by things most excellent and rare. But now and then things which are desirable are found relatively close at hand, and the intermediate struggle and employment of precurrent means is not necessary. This seems particularly to be the case with aesthetic values; the object of enjoyment is relatively present, immediate. As Schopenhauer said, "Art is always at its goal." Such immediacy of value, such enjoyment of an end without the necessity of securing the intervenient and precurrent means, may be compared to what is encountered when an atomic nucleus is stripped of its peripheral electrons. Regardless of spatial relationships, the end of a valuation is like the nucleus of an atom, in that it is in time the most enduring part. It is a valuation stripped down to its essential†, which is not merely here, but *now*.

The last point is enough to show that, although our treatment of valuations is psychological, it allows for the principle of enotation. This will appear more clearly in E, below.

(γ) According to R. B. Perry, the neo-realists are not all agreed upon the question of the independence of values from consciousness; he himself thinks that a value, which is the relation of an object to a valuing subject, acquires existence when an interest is generated regardless of any knowledge about it.[18] Appealing to what he calls the creative synthesis of the parts of the organism and physical entities, with the resulting perception of color, Spaulding argues that at least certain values are objective, in the sense that they are numerically distinct from and independent of both a per-

[18] R. B. Perry, in *The New Realism,* 1912, p. 140 and n. 1; *General Theory of Value,* pp. 122, 139 *f.*

ceiving consciousness and a receptive organism.[19] For us, valuation is a process not so much of creative synthesis, or integration, but of interaction between mind and objects, and interaction, as selective involves a neglected enotative reference. Sorley holds that moral values claim an objectivity, which we ought not to deny any more than we would deny the objectivity of the empirical world. In arguing for this claim, however, Sorley brings in concepts which carry open, enotative reference. He finds ground for affirming† the objectivity of moral values, not merely because of their claim to objectivity, but because of their universality† and ideality†.[20] But values may be talked about in such a way that valuation is assigned a status even prior† to the existent† world†; Urban holds that both existential and truth† judgments are forms of valuation.[21] For Urban, the only linkage of facts that is really ultimately intelligible is one which is interpretable in terms of value. The only thing that is self-explanatory is a will oriented towards value.[22] In this case the appeal must needs lie directly to inner personal experience, suggesting the more intuitive treatments of values.

(δ) In fact, for Urban, the ultimate goal is a form of contemplation which transcends will and thought alike.[23]

C. (a) Urban regards the postulate of the identity of the empirical with the metempirical will as constituting the essence of value.[24]

(d) Sometimes values are held to exhibit a universal quality but Urban, with more caution about the use of terms of indefinite or enotative reference, maintains that the objects of over-individual aesthetic value, if expanded with some degree of completeness† within a homogeneous† group, may by the exclusion† of opposing factors create the illusion of complete† universalization.[25]

(g) Picard agrees with the pragmatists who hold that truth is a contributory rather than an immediate value, and that there are

[19] E. G. Spaulding, *The New Rationalism*, 1918, pp. 499-501.

[20] W. R. Sorley, *Moral Values and the Idea of God*, 1921, pp. 150, 350 *f.*

[21] W. M. Urban, *Valuation, Its Nature and Laws*, p. 423. This work will hereafter be cited as *Valuation*.

[22] *idem*, *Intelligible World*, p. 185. On the idealistic arguments from our values and ideas to a World Mind, characterized by values and ideas, see G. P. Conger, *Course in Philosophy*, Chap. 28, and pp. 475 *ff.*, and *Jour. Phil.* 20, 1923, pp. 290 *ff.* For the word "denotative" in these discussions, the word "enotative" should be substituted.

[23] W. M. Urban, *Valuation*, p. 428.

[24] *ibid.*, pp. 406, 425.

[25] *ibid.*, pp. 345 *f.*, 409, 413, 427.

not† objectively existent† ontological norms. He thinks that it is better to assume† one's metaphysical position dogmatically, and make bold guesses at some transempirical reference of certain facts of observation than to attempt to deduce it from psychological considerations.[26]

(h) With a more matter of fact view, Urban points out that with regard to values, "practical absolutes"† may come at the limits of volition.[27]

(i) Urban thinks that the presupposition of valuation is a demand for continuity. The postulate† of validity† in values is that our experiences of feeling or will are in some way identical† or continuous with a reality† which transcends† our momentary† experience.[28]

(m) The difference between "thatness" and "whatness" becomes important, when Sorley argues that we unavoidably† presuppose *that* there is *a* value, although we may† not† know beforehand *which* of two or more possible† values is valid†.[29]

(p) R. B. Perry insists that value must be conceived not merely in terms of unity but of interest.[30] The unity of values, or of the highest value, is treated by Urban in ways involving and even suggesting enotative references. He admits that no† unified highest† value is demonstrable†, and that the postulate† of the identity† of the will with the metempirical† will is to be realized not in absolute†, but in sufficient identities—*i.e.*, identities sufficient for the continuity† of values according to certain inner† criteria.[31]

(q) Bosanquet, discussing the ground of values, maintains that the universe as a whole is self-directing and self-experiencing.[32]

(r) Sorley is virtually an absolutist. He makes a great point of the statement that in moral values there is presupposed a principle which is distinguished from mere impulse and declared to be an unconditioned good.[33] Bosanquet holds that the key to reality and

[26] M. Picard, *Values, Immediate and Contributory*, 1920, pp. 24, 113, 176, 183 *ff*.

[27] W. M. Urban, *Valuation*, pp. 348, 405, 413. [28] *ibid.*, pp. 400-2.

[29] W. R. Sorley, *Moral Values and the Idea of God*, p. 68.

[30] R. B. Perry, *General Theory of Value*, p. 50.

[31] W. M. Urban, *Valuation*, pp. 406, 411, 425.

[32] B. Bosanquet, *The Principle of Individuality and Value*, 1912, pp. xviii, xxv, and Chap. VI.

[33] W. R. Sorley, *op. cit.*, pp. 146 *ff*.

value is in the effort to self-completeness, which is the principle of individuality, and is most fully realized in self-consciousness.[34]

Urban thinks that religious constructions may become "practical absolutes." In fact, any form of existence† or truth† may, under certain circumstances, have absolute value. The will to live, the will to be a person, and the will to participate in social enterprises may under certain circumstances be unconditional.[35] His words "under certain circumstances" are enough properly to qualify his otherwise loose statement.

(s) Varisco holds that if values were not permanent they would not exist† as values.[36] But Urban sees that the ultimate† presupposition†, while it is eternity of value, is not the eternity of any† specific objects of value. It is chiefly in the attempts to universalize† specific ideals that incompatibilities appear.[37] Urban's work at this point serves as an important criticism of that of the type of argument which uses the indefinite reference of certain valuations as a means to the definite description of their ground. But Urban elsewhere tries to show the ultimate meaning of valuation in another way, which leads to a similar result.[38]

E. (I) Since valuations are monads, they have the monadic characteristics.[39] And since the justifiable ways of dealing with enotative concepts reflect monadic characteristics, we may expect to find that work with valuations is subject to these principles. The chief point here is that no value should be thought of as absolute†, or unconditioned. The duality or polarity often noticed in valuations is usually explicit rather than implicit.

(II) That there are various sublevels of valuation is not difficult to maintain. Mental organization is so varied and complex that there is room for all degrees of integration. The chief difficulty is not to provide places for them in the scale, but to recognize and distinguish them in the data. It must be remembered, however, that any so-called highest or most inclusive value should carry its enotative reference.

[34] B. Bosanquet, *Principle of Individuality and Value,* 1927, pp. xviii, and Chap. VI.

[35] W. M. Urban, *Valuation,* pp. 348, 405.

[36] B. Varisco, *The Great Problems,* transl. R. C. Lodge, 1914, p. 273.

[37] W. M. Urban, *Valuation,* pp. 409, 413.

[38] *ibid.,* p. 428.

[39] See G. P. Conger, *Epitomizations,* pp. 313 *ff.*

(III) All prepossessions, prejudices, etc., are sufficient to show that values are notably selective. But in accordance with what has just been said, the selecting interests are themselves relative.

(IV) Urban maintains that wherever there is the feeling of value, there must be at least the presumption of reality†, although this is to be distinguished from the narrower and later judgment of existence†.[40]

(V) Valuations may be said to exhibit certain boundary or limiting conditions, insofar as there are certain reactions which with regard to a given end belong at the threshold, and others which belong below the threshold. The boundary, however, is hard to fix and perhaps tapers off imperceptibly.

(VI) Spaulding maintains the Platonic view that justice and goodness and truth are eternal†. Time-† and space-conditioned† things approximate to the ideal in various ways, but never† attain it. The ideal remains, seemingly in accordance with the principle that the limit† is not† a member of the series of which it is the limit.[41] This is one reason for the indefiniteness of discussions of ethical and aesthetic values. The ideal has to be left undescribed and enotative, or, if it is indicated, something has to be left innotative between the ideal and our actual† achievement. Valuations may increase or decrease in range or depth, and encounter more or less definite limits. But to say that they become more than practical absolutes† is to disregard essentially open, enotative reference.

(VIII) Urban, following Wundt and Fechner, finds in connection with values the law of total† series, according to which a value is imputed to the whole which is not a sum of the separate parts†,[42] and that of "end-feelings," according to which the worth of a series of elements is determined by the final† moment of the series and its relation to the preceding moments.[42]

(IX) Differentiation of parts within valuations is indicated when Urban distinguishes in the operation of the "law of complementary value," the law of increase of value through contrast of elements.[43] Reductive or analytical interpretations treat such increases innotatively.

[40] W. M. Urban, *Valuation*, pp. 42 *f.*, 404 *ff.*

[41] E. G. Spaulding, *New Rationalism*, p. 498.

[42] W. M. Urban, *Valuation*, p. 180.

[43] *ibid.*, pp. 173-8, 180 *f.*

All this which has been said is, however, an interpretation of valuations in language and thought-forms, rather than an actual experience of valuation. A valuation, we said, includes ideas, somewhat as a molecule includes atoms; and it is certainly plausible to say that in both cases properties appear in the more inclusive whole which are not characteristic of the less inclusive parts. Is there, then, any more-than-thought, or more-than-thinkable property in valuations? If there is, after what we have seen in the present study, it will be reasonable to look for it, not in the indirectly mediated symbols of language or ideas, but in some immediate quality or process. This brings us to the problem of intuition.

CHAPTER XXI

INTUITION

"As intuition is helpless without intellect, it must always be accompanied and followed by conceptual thinking."

—W. E. Hocking, *Types of Philosophy*, 1929, p. 211.

Intuition is immediate apprehension. After this fundamental fact, the most important fact about it is that it is not confined to any one neuropsychological level, but may characterize the structures and processes of several. Mind is so highly coordinated that distinctions between reflexes, patterns, and end-reactions, as well as between sentiments and valuations can not be made with precision; psychology has for example in "instincts," "complexes," and "*Gestalten*" other data which belong to more than one level.

Intuitions do not have to be regarded as anything other than neuropsychological; no other-worldly or mystical cause for them is necessary. They may be, and probably are best traced to physiological, especially kinaesthetic and glandular processes. In fact, most of what used to be called the purely mental or spiritual processes have lately been recognized as actually belonging in a world of matter (which is not the opposite of spirit, after all). We need not hesitate to recognize intuitions in the ordinary neuropsychological structures and processes of different levels. We shall consider (1) vague, unattached feelings; (2) patterns, including perceptions, language-patterns, and ideas; (3) end-reaction complexes, including instincts and interests; (4) sentiments; (5) valuations; (6) selves; and (7) personality, with its inclusive adjustments.

1. *In the first place, intuited feelings may, either because of their unstudied freshness, or because of later conditionings, appear as unattached and vague.* Dewey, with his emphasis on the backgrounds and sources of the thinking process, says that apart from language and from imputed and inferred meanings, we continually engage in an immense multitude of immediate organic selections, rejections, welcomings, expulsions, appropriations, withdrawals, shrinkings, expansions, elations, dejections, attacks, wardings off, of the most minute vibratingly delicate nature. Even our most

highly intellectualized operations depend upon them as a fringe by which to guide our inferential movements. They give us our *sense* of rightness and wrongness, what to select, emphasize, slur over, ignore. Formulated discourse is mainly but a selected statement of what we want to retain among all these incipient starts, etc. These qualities are the stuff of intuition. They differ in degree. A reasoning person is one who makes intuitions more articulate, deliverable in speech, explicit.[1] Hocking regards feeling as the simpler state of mind from which intellect and will are differentiated. Wherever there is feeling there is cognition, *i.e.*, some knowledge or judgment about the objective world.[2]

2. *The term "intuition" is used in the sense of immediate perception, or* Anschauung, *of objects.* Lossky gives the name of intuitionism to the epistemological theory, according to which the subject immediately knows objects as they are in themselves, *i.e.*, as they exist apart from the act of knowing directed upon them.[3] For Hocking, intuition recognizes the presence of objects; intellect defines what they are. Intuition is perception of an object for itself; intellect is perception of its relations. Intuition is the perception of what is unique in an object; intellect is perception of the qualities which it has in common with other objects. Intuition is perception of the whole†; intellect, of its parts†.[4]

It is here that intuitions begin to acquire their so-called certainty. Santayana says he is concerned to rescue from oblivion "one of those preliminary facts which the science of things ignores, although it is the breath of its nostrils"—namely, intuition, with the inevitable unity and originality of the essence which that intuition defines from moment to moment.[5]

Perception is in some ways not different from thinking, and from language. All may be treated as patterns. We have developed in Chapter IV the view that ideas are either language-patterns or are sufficiently akin to them to be treated as of their level. This suggests again the time-honored problem of the relationships of intellect and intuition. It may be difficult to discuss the problem adequately. Watt

[1] J. Dewey, *Experience and Nature*, pp. 299 *f.* *Cf.* p. 85, and W. Köhler, *Gestalt Psychology*, pp. 326 *f.*, 367, 371.

[2] W. E. Hocking, *Types*, p. 178. *cf.* R. M. Eaton, *Symbolism and Truth*, p. 43.

[3] N. Lossky, in International Congress of Philosophy, Seventh, *Proc.*, 1931, p. 254.

[4] W. E. Hocking, *Types*, pp. 201 *f.*

[5] G. Santayana, *The Realm of Essence*, 1927, p. 145.

says that science rests upon thought, and thought lies in the cognitive level of mind; of this level, concepts are the proximate elements. How can we expect, he asks, to portray in this one level of mind the inner spontaneity and essence of all the later levels of integration. . . . We can† not† expect cognition to portray its own patterns completely†.[6] Moreover, as regards the patterns familiar to us as language-reactions, they seem to be incommensurable with intuitions, so that intuitions of any and all grades are typically only imperfectly expressed in words. William James saw that if we have intuitions at all, they come from a deeper level of our nature than the loquacious level which rationalism inhabits.[7] For Wittgenstein, propositions show the logical form of reality, but what can be shown can not be said.[8]

Among the attempts which have been made to interpret, in language and thought, the relationships of intuition and intelligence, we note that, according to some writers, a kind of intuition comes first, and when expanded and articulated, yields intelligence.[9] According to others, intuition is a concentration of processes of intelligence.[10] Sometimes there is emphasis upon the function of intuition as leading to ideas of unique value.[11]

Even more prominent is the view that intuition may synthesize and reconcile the dualities of thinking. This is associated historically with the Hegelian synthesis of opposites, and the various logics of absolutes, concrete universals, etc., founded upon it. Bergson

[6] H. J. Watt, *The Sensory Basis and Structure of Knowledge,* 1925, p. 236.

[7] W. James, *Varieties of Religious Experience,* pp. 73; *cf.* p. 456. *cf.* J. M. Baldwin, *Thought and Things . . . ,* Vol. 2, p. 418.

[8] L. Wittgenstein, *Tractatus,* p. 79.

[9] See J. M. Baldwin, *op. cit.,* Vol. 1, pp. ix *f.*; L. Lévy-Bruhl, *Primitive Mentality,* transl. L. A. Clare, 1923, pp. 60, 91, 444; *How Natives Think,* transl. L. A. Clare, 1925, p. 78; J. Piaget, *Language and Thought of the Child,* pp. 45, 127; F. Lorimer, *Growth of Reason,* pp. 150 *f.*; D. Hilbert, in *Die Naturwissenschaften,* 18, 1930, p. 961. W. James, *Pluralistic Universe,* p. 212; R. Otto, *Idea of the Holy,* p. 46; C. A. Bennett, *Philosophical Study of Mysticism,* p. 95 *f.*; M. R. Cohen, *Reason and Nature,* p. 187.

[10] See H. Bergson, *Introduction to Metaphysics,* p. 63. W. E. Hocking, in *Rev. de met. et de morale,* 29, 1922, p. 447; C. A. Bennett, in International Congress of Philosophy, Sixth, *Proc.,* p. 112.

[11] T. Dantzig, *Number, The Language of Science,* p. 180; C. K. Ogden and I. A. Richards, *Meaning of Meaning,* 1923, p. 170; J. Piaget, *op. cit.,* p. 45; H. H. Dubs, *Rational Induction,* pp. 366 *f.*; C. A. Bennett, *Philosophical Study of Mysticism,* p. 101.

would rely upon intuition to reconcile some of the opposites engendered in thinking.[12]

3. *It is said that intuition characterizes mental organizations more complex than patterns.* Rignano says that from intuition as a pure and simple observation of some fact or attribute we may pass step by step to intuition arrived at as a result of mental combination, produced in us spontaneously and in a flash from the impulse of a single affectivity, without any corrective control being exercised by a corresponding state of attention. He cites a case of intuition as a very rapid reasoning, almost instantaneous, a series of experiments carried out in the mind alone.[13]

Among organizations more complex than patterns, we may consider end-reactions, and take instincts and interests as at least in some respects typical of them. Van der Hoop, discussing Jung's theory of types, says that intuition is the spontaneous active expression of the instincts and of that part of the unconscious psychic life which is most closely related to them. Intuition itself is not actually either thought or feeling, but is, as it were, a primitive psychic function which contains elements of thought and feeling.[14]

Piaget says that in egocentric logic, little value is attached to proving, or even checking propositions. The vision of the whole† brings about a state of belief and a feeling of security far more rapidly than if each step in the argument were made explicit. Personal schemata of analogy are made use of, as are memories of earlier reasoning, which control the present course of reasoning without openly manifesting their influence. The child reasons as he draws, according to a sort of internal model.[15] As for later developments, Eaton says that when the cognitive act is taken in its wholeness† as the convergence of thought, sensation, and intuition upon an object, the feeling of the inadequacy of thought to reality† melts away.[16]

We have seen that it is at the level of end-reaction complexes, with the contributions of the distance-receptors and the differentiation of constituent patterns into precurrent and consummatory reactions, that mental organization begins to appear conspicuously as involving *time*. From this point of view, a selection, such as char-

[12] H. Bergson, *Introduction to Metaphysics,* p. 40.

[13] E. Rignano, *Psychology of Reasoning,* pp. 73, 104, 124.

[14] J. H. van der Hoop, *Character and the Unconscious,* transl. E. Trevelyan, 1923, p. 147.

[15] J. Piaget, *op. cit.,* pp. 47, 183.

[16] R. M. Eaton, *op. cit.,* pp. 308-10.

acterizes perception and thinking, "at any given instant" is a cross-section, a "temporal slab" of experience. And since the end-reaction complex is a more inclusive organization in time, the suggestion naturally occurs that intuition is a more inclusive apprehension in time. Intuition is, so to speak, a comprehension of means and end together. This applies easily to any retrospect of ends which we have already actually† achieved and of means which we took to achieve them. Such ends and means may be seized together, sensed as having been lived through, in what Santayana, following Plato, calls a "trope."[17] With regard to our future experience, our ends not yet achieved, there is more uncertainty. But sometimes, in the work of prophets and heroes, there does seem to be a kind of short-circuiting, a dealing with a future consummatory reaction, or end, without needing to run through the intermediate steps necessary to its actual achievement.[18] The question of the degree of certainty or assurance permissible here belongs with the general problem of the validity of intuitions, discussed below.

4. *At the level of sentiments, intuitions give the quality of personal relationships.* At this level, the organization of sentiments around persons, things, or ideas to be served as ends in time, with emotions appropriate to the various subordinate ends and means, makes a sentiment capable of an adjustment (or maladjustment) which is much too elaborate for any analysis in ideas or language. For this reason, the feeling of adjustment or maladjustment begins to take the place of articulated analysis, and we have the certainty and confidence characteristic of intuitive convictions about one's self or other persons. In ways deeper than cognition, in other ways than words, we respond more nearly totally to those chosen spirits who seem in some deep way akin to us. According to J. H. van der Hoop, intuition sees what is of personal importance. It is especially gifted in discovering all the various possibilities of individual development and activity, and is often capable of finding a solution where no other method could succeed.[19] According to Wieman, intuition is peculiarly fitted for those most complex and serious problems which arise in our dealings with other people.[20] On the other hand, Dewey warns us that the subconscious is surest to be wrong in

[17] G. Santayana, *Realm of Matter,* p. 27.
[18] *cf.* J. M. Baldwin, *op. cit.,* Vol. 1, p. 232.
[19] J. H. van der Hoop, *op. cit.,* pp. 147 *f.*
[20] H. N. Wieman, *Methods of Private Religious Living,* 1929, p. 194.

connection with intimate matters of self-regulation in health, morals, and social affairs—in matters most nearly connected with basic needs and relationships. Feelings, he says, are not the expression of a rectitude of organic action.[21]

5. *Since values are usually concerned with objects or ideas more remote and abstract, few men are as thoroughly certain of their valuations as they are of their sentiments.* We may have adjustments of the larger value structures to the structure of the cosmos, as we know it connotatively. Wittgenstein deals with a kind of immediately apprehended logical value. For Wittgenstein, everything properly philosophical belongs to what can only be *shown,* to what is in common between a fact and its logical picture.[22] Some have held that the aesthetic values develop in this way. Dewey says that an artist may select structures and forms by a kind of sympathetic vibration.[23] Sometimes, as we saw, the long machinery of means and instruments is avoided, and we are brought face to face with an end achievement or an ideal object. This is the artist's intuition, his absorption in beauty. Moral adjustments are also valuable; Hocking says that the mystical experience is achieved by an effort primarily moral rather than theoretical.[24]

6. *At the level of "selves" it may be said, at least roughly, that the "self as known" is known in thought, while the "self as knower" is apprehended immediately in intuition.*[25] Hocking thinks that self-knowledge is perhaps the best case for intuition. We do not perceive our minds by the bodily senses.[26] The "self as knower," if immediately apprehended, may by some be identified with the enotative or innotative background of our experience. It is easy in this way to argue that the world, even the enotative or innotative region, has the characteristics of a self. So long as one thinks on the problem, there seems no other place for the self as knower, except the innotative or enotative region. In a sense, the self as known is selected and the self as knower, which does the selecting, is neglected. It seems to remain, as it were, behind the scenes, beneath any diagram we draw. Since it does not belong to the connotative, nor even fully

[21] J. Dewey, *Experience and Nature,* p. 301.
[22] B. Russell, in L. Wittgenstein, *op. cit.,* pp. 11, 21.
[23] J. Dewey, *Experience and Nature,* p. 392.
[24] W. E. Hocking, *Types,* p. 385.
[25] *cf.* J. M. Baldwin, *op. cit.,* Vol. 2, p. 409.
[26] W. E. Hocking, *Types,* p. 195; *cf.* G. Santayana, *Realm of Matter,* p. 145; J. B. Pratt, *Matter and Spirit,* 1922, pp. 175 *ff.*

to the denotative, we may say that it belongs to the enotative or innotative. But if it does, it is unique, and, along with the realists, we hold that it does not change the world as known when we turn to consider the knower along with that world. It gives us the beginning of a hold upon the enotative—by its confidence that the enotative although open, also is in some sense our experience. Perhaps the self as knower is uniquely innotative, like the point which forms the center of a circle. The self as known is like the circle with chords, radii, etc., representing a field of connotative knowledge. Between this and the momentarily excluded and enotative not-self, the non-self, which may be subconscious or unconscious, is perhaps like the innotative boundary region of a circumference line.

At the level of selves, also, it appears that "self-forgetting" means a shift in the self as known, more than in the self as knower, or doer. Unselfishness is not selflessness. Self-forgetting is not self-annihilation; it is, rather, a more inclusive personal adjustment.

7. *A personality can, properly speaking, not be adjusted to itself, but must find something outside itself.* Otherwise there is the old cleft between the self as knower and the self as known. Evidently a personality as a whole must be extraverted, not introverted. Of the great adjustments at the level of personality, we have considered the aesthetic adjustment as a sentiment or value. The ethical adjustment to other persons is in some respects a problem of selves, and beyond that need not concern us here, as it may be more appropriately treated in a future study. At the level of personality, comes the great adjustment to the universe which is best called religion; the particular religions are its fragments or refractions. This field of personal adjustment to the universe seems appropriate to intuition, rather than primarily to thinking. For one thing, religious thinking is paradoxical, as if the subject were too great for the forms of thought. Again, there is abundant testimony, all the way from the great mystics to the recent philosophers, to the effect that knowledge of God can not be confined to conceptual forms.

R. Otto says that for Schleiermacher the "overplus" can not be apprehended by mere cognition of the world and the cosmic system, in the form which it assumes for science. It can be grasped and experienced in intuitions which assume shape in definite statements and propositions, distinguished from theoretical propositions by their free and felt, not reasoned, character.[27] For Hocking,

[27] R. Otto, *Idea of the Holy*, pp. 150 *f.*

the ultimate evidence for the selfhood of the Whole† is not primarily the evidence of argument, nor of analogy, but immediate experience interpreted by dialectic.[28] Wieman maintains that ideas are inadequate for adjustment to cosmic processes, and that we must strive for more inclusive integrations.[29]

8. *Can there be a logic of personal values and of the intuitions by which they are immediately apprehended?* Let us take for granted that the term "logic" may be used here inclusively, to cover any and all techniques by which we may win adjustment to the cosmos adequate for personality or society. When we turn to the history of logic, we find that in three other cases the results of a method have been used before the technique of the method was perfected. Deductive reasoning is found in Plato, before Aristotle had made plain the machinery of the syllogism. There are experimental inquiries in the early modern period, before Francis Bacon and long before John Stuart Mill had worked out a technique of induction. And the logic of instrumentalism, when it comes to the aid of pragmatism, comes to the aid of what is avowedly a new name for some old ways of thinking. Perhaps this is the case with intuitionism now; the method is being used, although the technique is not clear. Of the four principal methods of obtaining knowledge (rationalism, empiricism, pragmatism, and intuitionism), the two oldest have their techniques, the third has at least the beginnings of its technique, and the fourth has at least strong appeal. "For my own part," said William James, "I have finally found myself obliged to give up the logic, fairly, squarely, and irrevocably. It has an imperishable use in human life, but that use is not to make us acquainted with the essence of reality."[30] Notable passages expressing similar confidence in non-intellectual or intuitive methods of apprehending reality may be cited from the works of Alexander,[31] Hocking,[32] Santayana,[33] and Urban.[34] Dewey says that our affections, when they are enlightened by understanding, are organs by which we enter into the meaning of the natural world as genuinely as by knowing, and with greater fulness and intimacy.[35]

[28] W. E. Hocking, *Types*, p. 442. [29] H. N. Wieman, *op. cit.*, p. 195.

[30] W. James, *Pluralistic Universe*, p. 212.

[31] S. Alexander, *Space, Time, and Deity*, Vol. 2, p. 377.

[32] W. E. Hocking, *Types*, pp. 203 *ff.*

[33] G. Santayana, in G. P. Adams and W. P. Montague, editors, *Contemporary American Philosophy*, Vol. 2, p. 257.

[34] W. M. Urban, *Intelligible World*, p. 468.

[35] J. Dewey, *Quest for Certainty*, 1930, p. 282.

We may take it for granted that few writers would now attempt to answer all the problems by intuition alone. The trend is toward a federation of methods, such as has been championed by Montague.[86] It may be possible, however, in accordance with the epitomization hypothesis, to exhibit the four methods in a more interrelated and organic connection. We take it that the deductive inferences of rationalism can go on within the larger units of mental organization, just as the process of biological reproduction goes on within a social group. Similarly, inductions proceed within larger mental organizations just as, presumably, primitive multicellular organisms and germ-cells are integrated in social groups. Pragmatism requires at least end-reactions, and, as we have seen, intuitions help in adjusting them to the cosmos. It is, in fact, on such an organic unity of the methods (exhibited in its own account of the neuropsychological and logical realms) that the epitomization hypothesis must itself rest its claim to validity. Its various realms and levels have *some* features such as would be seen if the periodic table of the chemical elements were magnified to giant size. Like the periodic table, the epitomization hypothesis depends for its validity upon induction, deduction, practical consequences (in the case of the hypothesis, not yet discussed) and upon intuitive appreciation. There is a great difference with regard to the last-named, because the hypothesis calls for the adjustment of the more inclusive structures of mind and for their distinctive organizations in time. Those who question the method of analogy as used in the hypothesis should consider the analogies exhibited in the periodic table.

9. *It must be admitted that intuitionism lacks systematic technique.* Any reliance upon the suggestions of intuition, or any systematic cultivation of them is open to the criticisms and dangers of autosuggestion. And reliance upon intuition as a way of synthesizing and reconciling the oppositions of thinking can hardly be urged, in view of the present state of proposed theories and techniques. The great difficulties with such intuitionist views are, first, their doubtful accuracy as to matters of empirical fact (although there is more to be said for them than is often supposed); second, their almost inevitable idealistic metaphysics; and third, the fact that in spite of themselves, they remain essentially intellectualistic, dialectical to the end. Either the intuitionists do not shake themselves free from thinking, or else, when they do, they flounder helplessly in

[86] W. P. Montague, *The Ways of Knowing,* 1925, pp. 224 *ff.*

a world which they then seem only vaguely to comprehend.[87] The fact that they do not shake themselves free from the bounds of thought means that the dualities which they perhaps reconcile are more like explicit than implicit dualities, and more like contraries than contradictories. This difficulty is evidently felt by Ouspensky, when he says that the axioms which his higher intuitive logic, the logic of infinity†, the logic of ecstasy, or *Tertium Organum,* embraces, can not be formulated in our language; if we attempt such formulations, they look like absurdities. Thus, *A* is both *A* and *not-A*. Everything† is both *A* and *not-A*. Everything is All†.[88]

10. *What, after all, can we think or what can we do, about intuitional ways of knowing?* In the first place, there is a point, which, if we may not take for granted, we have taken, whether it is granted or not; it is the point of view. Several thinkers in contemporary philosophy make much of the utter uniqueness of the concrete here-now, the inevitable, intrinsically vivid present moment of experience. Doubtless its importance can be exaggerated, as when it is made to support a whole epistemology, or, worse still, a whole metaphysics. But at any rate, it is the point of view, and about it there is something immediate, intuitional, and undeniable. No matter how much our scenes are shifted, it is out of the question for us to attempt to approach the enotative from any point of view which lacks this quality. It is our predicament, our locus, from which everything has to be estimated. We have to see the world *sub specie actualis.*

From our locus, the here-now, the threads of thought, so to speak, stretch out and back, but the long studies of this book should make it clear that there is only questionable anchorage for them in the enotative. Our reliance must be in the denotative and connotative. In other words, stored up in the denotative and connotative are whatever resources we have for our adventuring into the enotative.

This means that, at least in a way, our resources are limited. The cosmos which we are able to apprehend in perception and comprehend in thought is an island, and all around it is an ocean of the unknown. We can even, as it were, contract the boundaries of our selected island (as in Husserl's phenomenological reduction and some procedures of logical positivism), but according to our realistic presuppositions, we can not quite annihilate it. And the connotative

[87] *cf.* W. E. Hocking, *Types,* p. 201.
[88] P. D. Ouspensky, *Tertium Organum,* pp. 261 *f.*

knowledge gathered by other minds, if not by our own, even with allowance for defects, mistakes, and future revisions, is enough to show that, island though it is, the cosmos, after all, is not so much like the island of Hawaii, well-nigh lost amid the vast expanse of surrounding water, as it is like the island of North America, ample enough so that the ocean outside need not trouble us. Even the waters within, like the Great Lakes, corresponding to the gaps in our thinking which are left innotative, are minor rather than major features of the continent. And our island or continent of the cosmos invites exploration, cultivation, and perhaps even *settlement*. Traditionally we have looked too much to the enotative, the transcendent, for our grounds and sanctions. The task of metaphysics is to explore our island-continent; the task of philosophy of religion and of ethics is to show what are the conditions of our settling down and being at home in it.

Now a systematic exploration, such as has been attempted in the epitomization hypothesis, reveals throughout the cosmos, both existent and subsistent, a recurring pattern of structures and processes. The significance of it can be variously read off. Structure (with process implied) as a metaphysical category is perhaps less tangible than substance, but after all it is more evident, and at the same time not less dependable. Structure as a category gains something from its concreteness; it is perhaps less fundamental than order, but it is more definite and clear cut. But it gains something, too, from its abstractness; it is perhaps less contentful than law, but it is more plastic, and seems, at least, less likely to suffer from possibilities of future revision.

The present moment is unique, but it is not isolated. It links with the cosmic structure, or structure of structures, and we do not live in a present unsupported by a past. If we are to rely upon anything, then, it is upon these cosmic facts, connotatively known. Boodin holds that we may proceed by a sort of intuition in our creative activity, and be guided by a feeling of fitness. There is a structure which presupposes to a certain activity and guides this activity in its trial-and-error procedure.[39]

Relying upon this structure, which we thread by our intelligence, we may acquire from it not merely a kind of familiarity with nature, and even a fitness for its suggestions, but a kind of momentum to carry us forward from the past through the present into the

[39] J. E. Boodin, *Cosmic Evolution*, p. 229.

future. Our experience is weighted by the connotative, but it is directed toward the enotative. We may live forward progressively by incorporating the enotative into the connotative as we proceed. This may be undertaken along lines of aesthetic valuation; Whitehead says that logic is necessary, but aesthetic harmony is a living ideal, moulding the general flux in its broken progress toward finer, subtler issues.[40]

This reliance implies an attitude as regards the place of our valuations in the cosmos or, in other words, as regards the conservation of values. According to the orthodox supernaturalistic views, values, taken literally—or, as we may say, our values and the ideas which interpret them—are sanctioned and conserved. According to some more liberal views, especially the pragmatic, values in one way or another, are sanctioned and conserved, but the ideas which are used to develop them at any particular time are secondary and open to revision. According to extreme empirical views, ideas are sanctioned and conserved, but values are not. According to skepticism, neither values nor ideas are conserved. According to the epitomization hypothesis, as thus far developed, at least the form, the monadic characteristics of valuations are cosmic, and are found at all other levels of the connotatively known cosmos. This does not yet attempt to answer the questions of *content* of valuations, but it gives us with regard to the enotative future at least a formal confidence.

In fact, with this formal confidence, we need not be troubled, if the way of advance is not fully determined nor explicitly stated. In the largest sense, we may in fact *live* with an indefinite and even with an enotative reference. This seems at first like purposelessness, but it is of the essence of ideality. It is, after all, only a narrow and a low ideality which can define its goal. It is true that some definition of goals, and some attainment, too, are morally indispensable; but morality itself, unless it is more than this, does not avail to fill up the measure of value. As Dewey puts it, "from the standpoint of its *definite* aim, any act is petty in comparison with the totality of natural events. . . . In a genuine sense, every act is already possessed of infinite import. . . . The consciousness of this encompassing infinity of connections is ideal. . . . This ideal is not a goal to be attained. It is a significance to be felt, appreciated. Though consciousness of it can not become intellectualized (identified in objects of a distinct

[40] A. N. Whitehead, *Science and the Modern World*, 1925, p. 27.

character), yet emotional appreciation of it is won only by those willing to think."[41] That is, we should say, those who are willing to think of the structures as they have been detected, and to use such thoughts as guideposts for their emotional appreciations.

We have seen from the preceding sections of this chapter enough to indicate that intuition, at one or another of its several levels, can give us apprehension of the uniqueness of the present moment, and can be of indispensable help to us in our comprehension of the past. It becomes clear, when we study the more inclusive structures and processes of the higher levels of mind as well as of the world, that sometimes the more inclusive data have to be recognized, especially in their characteristic organizations stretching out in time. Somewhat as every particle of a crystal is a crystal, so every monad of the universe is a universe; the macrocosm splits into uncounted myriads of microcosms. And just as a crystal may be studied along various planes and faces, to make it more or less inclusive, so the universe, especially in time, may be comprehended in many of its parts or in few. Intuition is the instrument of the more inclusive apprehension which we call comprehension. This in itself is no narrow field and no small achievement. It needs to be cultivated and emphasized. But even intuition, in spite of assertions to the contrary, can not afford us apprehension or comprehension of the enotative future. All that intuition really does here is to afford us *prehension*;[42] by intuition we lay hold upon the unknown future, tentatively, but with a formal confidence, between which and random trial and error there is, precisely, a *world* of difference.

All that we can hope to have done in the *Epitomizations* book and now in this book is to establish some principles according to which thinking is limited to various horizons, and some principles whereby intuitions afford us a formal confidence and resolute grasp when we try to carry our connotative knowledge into the enotative future. We have still to face the problem not of the form, but of the content of our values and intuitions. With what particular concrete, historic, or actual values are we justified in proceeding with confidence?

This involves, in the first place, the difficulty that men do not now and may never agree in their intuitions. In fact, it is hard enough for a man to find agreement among his own intuitions at different times

[41] J. Dewey, *Human Nature and Conduct*, 1922, pp. 262 *f.*

[42] It will be observed that the word is used here in a sense different than that of A. N. Whitehead, *Science and the Modern World*, pp. 97 *ff.*, etc.

or in different circumstances. Is there, then, any hope at all of arriving at agreement as regards the content of the intuitions of different persons? Hocking, for instance, notes that intuitions of different minds need not be identical.[43] The answer here seems to be, not absolutely, but progressively. Men differ from one another in all mental operations and their results. They now differ less in perceptions than in ideas; empiricism, establishing agreement in the first, for a while introduced more disagreement in the second, but is slowly fashioning its ideas into a new unity. The whole enterprise of science and most of modern philosophy proceeds on the presupposition that disagreements can be progressively minimized, and that pluralism is essentially temporary, formal, or verbal. One reason why there has not been more agreement in the intuitions of different men is because intuition has been such an uncharted, hit or miss affair. But in proportion as we all come better to understand the orderly monadic structure of the connotatively known world, we may expect a better agreement, not merely about its details, but about its wider perspectives.

11. *We must study the content as well as the form of intuitions.* The next step in the argument which we are developing through a series of books should be a more detailed study of the concrete content of the more inclusive valuational and personal structures; the larger historical, social structures; the adjustments of these two sets of structures to one another; and the light which all this throws upon the nature of the cosmos. We shall find that the more inclusive valuational and personal structures appear in history and society, in one way or another, as religions. Our task will then be to study the data of the principal religions, with their accompanying ethics. This will be undertaken first in a book to be called *The Unity of the Faiths.*

[43] W. E. Hocking, *Types,* p. 204.

INDEX

In this index, an asterisk (*) indicates that a name, word, or topic is found or discussed at least two more times in the chapter in which the given page number falls. The later discussions of authors are readily located by following the citations in the footnotes. The letters used in index entries are general references to corresponding subdivisions, where they occur, in Chapter II and Chapters V-XX, inclusive, according to the plan on page 96. This material is usually not indexed in further detail. The instances of horizon concepts indicated in the text by daggers, as explained on page 66, are usually not indexed.

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